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03/22

Endogenous Uncertainty

1 Aggregate activity and information

During a recession, a dominant question is the timing of the recovery. A renewal of activity by some agents provides good news to other agents but when investment decisions imply long range planning and irreversible expenditures agents may play a game of waiting for news, and that delays the recovery. In this context, more activity generates more information. However, a symmetric situation may exist in which a high level of activity in a regime of "business as usual" hides a downturn of the fundamental which becomes known only after a sufficiently large reduction of aggregate activity. In this case, information is inversely related with aggregate activity. Simple models for each case are presented here.

1.1 The model of Veldcam (2005)

The economy is in one of two states, the "good" ("bad") state with high (low) productivity of projects. The productivity is the probability that a project (which lasts one period) is successful at the end of the period. Failure produces zero. The outcome of projects is observable but the state is not. It switchesaccording to a Markov process. When the belief (probability in the public information) is higher, there are many projects (a discrete number). Hence, the sample of observation is higher at the end of the period. More activity generates more information. There is more information in a boom than in a trough (which takes place when the belief is low).

If the economy is an a boom with a high number of projects, and at the end of the period, a sufficiently large number of these projects has failed, then the belief (in the good state) is reduced by a large amount. The number of projects (aggregate activity) in the next period jumps down. If the belief is low, the amount of information that is revealed at the end of the period is small and the belief does not change much. It may increase when the number of observed successes is higher than expected (one also has to take into account the probability of switching), or decrease in the other case, but the changes of the belief may be sufficiently small to generate a protracted recovery. There is an asymmetry between the turns, up and down, of aggregate activity, a fast "crash" and a slow recovery.

The model is analytical, with lenders and borrowers (entrepreneurs) and a choice for the entrepreneurs (in finite fixed number) between starting a project (and borrowing) and an outside opportunity with a fixed payoff. One can probably simplify the model (which in any case is going to be highly stylized, to say the least) in order to focus on the essential property. The model has no delay option of the investors. A fondamental property of the model is that the economy can have large negative shocks (in the Markov process). The crash occurs because a large negative exogenous shock is perceived very quickly in the boom. Without these large negative shocks, there is no crash. A standard critique of RBC models is that they assume these large negative shocks in the productivity of production.

What are these shocks.

We will see that in a model of Caplin and Leahy (1994), there can be a crash when the state is constant.

1.2 Cycles and learning with a random evolution of the fundamental

In Veldcamp (2005), the fundamental (parameter determining the productivity of action) switches back and forth between two fixed values, high and low, according to a Markov process. The essential assumption is that information (about the fundamental) increases with the level of activity. The same assumption is made here in a context where the fundamental evolves according to a random walk (which can include a drift and/or a return to a mean). The model of Fajgelbaum, Schaal and Taschereau-Dumouchel (2016), hereafter FST, is presented here in a reduced form.

1.3 Exogenous signals

Let us begin with a preparatory exercise. The state of nature follows a random walk with the normal distribution

$$\theta_{t+1} = \theta_t + \epsilon_t, \qquad \epsilon_t^{\theta} \sim \mathcal{N}(0, 1/\gamma_\epsilon). \tag{1}$$

If there is no information, the variance of θ_t increases linearly with time, $\gamma_{t+1} = \gamma_t + \gamma_\theta$. Note that θ_t follows a random walk with no regression to a "long-run value".

Suppose that in each period, agents observe the signal on θ, with a normal noise:

$$y_t = \theta_t + \epsilon_t^y, \qquad \eta \sim \mathcal{N}(0, 1/\gamma_y).$$

After the observation of the signal y, the precision (inverse of the variance) on θ is equal to

$$\gamma_t' = \gamma_t + \gamma_y.$$

From one period to the next, we have two updates: the signal y augments the precision on θ and the random change from period t to period $t+1$ reduces the precision. None of these two steps "wins" over the other. The precision cannot become infinite because of the changes of θ between consecutive periods. The precision cannot go to zero because it is bounded below by the precision of the signal y. These two mechanisms are probably at work for a wide class of processes. Here, given the normality assumption, we can write

$$\gamma_{t+1} = \frac{\gamma_t' \gamma_\epsilon}{\gamma_t' + \gamma_\epsilon} = \gamma_\epsilon \frac{\gamma_t + \gamma_y}{\gamma_\epsilon + \gamma_t + \gamma_y} = \Gamma_0(\gamma_t). \tag{2}$$

From (2), the function $\Gamma_0(\cdot)$ is increasing with $\gamma_0(0) = 0$, $\Gamma_0(\cdot) < \gamma_\epsilon$. It has therefore a unique fixed point. Its graph is represented in Figure 1. For given γ_θ and γ_y, when $t \to \infty$, the precision γ_t converges to the fixed point of Γ_0.

Note that if the precision of the information signal γ_y increases, the function Γ_0 increases and its graph is shifted upwards (see Figure 1), with a higher precision in the limit. That property is now exploited to extended the model to the case of endogenous uncertainty with multiple equilibria.

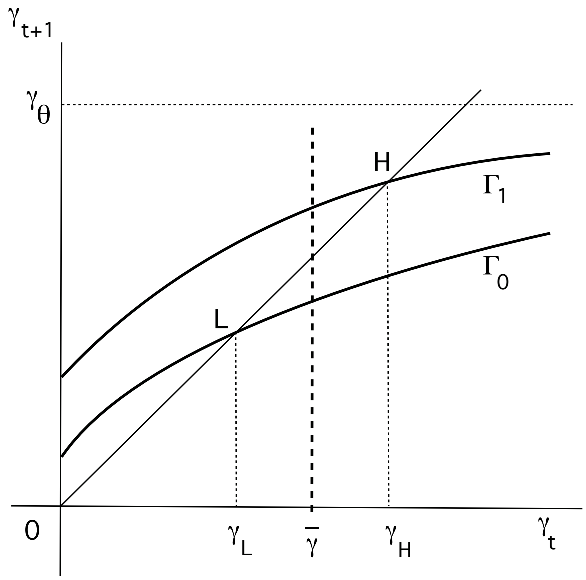

Figure 1: Evolution of the precisions

1.4 Endogenous uncertainty and traps

The issue is the following. We would like a model where information increases with economic activity and activity increases with information. That will generate a strategic complementarity and multiple equilibria with different levels of "endogenous uncertainty".

Assume that there is a mass of firm, equal to θ, that each take action 1 in any period where the uncertainty on θ is not too large, *i.e.*, when $\gamma_t > \bar{\gamma}$, for some value $\bar{\gamma}$ that will be chosen later. One could also assume a mass one of firms and an output that is equal to θ multiplied by the mass of action. If $\gamma_t < \bar{\gamma}$ no firm invests. If $\gamma_t > \bar{\gamma}$, the total level of activity is θ, but this level is observed through some noise. In this case, to the signal y one adds the signal

$$y' = \theta + \eta, \quad \text{with} \quad \eta \sim \mathcal{N}(0, 1/\gamma_\eta).$$

In Figure 1, we have now two regimes. For $\gamma_t < \bar{\gamma}$, the curve Γ_0 applies: γ_t converges to γ_L.

For $\gamma_t > \bar{\gamma}$, the updating formula (2) is trivially updated to

$$\gamma_{t+1} = \gamma_\epsilon \frac{\gamma_t + \gamma'_y + \gamma_\eta}{\gamma_\epsilon + \gamma_t + \gamma 1_y} = \Gamma'(\gamma_t), \quad \text{with} \quad \gamma'_y = \gamma_y + \gamma_\eta. \tag{3}$$

We have seen that an increase of the precision of γ_y shifts the curve Γ_0 upwards, here to Γ_1. For $\gamma_t > \bar{\gamma}$, we are in a high regime where γ_t converges to the higher value γ_H.

In the "low regime", agents do not produce because the uncertain is too high, and the uncertainty remains high because agents do not produce. In the regime with low uncertain, agents produce and the production provides information such that although θ_t varies from period to period, the uncertainty remains sufficiently low for the agents to keep producing.

The model and Figure 1 exhibit "uncertainty traps" (Fajgelbaum *et al.*, 2014). Fajgelbaum *et al.* construct a model with delays, but as shown here, the property has nothing to do with delays (which amplify the difference the low and the high regime). This property of "uncertainty traps" has already been analyzed by Pagano (1989) in the context of financial markets. An asset may be subject to price volatility and therefore be illiquid because there is only a small mass of agents who are active in that market. And there is a small mass because risk-averse agents do not enter that highly volative market. For the same structure, there is another equilibrium where the market is less volatile because, etc...

One can also assume that the fundamental is determined by a first-order auto-regressive process of with a stable long-run distribution. See Exercise **??**.

In the previous model, the cutoff point $\bar{\gamma}$ is fixed. Regimes depend on the initial value γ_1 and are permanent. On can easily modify the payoff of agents such that the cutoff depends both on the variance and the mean of θ in the public information.

Payoff of mean and variance

Assume that agents have a constant absolute risk aversion, a and a payoff of action 1 equal to the expected value of $(1 - e^{-a\theta})/a$. When the distribution of θ is (m, σ^2), the certainty equivalent of that payoff is $(1 - e^{-az})/a$ where $z = m - a\sigma^2/2$ is the certainty equivalent of the random return θ. The net payoff of investment (taking action 1) is positive if

$$z > -\frac{1}{a} Log(1 - ca).$$

Noting that $\gamma = 1/\sigma^2$, and changing the notation, an agent takes action 1 if and only if for some a and b,

$$m \geq \frac{a}{\gamma} + b. \tag{4}$$

Note that b does not have to be positive. The curve (Γ) marks the frontier between high and low activity. It embodies a trade-off between the mean m of θ and its precision γ. Above the frontier, the mean m is sufficiently high for a high activity and the precision increases toward $\bar{\gamma}$. Below the frontier, the mean decreases toward $\underline{\gamma}$. The vertical movements of m are driven by the evolution of θ and the learning about θ in equations.

Asymmetric regimes

One can make an argument that in a recession, the threshold c for activity gradually decreases. For example, part of the output could be capital, which does not have to be modeled here. Such a modelization adds significant complexity. It does not provide additional insight on the essential mechanism of the model.[1]

The impact of a depreciating capital stock can be modeled here by assuming that c gradually decreases with time in the low regime and increases with time in the high regime. In the previous figure, the

[1]The model of FST-M indeed has capital and labor and also delays in taking actions. That is why it is so complicated. All these efforts are made for a reproduction of some empirical properties of a business cycle. But I that exercise seems more like playing a game. The fitting of some properties with business cycle features is proof of the skills of the authors but does not provide a scientific validation : there are many other things that go on in a business cycle, and the current mechanism may be only a part, possibly a small part, of what drives a business cycle.

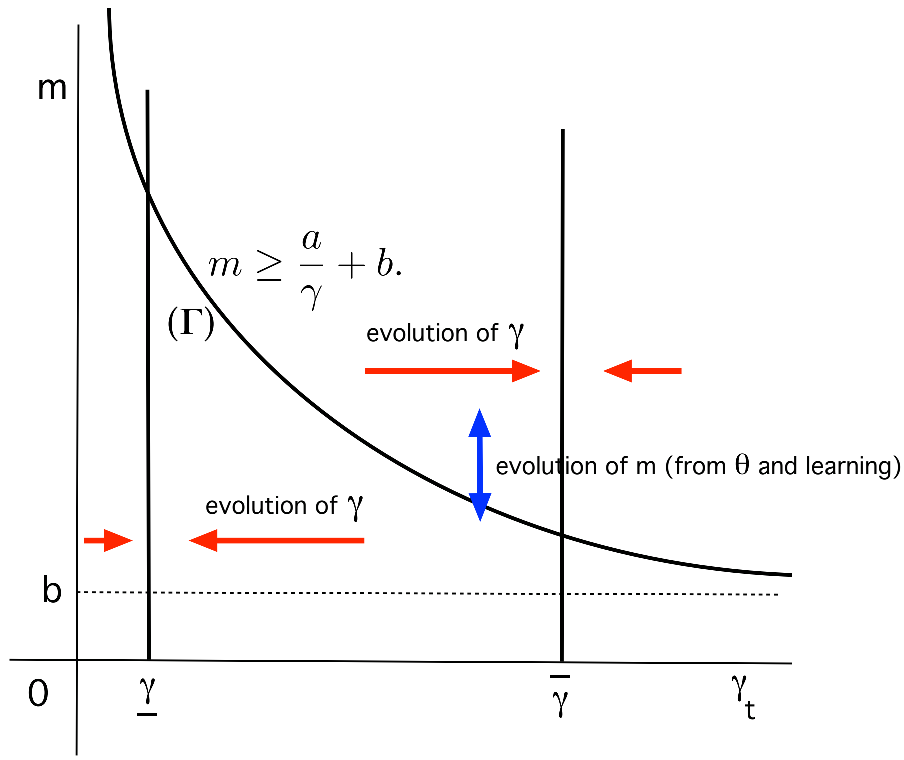

Figure 2: Evolution of the precisions

(γ) curves shifts gradually downwards in a low regime, when the point (m, γ) is below the (Γ) curve. The reverse is true above the frontier.

The timing of policy

Because of the information externality, the equilibrium is not a Pareto optimum. A policy of subsidization to lower the cost of action may be beneficial. This is not the place for a formal analysis but one can note that a subsidy is equivalent to a decrease of the parameter b, and in the Figure, a downward shift of the (Γ) frontier. In this model m and γ are public information. In a low regime, below the frontier, the policy is likely to be more effective when the (m, γ) point is near the frontier.

Extension with capital

In the present model, which is a pedagogical tool, the fundamental parameter, θ, follows a random walk (equation (1)). The economy could stay permanently in a boom or a recession. One can easily generate cycles by extending the model and include in the random evolution of θ an endogenous drift that represents a "capital effect". As an example, introduce the variable, say k_t which increases with y_t replace equation (1) by

$$\theta_{t+1} - \theta_t = \epsilon_t + \alpha(k^* - k_t). \tag{5}$$