Spring 2024
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Problem Set 1

1. Browse the paper by Nyarko (1991) which is posted on the website. Focus on Section IV. Following the paper, assume that the demand curve in period t is of the form

$$q_t = a_\omega - b_\omega p_t + \epsilon_t,$$

where the parameters (a, b) depend on the state of nature ω, and ϵ_t is a normal random variable with mean 0 and variance σ^2. The agent is a monopolist who in each period, optimizes profit myopically, without experimentation. The cost of production is 0.

Following the paper, the true state is $\omega^* = (28.5, 5.25)$. However, (simplifying the paper), the monopolist believes that the state is in the set $\{\omega_1, \omega_2\}$, where $\omega_1 = (20, 1)$ and $\omega_2 = (16, 4)$. Let μ_t be the belief of the monopolist at the beginning of period t with $\mu_1 = 1/2$. The monopolist updates the belief at the end of each period, using Bayes' rule.

   (a) Explain how this exercise is a simplification of the paper by Nyarko.

   (b) Simulate the evolution of beliefs for a large number of periods. (You may have to experiment with the value of σ^2.

   (c) In view of the simulations, comment on the section IV of the paper.

2. Recall the Vives model. Agent t has a payoff function $-E(\omega - x_t - \omega)^2]$, where, to simplify here, ω has diffuse prior (infinite variance), and η_t are independent normally distributed r.v. with mean b and precision ρ_η. Each agent has a signal $s_t = \omega + \epsilon_t$ with $\epsilon \sim N(0, 1/\rho_\epsilon)$ as in the standard model.

   (a) Assume that agents are mistaken about the distribution of η_t. They believe that $b = 0$ whereas the true value is positive.

   (b) Assume that agents optimize without looking at the history of past actions. Let μ_t be the expected value of ω or an outside observer, an econometrician, who knows the history of past actions, $h_t = \{x_1, ..., x_{t-1}\}$. Comment on the case b small.

   (c) Assume now that any agent t observe the history h_t of past actions. Analyze the evolution of μ_t. Compare with the previous question. At some point, you may use the proposition of Vives (p. 8 of Notes 1).

(d) 3. (Optional). Simulate the evolution of μ_t at a function of t for the particular realization of all noise terms equal to 0. (You may normalize other parameters to 1). Plot on a diagram the values of μ_t and ρ_t (the precision of the estimate of ω at the beginning of period t. Comment. (You may anticipate the answer in the previous question).

(e) In view of your answer in question 2, what may be a "flaw" in the the assumption about b, in that question? Is the exercise nevertheless informative? Comment.

3. Consider the model of social learning with two states, 0 and 1, two actions 0 and 1, with a cost of investment equal to c. The prior belief (probability of state 1) is μ_1. Any agent t has a signal $s_t = \omega + \epsilon_t$, where $\epsilon_t \sim N(0, 1/\rho_\epsilon$. Assume that the true state is 1.

(a) Does the belief converge? If yes, what is the limit? Does the sequence of actions eventually become a herd?

(b) Assume now that in each round, with probability π, the acting agent is a "noisy" agent who does not invest, because his cost is greater than 1. Rational agents know the model and the probability π. Answer the previous question.

(c) Assume that now that rational agents are not aware of the existence of noisy agents. (They think that $\pi = 0$). To make the case interesting, assume that π is very small. Answer the previous question and comment on the case where π is very small. You may give an informal answer.