Cascades and Herds

PSE

(addendum)

March 13, 2024

The 2x2x2 Model

- Minimum elements of the model
  - What is the minimum number of states needed for a SL model ?
  - What is the minimum number of signal values in the canonical SL model?
  - What is the minimum number of possible actions?

The 2x2x2 Model

- Minimum elements of the model
  - What is the minimum number of states needed for a SL model ?
  - What is the minimum number of signal values in the canonical SL model?
  - What is the minimum number of possible actions?
- The model
  - 2 states $\omega \in \{0, 1\}$.
  - 2 signal values $P(s = \omega) = q \in (0, 1)$. (symmetric binary signal, SBS)
  - $x \in 0, 1$, payoff $U = x(\omega - c)$, $\qquad c \in (0, 1.)$

Social learning in the 2x2x2 Model

- $\mu_t = P(\omega = 1|h_t), \qquad \tilde{\mu}_t = P(\omega = 1|h_t, s_t).$

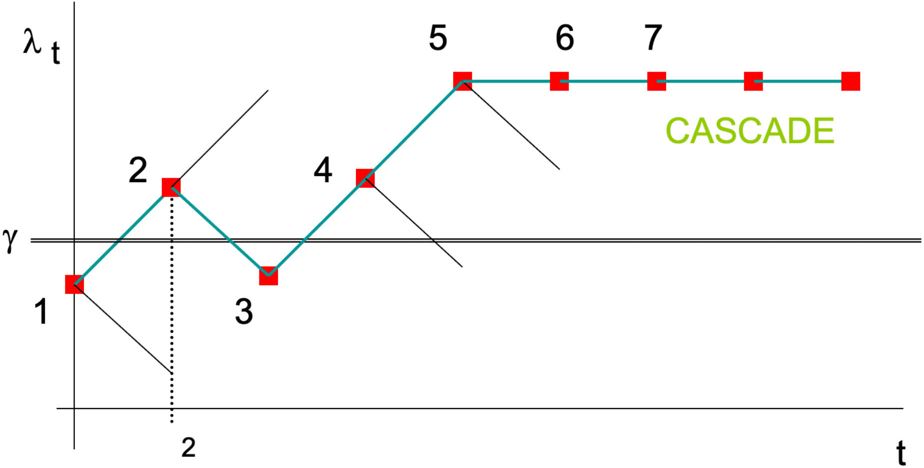
- Use LR : with $s_t = 1$,

$$\tilde{\ell}_t = \frac{\tilde{\mu}_t}{1-\tilde{\mu}_t} = \frac{P(\omega_1|s=1)}{P(\omega_0|s=1)} = \frac{\frac{P(s=1|\omega_1)P(\omega_1)}{P(s)}}{\frac{P(s=1|\omega_0)P(\omega_0)}{P(s)}} = \ell_t \frac{P(s=1|\omega_1)}{P(s=1|\omega_0)}.$$

- Use LLR: $\lambda_t = Log(\frac{\mu_t}{1-\mu_t})$: $\quad \tilde{\lambda}_t = \lambda_t \begin{cases} +a \text{ if } s_t = 1, \\ -a \text{ if } s_t = 0. \end{cases}$, with $a = Log\frac{q}{1-q}$.

- Action $x_t = 1$ iff $\tilde{\lambda}_t > \gamma = Log(c/(1-c))$.

Cascade representation



Cascade representation

- Convergence.
- Right and wrong cascades

Martingales and social learning

- Definition: $\mu_t = E_t[\mu_{t+1}]$. (Super-martingale: $\mu_t \leq E_t[\mu_{t+1}]$).
- MCT: If μ_t is a bounded martingale, there is μ^* such that $\mu_t \to \mu^*$ in probability. Intuition:
  - There cannot be (random) predictable up- or down-turns.
  - One cannot make money in an efficient market (proof by Doob with upward crossing Lemma)
  - Other intuition (in $\mathcal{L}^2$): consecutive changes of μ_t are uncorrelated and $E[(X_{t+T} - X_t)^2]$ is bounded, therefore, $\sum_{\tau \geq t} E[(X_{t+1} - X_t)^2] \to 0$ at $t \to \infty$.
  - Many proofs
- The standard Bayesian process of updating after a signal is a martingale, if **the agent has the right distribution of the signal**.
- Great tool for analysis of convergence, but many cases where SL is not a martingale.

Tools

Two states $\omega \in \{0, 1\}$. Bayesian learning.

In state 1,

- $\ell_t = \dfrac{P(\omega = 0|t_t)}{P(\omega = 1|t_t)}$ is a martingale.

- $P(\omega = 1|h_t$ cannot converge to 0.

(Symmetric result for state 0)

Signals and private beliefs

- 2 states $\{\omega_0, \omega_1\}$ with equal probabilities. Private signals with distributions, cdf $F^{\omega}(s)$.
- Call p the probability of ω_1 for an agent with signal s. When an agent receives the signal s, by Bayes' rule, the likelihood ratio between the two states is

$$\frac{F'^{\omega_1}(s)}{F'^{\omega_0}(s)} = \frac{p}{1-p}. \tag{1}$$

  In state ω, F^{ω} generates a distribution of signals, for which the belief is given by the previous equation.
- The reverse applies: belief is a signal. The distribution of this signals must satisfy the previous equation.

Continuum of beliefs

- Individual beliefs (equivalent to private signals) are distributed according to the *c.d.f.* $F^{\omega}(\mu)$

  First order stochastic dominance: if $\omega_1 > \omega_0$, $F^{\omega_0}(s) > F^{\omega_1}(s)$

Observations

States of Nature		$x_t = 1$	$x_t = 0$
	$\omega = \omega_1$	$1 - F_t^{\omega_1}(\gamma)$	$F_t^{\omega_1}(\gamma)$
	$\omega = \omega_0$	$1 - F_t^{\omega_0}(\gamma)$	$F_t^{\omega_0}(\gamma)$

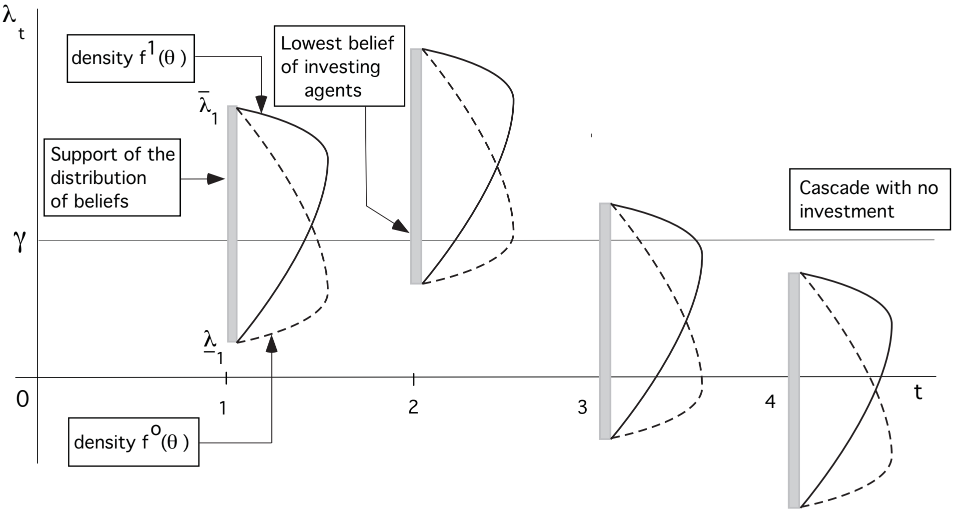
with $\gamma = Log\left(\frac{c}{1-c}\right)$.

- Social learning

$$\lambda_{t+1} = \lambda_t + \nu_t, \text{ with } \nu_t = Log\left(\frac{P(x_t|\omega_1)}{P(x_t|\omega_0)}\right). \tag{2}$$

Representation

(Replace θ by ω).



Conditions for informational cascades

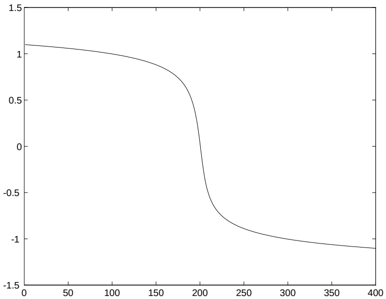
- Two (necessary) conditions:
  - Bounded beliefs: At some point the "most convinced" person cannot overwhelm the public belief.
  - Discrete action: at some point, switching action requires a quantum difference in belief.
- Exercise: two actions and private signals $s_t = \omega + \epsilon_t$, with $\epsilon_t \sim N(0, \sigma^2)$.

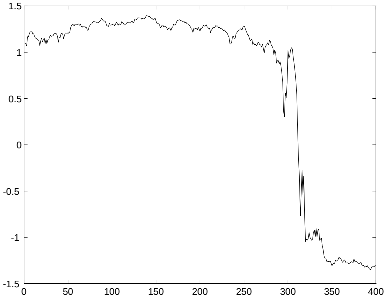
Exercise: Crashes and booms with a continuum of agents

- Model: Two states ω_0 and ω_1, $s_t = \omega + \epsilon_t$ with ω and ϵ Gaussian; $x_t \in \{0, 1\}$.
- Belief (LLR) of agent with signal s $$\lambda(s) = \lambda_t + \frac{\omega_1 - \omega_0}{\sigma_\epsilon^2}\left(s - \frac{\omega_0 + \omega_1}{2}\right).$$
- Cutoff for investment ($x_t = 1$): $$s > s^*(\lambda_t) = \frac{\omega_0 + \omega_1}{2} - \frac{\sigma_\epsilon^2}{\omega_1 - \omega_0}\lambda_t.$$
- Model with one agent. Discussion
- Model with a continuum of agent in each period: $X_t = 1 - F(s^*(\lambda_t) - \theta; \sigma_\epsilon)$.
- Observed aggregate activity $$Y_t = 1 - F(s^*(\lambda_t) - \theta; \sigma_\epsilon) + \eta_t.$$

Crashes and booms (2)

- Observed aggregate activity $Y_t = 1 - F(s^*(\lambda_t) - \theta; \sigma_\epsilon) + \eta_t$.
- $\lambda_{t+1} = \lambda_t + Log\Big(\frac{f(x_t - (1 - F(s^*(\mu_t) - \omega_1; \sigma_\epsilon)); \sigma_\eta)}{f(x_t - (1 - F(s^*(\mu_t) - \omega_0; \sigma_\epsilon)); \sigma_\eta)}\Big)$, (f density function).
- On the left, $\eta_t \equiv 0$.
- On the right, the evolution of the public belief is represented for random realizations η_t.





Cascades and bounded private beliefs

- Previous model: distribution of private beliefs with cdf $F^\omega(s)$ and density $f^\omega(s)$.
- Assume that $f(s) > 0$ for $s \in (a, b)$ with $0 < a < b < 1$ and $f(s) = 0$ otherwise. Private beliefs are bounded.
- Payoff: agent choose the state that is more likely (equivalent to $c = 1/2$).
- Update: $\frac{\tilde{\mu}}{1-\tilde{\mu}} = \frac{\mu}{1-\mu}\frac{s}{1-s}$. Invest $(x = 1)$ if $s > 1 - \mu$.
- Cascade set with investment $a > 1 - \mu$ which is equivalent to $\mu > 1 - a$.
- Cascade set with no investment $\mu < 1 - b$.
- MCT $\Longrightarrow \mu_t \to \mu^*$.
- The limit μ^* cannot be in the interval $(1 - b, 1 - a)$.
- A cascade occurs with probability one.

Cascades and herds

- Two states $\omega \in \{0, 1\}$, two actions (to simplify).
- No assumption on private beliefs (bounded or unbounded).
- **Definitions**
  - A herding agent ignores the private signal (and takes a decision based on the public information).
  - A herd is when all agents are taking the same action.
- **Properties**
  - A informational cascade generates a herd.
  - For any distribution of private beliefs, with probability 1, there is some T such that a herd begins at time T.
- Intuition

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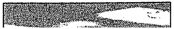
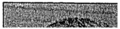
  - Standard Dixit-Stiglitz type model of GE with two possible values for aggregate productivity parameter θ_0, θ_1.
  - Share of agents with sure belief in θ_1.
  - Non Bayesian learning. Transition probabilities between the two sure beliefs. The transition probabilities depend on the aggregate output.

Part two: Delays

Motivation

The Economics of Wait and See

By SYLVIA NASAR

The uncertainty in Washington is clearly one reason for the hesitation. But even as executives keep one eye on Congress and another on the White House, many say their caution comes as much from their own modest order books and economic forecasts as from the political outlook. This wariness comes at a stage in the business cycle when attitudes normally turn more ebullient and is likely to help turn predictions for only moderate growth into reality.

"We quite frankly are waiting to see," said Allen I. Questrom, chairman and chief executive of Federated Department Stores. "We are trying to understand where it's all going. We're just not seeing a lot of activity. We have to be conservative. There's a sense of uneasiness about the future."

Questions

- "Waiting to see" what?
- "We're just not seeing a lot of activity."
- Activity of others. But are they also waiting?
- There could be a variety of mechanisms (models):
  - Some sectors have unknown excess capacity which has to be depleted, while other sectors are waiting for this.
  - Here, all the activity is driven by waiting agents. Focus in on the interaction between activity and information.
- Anticipated property of the model: Penguins.
- The model with turn out to be an extension of BHW with endogenous timing of investment.

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Toy model

- 2 states $\{\omega_0, \omega_1\}$
- 2 agents with option to make one investment (fixed size) in period t, $x \in \{0, 1\}$ and payoff $E_t[\delta^{t-1}(\omega - c)]$.
- In state 0, one agent; in state 1, 2 agents. $P(\omega = 1) = \mu_1$.. Remark.
- Fundamental assumption:
$$0 < \mu - c < \delta\mu(1 - c).$$

Justification of the two inequalities

Equilibrium: necessary properties

- PBE equilibrium, symmetric strategies.
- Strategy in period 1: probability to invest z.
- $z = 1$ cannot be an equilibrium strategy.
- $z = 0$ cannot be an equilibrium strategy. (A little more tricky, but simple with two periods).
- If there is an equilibrium, $0 < z < 1$.
- If there is no investment in period 1, there is no investment after.

Equilibrium: arbitrage and existence

- Since $0 < z < 1$, payoff of no delay = payoff of delay.
- Payoff of delay = $\delta\mu z(1 - c)$.
- Equilibrium:
$$\mu - c = \delta\mu z(1 - c).$$
- Because of the fundamental assumption, unique value $0 < z < 1$.
- Value of the game: $\mu - c$.
- Comments

Interpretation of the arbitrage condition

- $$\begin{aligned}\frac{1-\delta}{\delta}(\mu - c) &= \Big(\mu z(1-c) - (\mu - c)\Big) \\ &= P(x=0|\mu)\Big(c - P(\omega_1|x=0,\mu)\Big)\end{aligned}$$

- Information and time discount.
  - $0 < z < 1$ only if δ is in the interval $[\delta^*, 1)$, with $\delta^* = (\mu - c)/(\mu(1-c))$.
  - If $\delta \to \delta^*$, then $z \to 1$.
  - If $\delta \to 1$ and the period length is vanishingly short, information comes in quickly but there is a positive probability that it is wrong.

Other properties

- Optimism and investment level: mechanism on information and arbitrage, $\neq$ Tobin-q.

- Observation noise
  - if an investment is made, the other agent sees it with probability $1-\gamma$ and sees nothing with probability γ, (γ small).

- Large number of agents: N agents in state 1 and 1 agent in state 0.
  - Exercise

- Non symmetric equilibrium
  Two agents, A and B, (see each other bu uncertain whether the other has an option). B always delays and does not invest ever if no investment in the first period.
  - If A has an option, no delay, payoff $\mu - c$.
  - Payoff of B is $\delta\mu(1-c)$, higher than in the symmetric equilibrium. Asymptotically ($\delta \to 1$), payoff $\mu(1-c)$ equivalent of instant revelation.

General model with any random number of identical agents (1)

- Two states.
- All agents have the same belief.
- The number of agent is random two distributions, and the distribution with the high state dominates the other in the first-order.
- Key property: in any subgame beginning in period t, the payoff of the game is the payoff of investing right away, $\mu_t - c$.
- To solve for the equilibrium, is is sufficiently to consider delays for one period. The trade-off is between no delay with payoff $\mu_t - c$, and delay with no delay in the next period with payoff $\mu_{t+1} - c$.

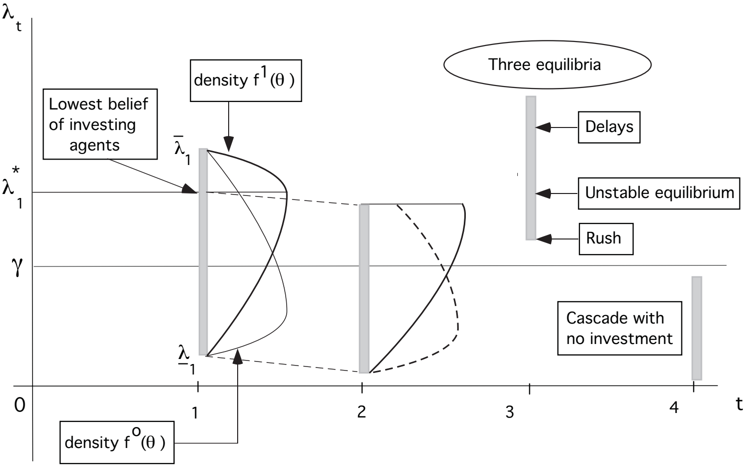
General model with any random number of identical agents (2)

- Arbitrage equation

$$\mu_t - c = \delta E_t[(max(\mu_{t+1} - c, 0)]$$

- Results are the same (extended).
- In particular, if there is no investment in a period, there is no investment after.

Heterogenous beliefs and fixed number of agents : Illustration



An example of evolution of beliefs in a model with delays

λ_t is the LLR between state 1 and state 0.

Proposition

Let μ^* be an equilibrium strategy in a game with $n \geq 2$ remaining players: the belief of the marginal agent who does not delay.

Then μ^* is solution of the arbitrage equation between the opportunity cost and the option value of delay

$$r(\mu^* - c) = Q(\mu^*), \qquad \text{with } \delta = \frac{1}{1+r},$$

$$Q(\mu^*) = \sum_{k=0}^{n-1} P(x = k|\mu^*, F^{\omega}, n) Max\Big(c - P(\omega = \omega_1 | x = k; \mu^*, F^{\omega}, n), 0\Big), \tag{3}$$

where x is the number of investments by other agents in the period.

- Q is a "regret function" that measures the option value of delay: the expected price that the agent would pay to undo the investment after receiving bad news at the end of the period.