

# Endogenous delays

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## *The Economics of Wait and See*

By SYLVIA NASAR



The uncertainty in Washington is clearly one reason for the hesitation. But even as executives keep one eye on Congress and another on the White House, many say their caution comes as much from their own modest order books and economic forecasts as from the political outlook. This wariness comes at a stage in the business cycle when attitudes normally turn more ebullient and is likely to help turn predictions for only moderate growth into reality.

"We quite frankly are waiting to see," said Allen I. Questrom, chairman and chief executive of Federated Department Stores. "We are trying to understand where it's all going. We're just not seeing a lot of activity. We have to be conservative. There's a sense of uneasiness about the future."

# Questions

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- “Waiting to see” what?
- “We’re just not seeing a lot of activity.”
- Activity of others. But are they also waiting?
- There could be a variety of mechanisms (models):
  - Some sectors have unknown excess capacity which has to be depleted, while other sectors are waiting for this.
  - Here, all the activity is driven by waiting agents. Focus in on the interaction between activity and information.
- Anticipated property of the model: Penguins.
- The model with turn out to be an extension of BHW with endogenous timing of investment.

## Additional references

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- Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten and Stephen J. Terry (2018). “Really Uncertain Business Cycles,” *Econometrica*, 86, 1031-65.
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# Toy model

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- 2 states  $\{\omega_0, \omega_1\}$
- 2 agents with option to make one investment (fixed size) in period  $t$ ,  $x \in \{0, 1\}$  and payoff  $E_t[\delta^{t-1}(\omega - c)]$ .
- In state 0, one agent; in state 1, 2 agents.  $P(\omega = 1) = \mu_1$ . **Remark.**
- Fundamental assumption:

$$0 < \mu - c < \delta\mu(1 - c).$$

Justification of the two inequalities

## Equilibrium: necessary properties

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- PBE equilibrium, symmetric strategies.
- Strategy in period 1: probability to invest  $z$ .
- $z = 1$  cannot be an equilibrium strategy.
- $z = 0$  cannot be an equilibrium strategy. (A little more tricky, but simple with two periods).
- If there is an equilibrium,  $0 < z < 1$ .
- If there is no investment in period 1, there is no investment after.

## Equilibrium: arbitrage and existence

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■ Since  $0 < z < 1$ , payoff of no delay = payoff of delay.

■ Payoff of delay =  $\delta\mu z(1 - c)$ .

■ Equilibrium:

$$\mu - c = \delta\mu z(1 - c).$$

■ Because of the fundamental assumption, unique value  $0 < z < 1$ .

■ Value of the game:  $\mu - c$ .

■ Comments

## Interpretation of the arbitrage condition

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$$\begin{aligned}\frac{1-\delta}{\delta}(\mu-c) &= (\mu z(1-c) - (\mu-c)) \\ &= P(x=0|\mu)(c - P(\omega_1|x=0, \mu))\end{aligned}$$

■ Information and time discount.

- $0 < z < 1$  only if  $\delta$  is in the interval  $[\delta^*, 1)$ , with  $\delta^* = (\mu - c)/(\mu(1 - c))$ .
- If  $\delta \rightarrow \delta^*$ , then  $z \rightarrow 1$ .
- If  $\delta \rightarrow 1$  and the period length is vanishingly short, information comes in quickly but there is a positive probability that it is wrong.



## Other properties

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- Optimism and investment level: mechanism on information and arbitrage,  $\neq$  Tobin-q.
- Observation noise
  - if an investment is made, the other agent sees it with probability  $1 - \gamma$  and sees nothing with probability  $\gamma$ , ( $\gamma$  small).
- Large number of agents:  $N$  agents in state 1 and 1 agent in state 0.
  - Exercise
- Non symmetric equilibrium

Two agents,  $A$  and  $B$ , (see each other but uncertain whether the other has an option).  $B$  always delays and does not invest ever if no investment in the first period.

  - If  $A$  has an option, no delay, payoff  $\mu - c$ .
  - Payoff of  $B$  is  $\delta\mu(1 - c)$ , higher than in the symmetric equilibrium. Asymptotically ( $\delta \rightarrow 1$ ), payoff  $\mu(1 - c)$  equivalent of instant revelation.

# General model with any random number of identical agents (1)

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- Two states.
- All agents have the same belief.
- The number of agent is random two distributions, and the distribution with the high state dominates the other in the first-order.
- Key property: in any subgame beginning in period  $t$ , the payoff of the game is the payoff of investing right away,  $\mu_t - c$ .
- To solve for the equilibrium, it is sufficient to consider delays for one period. The trade-off is between no delay with payoff  $\mu_t - c$ , and delay with no delay in the next period with payoff  $\mu_{t+1} - c$ .

## General model with any random number of identical agents (2)

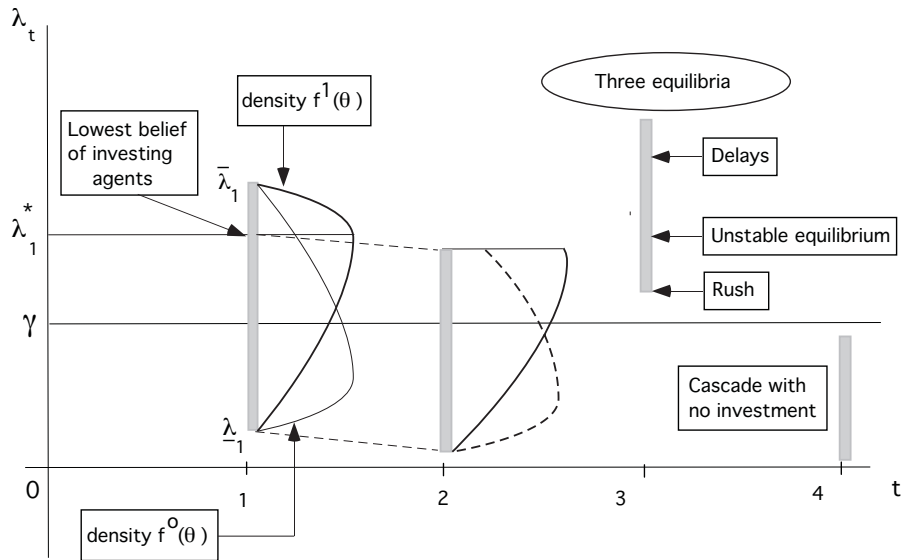
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- Arbitrage equation

$$\mu_t - c = \delta E_t[(\max(\mu_{t+1} - c, 0))]$$

- Results are the same (extended).
- In particular, if there is no investment in a period, there is no investment after.

# Heterogenous beliefs and fixed number of agents : Illustration



## Proposition

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Let  $\mu^*$  be an equilibrium strategy in a game with  $n \geq 2$  remaining players,  $\underline{\mu} < \mu^* < \bar{\mu}$ . Then  $\mu^*$  is solution of the arbitrage equation between the opportunity cost and the option value of delay

$$(1 - \delta)(\mu^* - c) = \delta Q(\mu^*), \quad \text{with}$$

$$Q(\mu^*) = \sum_{k=0}^{n-1} P(x = k | \mu^*, F^\omega, n) \text{Max} \left( c - P(\omega = \omega_1 | x = k; \mu^*, F^\omega, n), 0 \right), \quad (1)$$

where  $x$  is the number of investments by other agents in the period.