Coordination of expectations

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PSE

March 20, 2024

References

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  `(Background)`

- Morris, Stephen and Hyun Song Shin (2002). "Social Value of Public Information," *American Economic Review*, 92(5), 1521-1534.
  `(Main paper here)`

- Evans, George W., Roger Guesnerie and Bruce McGough (2018). "Eductive Stability in Real Business Cycle Models," *The Economic Journal*, 129, 821-852.
  `(Another perspective)`

Model

- Static. Continuum of mass one of agents, each with payoff

$$u_i(a_i - \omega) = -(1-r)(a_i - \omega)^2 - rL_i. \tag{1}$$

  - First term: matching the fundamental.
  - Second term:
    - \+ deviation from others–the "beauty contest" of Keynes: $L_i = \int_0^1 (a_j - a_i)^2 dj.$
    - \+ **Strategic complementarity**

- Other expression (equivalent for the individual decision, but not for social welfare):

$$\tilde{L}_i = (a_i - \bar{a})^2, \text{ with } \bar{a} = \int_0^1 a_j dj. \tag{2}$$

$$L_i = \int a_j^2 dj - 2a_i \int a_j dj + a_i^2, \qquad \tilde{L}_i = (\int a_j dj)^2 - 2a_i \int a_j dj + a_i^2.$$

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$$\text{Optimal action} \quad a_i = (1-r)E_i[\omega] + rE_i[\bar{a}]. \tag{3}$$

Public information

- State ω has a prior distribution that is "uniform" on the real line (equivalent to a normal with infinite variance)
    - Useful "trick": facilitates solution and focus on information that is additional to the prior.
- Public signal:

$$y = \omega + \eta, \qquad \eta \sim \mathcal{N}(0, 1/\rho_\eta). \tag{4}$$

- Given the public signal, all agents have a posterior on ω that is $\mathcal{N}(y, 1/\rho_\eta)$.
- All agents have the same information. No deviation from each other. Individual actions:

$$a_i(y) = y \tag{5}$$

Private information

- $$\text{Private signals} \quad s_i = \omega + \epsilon_i, \qquad \epsilon_i \sim \mathcal{N}(0, 1/\rho_\epsilon). \tag{6}$$

- $$E[\omega|s_i] = \frac{\rho_\eta y + \rho_\epsilon s_i}{\rho_\eta + \rho_\epsilon} \tag{7}$$

- For the equilibrium, as we are used to linear rules for
  (i) learning with Gaussian distrib.
  (ii) quadratic loss function, look for a linear solution.

- $$a_i = \kappa s_i + (1 - \kappa) y. \tag{8}$$

  (comment on the sum of coeff $= 1$)

Linear solution

- $$a_i = (1-r)E_i[\omega] + rE_i[\bar{a}].$$
- $$a_i = \frac{\rho_\eta y + \rho_\epsilon (1-r) s_i}{\rho_\eta + \rho_\epsilon (1-r)}. \tag{9}$$

Thinking about others

- If someone observes a public signal that is worse than her private signal, then her expectations of others' expectations of ω is lower than her expectation of ω (i.e., it is closer to the public signal than her own expectation).
  - This in turn implies that if we look at the nth order expectations about ω (e.e., someone's expectation of others' expectations of others' expectaton of [n times] of ω, then this approaches the public signal as n becomes large.
  - Higher-order expectations depend only on public signals.

Iterative method(1)

- $$a_i = (1-r)E_i[\omega] + rE_i[\bar{a}],$$

  with

  $$E_i[\bar{a}] = E_i[\int a_j dj] = (1-r)E_i\Big[\int E_j[\omega]dj\Big] + rE_i\Big[\int djE_j[\int a_k dk]\Big].$$

$$a_i = (1-r)E_i[\omega] + r\left((1-r)E_i\Big[\int E_j[\omega]dj\Big] + rE_i\Big[\int djE_j[\int a_k dk]\Big]\right) \tag{10}$$

- Call the operator $\tilde{E} = \int djE_j$.

$$\begin{aligned} a_i &= (1-r)E_i[\omega] + r(1-r)E_i\Big[\tilde{E}[\omega]\Big] + r^2(1-r)E_i\Big[\tilde{E}[\tilde{E}[\omega]\Big] + \ldots \\ &= (1-r)\textstyle\sum_{k=0}^{\infty} r^k E_i\Big[\tilde{E}^k[\omega]\Big]. \end{aligned} \tag{11}$$

Iterative method(2)

$$E_i[\omega] = \frac{\rho_\eta y + \rho_\epsilon s_i}{\rho_\eta + \rho_\epsilon}.$$

$$\tilde{E}[\omega] = \int E_i[\omega] di = \frac{\rho_\eta y + \rho_\epsilon \omega}{\rho_\eta + \rho_\epsilon}, \qquad E_i\left[\tilde{E}[\omega]\right] = \frac{\rho_\eta y + \rho_\epsilon \left(\frac{\rho_\eta y + \rho_\epsilon s_i}{\rho_\eta + \rho_\epsilon}\right)}{\rho_\eta + \rho_\epsilon}.$$

$$\tilde{E}^2[\omega] = \frac{\left((\rho_\eta + \rho_\epsilon)^2 - \rho_\epsilon^2\right) y + \rho_\epsilon^2 \omega}{(\rho_\eta + \rho_\epsilon)^2}.$$

- Lemma $\quad \tilde{E}^k[\omega] = (1 - \mu^k) y + \mu^k \omega, \quad E_i\left[\tilde{E}^k[\omega]\right] = (1 - \mu^{k+1}) y + \mu^{k+1} s_i$, with
$$\mu = \frac{\rho_\epsilon}{\rho_\eta + \rho_\epsilon}.$$

- Substituting in (11), one finds the solution (9).

Welfare effect of public information

$$W = -\int_0^1 (a_i - \omega)^2 di, \qquad a_i = \omega + \frac{\alpha\eta + \beta(1-r)\epsilon_i}{\alpha + \beta(1-r)}.$$

- Given ω, expectation of welfare (about y and s_i):

$$\begin{aligned} E[W|\omega] &= -\frac{\alpha^2 E[\eta^2] + \beta^2(1-r)^2 E[[\epsilon_i^2]}{(\alpha+\beta(1-r))^2} \\ &= -\frac{\alpha + \beta(1-r)^2}{(\alpha+\beta(1-r))^2}. \end{aligned}$$

$$\frac{\partial E[W|\theta]}{\partial \beta} > 0, \qquad \frac{\partial E[W|\theta]}{\partial \alpha} \quad \text{iff} \quad \frac{\beta}{\alpha} < \frac{1}{(2r-1)(1-r)}.$$

Interpretation

- If $\beta = \rho_\epsilon$ is small with respect to $\alpha = \rho_\eta$, the public signal pulls agents too much away from ω because agent have a dominant taste for "begin together" $(r > 1/2)$.

- The law of iterative expectations does not apply.

$$\tilde{E}[\omega] = \int E_i[\omega]di = \frac{\rho_\eta y + \rho_\epsilon \omega}{\rho_\eta + \rho_\epsilon}, \qquad E_i\left[\tilde{E}[\omega]\right] = \frac{\rho_\eta y + \rho_\epsilon \left(\frac{\rho_\eta y + \rho_\epsilon s_i}{\rho_\eta + \rho_\epsilon}\right)}{\rho_\eta + \rho_\epsilon}.$$

$$\tilde{E}^2[\omega] = \frac{\left((\rho_\eta + \rho_\epsilon)^2 - \rho_\epsilon^2\right) y + \rho_\epsilon^2 \omega}{(\rho_\eta + \rho_\epsilon)^2} \neq \tilde{E}[\omega].$$

- If $\tilde{E}^k[\omega] = \tilde{E}[\omega]$, in (11), $\quad a_i = (1-r)\sum_{k=0}^{\infty} r^k E_i\left[\tilde{E}[\omega]\right] = \frac{\rho_\eta y + \rho_\epsilon s_i}{\rho_\eta + \rho_\epsilon} = \omega + \frac{\rho_\eta \eta + \rho_\epsilon \epsilon_i}{\rho_\eta + \rho_\epsilon}$,

  which is socially efficient: recall $E[W|\omega] = -\int_0^1 (a_i - \omega)^2 di = -1/(\rho_\eta + \rho_\epsilon)$.

  In the private payoff, agents want to have their action as close as possible to the mean. This term appears only because of the differences in information.

The eductive approach in the coordination of expectations

- In the standard RE, the equilibrium is a Nash-equilibrium.
- Example: the Muth model of agricultural price:
  - Strategic substitutability, one Nash equilibrium:
    The more others produce, the lower the price, therefore my response should be lower (the opposite direction).
  - Strategic complementarity: if other run, I should run faster. Multiple equilibria.
- In the eductive approach, thinking about others:
  - Muth model: the price cannot be more than p_1, therefore, I should produce not more than x_1. With that maximum supply (of all producers), the price cannot be lower than p_2. Therefore, one cannot produce less than x_2, etc...
  - Does the process (in thinking) converge?