Cascades and Herds

PSE

March 11, 2024

The 2x2x2 Model

- Minimum elements of the model
  - What is the minimum number of states needed for a SL model ?
  - What is the minimum number of signal values in the canonical SL model?
  - What is the minimum number of possible actions?

The 2x2x2 Model

- Minimum elements of the model
  - What is the minimum number of states needed for a SL model ?
  - What is the minimum number of signal values in the canonical SL model?
  - What is the minimum number of possible actions?
- The model
  - 2 states $\omega \in \{0, 1\}$.
  - 2 signal values $P(s = \omega) = q \in (0, 1)$. (symmetric binary signal, SBS)
  - $x \in 0, 1$, payoff $U = x(\omega - c)$, $\quad c \in (0, 1.)$

Social learning in the 2x2x2 Model

- $\mu_t = P(\omega = 1|h_t), \qquad \tilde{\mu}_t = P(\omega = 1|h_t, s_t).$

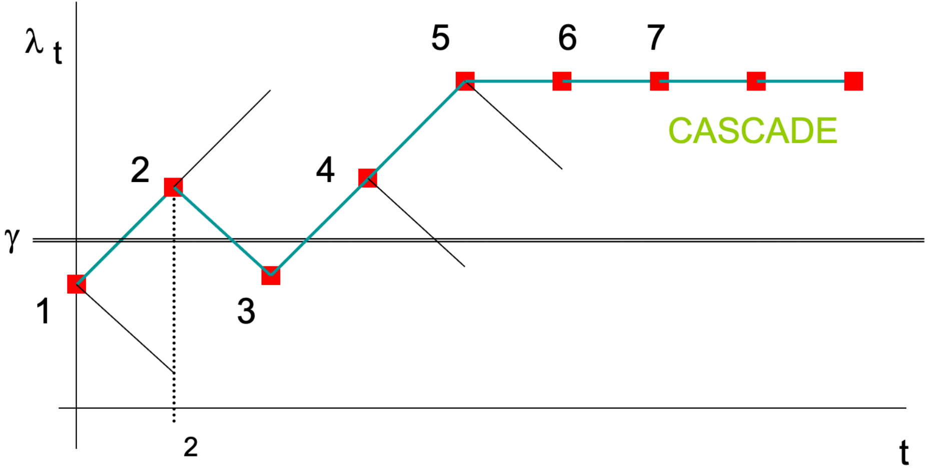
- Use LR : with $s_t = 1$,

$$\tilde{\ell}_t = \frac{\tilde{\mu}_t}{1-\tilde{\mu}_t} = \frac{P(\omega_1|s=1)}{P(\omega_0|s=1)} = \frac{\dfrac{P(s=1|\omega_1)P(\omega_1)}{P(s)}}{\dfrac{P(s=1|\omega_0)P(\omega_0)}{P(s)}} = \ell_t \frac{P(s=1|\omega_1)}{P(s=1|\omega_0)}.$$

- Use LLR: $\lambda_t = Log(\dfrac{\mu_t}{1-\mu_t})$: $\quad \tilde{\lambda}_t = \lambda_t \begin{cases} +a \text{ if } s_t = 1, \\ -a \text{ if } s_t = 0. \end{cases}$, with $a = Log\dfrac{q}{1-q}$.

- Action $x_t = 1$ iff $\tilde{\lambda}_t > \gamma = Log(c/(1-c))$.

Cascade representation



Cascade representation

- Convergence.
- Right and wrong cascades

Continuum of beliefs

- Individual beliefs (instead of signals) are distributed according to the *c.d.f.* $F^{\omega}(\mu)$
  First order stochastic dominance: if $\omega_1 > \omega_0$, $F^{\omega_0}(s) > F^{\omega_1}(s)$

Observations

States of Nature

	$x_t = 1$	$x_t = 0$
$\omega = \omega_1$	$1 - F_t^{\omega_1}(\gamma)$	$F_t^{\omega_1}(\gamma)$
$\omega = \omega_0$	$1 - F_t^{\omega_0}(\gamma)$	$F_t^{\omega_0}(\gamma)$

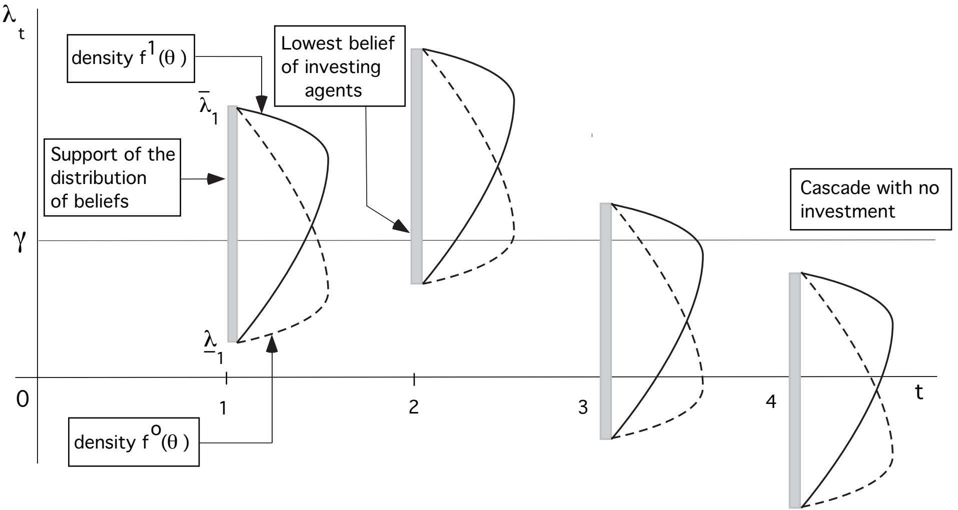
with $\gamma = Log\left(\frac{c}{1-c}\right)$.

- Social learning

$$\lambda_{t+1} = \lambda_t + \nu_t, \text{ with } \nu_t = Log\left(\frac{P(x_t|\omega_1)}{P(x_t|\omega_0)}\right). \quad (1)$$

Representation

(Replace θ by ω).



Unbounded support of private beliefs

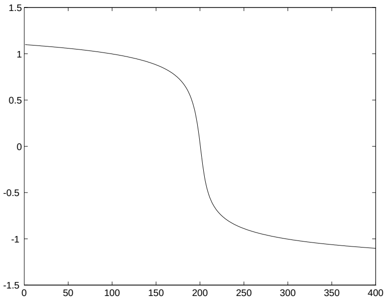
- There is no cascade.
- Discussion of the evolution of beliefs

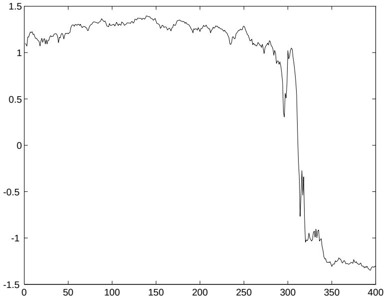
Crashes and booms

- Model: Two states ω_0 and ω_1, $s_t = \omega + \epsilon_t$ with ω and ϵ Gaussian; $x_t \in \{0, 1\}$.
- Belief (LLR) of agent with signal s $\quad \lambda(s) = \lambda_t + \frac{\omega_1 - \omega_0}{\sigma_\epsilon^2}\left(s - \frac{\omega_0 + \omega_1}{2}\right).$
- Cutoff for investment ($x_t = 1$): $\quad s > s^*(\lambda_t) = \frac{\omega_0 + \omega_1}{2} - \frac{\sigma_\epsilon^2}{\omega_1 - \omega_0}\lambda_t.$
- Model with one agent. Discussion
- Model with a continuum of agent in each period: $X_t = 1 - F(s^*(\lambda_t) - \theta; \sigma_\epsilon)$.
- Observed aggregate activity $\quad Y_t = 1 - F(s^*(\lambda_t) - \theta; \sigma_\epsilon) + \eta_t.$

Crashes and booms with a continuum of agents

- Observed aggregate activity $\quad Y_t = 1 - F(s^*(\lambda_t) - \theta; \sigma_\epsilon) + \eta_t.$
- $\lambda_{t+1} = \lambda_t + Log\left(\dfrac{f(x_t - (1 - F(s^*(\mu_t) - \omega_1; \sigma_\epsilon)); \sigma_\eta)}{f(x_t - (1 - F(s^*(\mu_t) - \omega_0; \sigma_\epsilon)); \sigma_\eta)}\right),$ $\quad$ (f density function).
- On the left, $\eta_t \equiv 0$.
- On the right, the evolution of the public belief is represented for random realizations η_t.





Signals and private beliefs

- 2 states $\{\omega_0, \omega_1\}$ with equal probabilities. Private signals with distributions, cdf $F^{\omega}(s)$.

- Call p the probability of $\$state_1$ for an agent with signal s. When an agent receives the signal s, by Bayes' rule, the likelihood ratio between the two states is

$$\frac{F'^{\omega_1}(s)}{F'^{\omega_0}(s)} = \frac{p}{1-p}. \tag{2}$$

  In state ω, F^{ω} generates a distribution of signals, for which the belief is given by the previous equation.

- The reverse applies: belief is a signal. The distribution of this signals must satisfy the previous equation.

Cascades and bounded private beliefs

- Previous model: distribution of private beliefs with cdf $F^{\omega}(s)$ and density $f^{\omega}(s)$.
- Assume that $f(s) > 0$ for $s \in (a, b)$ with $0 < a < b < 1$ and $f(s) = 0$ otherwise. Private beliefs are bounded.
- Payoff: agent choose the state that is more likely (equivalent to $c = 1/2$).
- Update: $\frac{\tilde{\mu}}{1-\tilde{\mu}} = \frac{\mu}{1-\mu}\frac{s}{1-s}$. Invest $(x = 1)$ if $s > 1 - \mu$.
- Cascade set with investment $a > 1 - \mu$ which is equivalent to $\mu > 1 - a$.
- Cascade set with no investment $\mu < 1 - b$.
- MCT $\Longrightarrow \mu_t \to \mu^*$.
- The limit μ^* cannot be in the interval $(1 - b, 1 - a)$.
- A cascade occurs with probability one.

Cascades and unbounded private beliefs

- Assumption $f(s) > 0$ on $(0, 1)$ (to simplify)
- Assume that $f(s) > 0$ for $s \in (a, b)$ with $0 < a < b < 1$ and $f(s) = 0$ otherwise. Private beliefs are bounded.
- Payoff: agent choose the state that is more likely (equivalent to $c = 1/2$).
- Update: $\tilde{\ell} = \ell \frac{s}{1-s}$. Invest $(x = 1)$ if $s > \frac{1}{1+\ell}$.
- Cascade set with investment $a > \frac{1}{1+\ell}$ which is equivalent to $\mu > 1 - a$.
- Cascade set with no investment $\mu < 1 - b$.
- MCT $\Longrightarrow \mu_t \to \mu^*$.
- The limit μ^* cannot be in the interval $(1 - b, 1 - a)$.