Ec741
Fall 2017

Problem set 1

One of the most useful exercises in the training for research on theoretical models is to build a problem from a research paper. Such a problem must extract the main mechanisms of the paper and build from the sequence of questions a simple model and its analysis. Often the author(s) of the paper have started from a such simple model.

Here the problem set is built from:

Murphy, K. M. A. Shleifer and R. W. Vishny (1989). "Industrialization and the Big Push," *Journal of Political Economy*, **97 (3)**, 1003-1026.

The paper is motivated by

- Rosenstein-Rodan, Paul N., (1943). "Problems of Industrialisation of Eastern and South-eastern Europe," *Economic Journal*, **53**, 202-11.
- ———— (1961). "Notes on the Theory of the 'Big Push,' " in *Economic Development for Latin America*, Howard S. Ellis and Henry C. Wallich, eds, New York: St. Martin's.

Rosenstein-Rodan was in the department at Boston University at the time when the main (only?) strength was in development economics. The paper by MSV has actually nothing to do with industrialization but it is a very good theoretical model of investment and imperfect information with payoff externalities and multiple equilibria. It is related to issues of payoff externalities in economies with networks.

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There are two periods. There is a continuum of goods indexed on $(0,1)$ and a large number of individuals who can be represented by a single agent with the utility function

$$U = -x_0 + \frac{1}{1+\rho}C, \quad \text{with} \quad C = \Big(\int_0^1 x_i^{\frac{\epsilon-1}{\epsilon}} di\Big)^{\frac{\epsilon}{\epsilon-1}}. \tag{1}$$

The quantity x_0 represents an investment good which is withdrawn from consumption in the first period, and C measures the total consumption in the second period.

Since the function C has constant returns to scale in its arguments, the index of welfare will be identical with the present discounted value of output. The parameter ρ (which may be zero), is a subjective discount rate between the two periods.

Each good is produced by one firm. The labor supply in period 2 is fixed and equal to L. Each firm is owned by individuals who receive the firms' profits as a lump-sum payment. Each firm makes a zero-one decision in the first period.

- If the firm does not invest, its marginal cost of production (in terms of labor) in the second period is equal to 1.

- If the firm invests, its marginal cost is reduced to $\alpha < 1$. The cost of investment is equal to a fixed amount c. Denote by $\Delta\pi$ the increase of profits which is generated by the improvement of productivity. The net impact of investment on profits is equal to $\Delta\pi - (1+r)c$, where r is the equilibrium interest rate. The interest rate will be defined between the investment good x_0 in the first period and a good which remains to be chosen in the second period.

1. Let w be the wage rate in period 2. Show that the demand for each good is of the form

$$x_i = \frac{C}{P} \frac{1}{\left(\frac{p_i}{P}\right)^{\epsilon}}, \tag{1}$$

with a price index P that you will determine. Analyze the relation between p_i and w for the firms that invest and those that do not.

2. We normalize the prices such that $P = 1$. Let q be the equilibrium price of a bond that delivers 1 in period 2. Determine the value of q and of the interest rate of such a bond.

3. Given the equilibrium in the second period, one can compute the level of profit for each firm. We can then determine for each firm whether the investment decision is optimal. Suppose that a fraction λ of firms invest in the first period, and call $\pi_{i,\lambda}$ the firm's profit in the second period measured in units of total consumption, where $i = 0$ for a non investing firm and $i = 1$ for an investing firm. Each firm takes the behavior of others as given, maximizes its expected profit:

$$V = Max\Big(\pi_{0,\lambda},\ \pi_{1,\lambda} - (1+r)c\Big).$$

In an equilibrium the optimal investment decision of each firm in the first period is made with the perfect expectation of the equilibrium with monopolistic competition in the second period. (When random strategies are admitted, firms will be risk-neutral and have rational expectations). We begin the analysis with the simple case (which is also the most interesting here), where all firms make the same decision in the first period.

a. Suppose now that no firm invests, $\lambda = 0$. Using a price normalization $P = 1$, what is the aggregate income in period 2? Show that

$$\pi_{1,0} = \alpha^{1-\epsilon}\pi_{0,0}. \tag{2}$$

b. Suppose that all firm invest, $\lambda = 1$. Using a price normalization $P = 1$, what is the aggregate income in period 2? Show that

$$\pi_{1,1} - \pi_{0,1} = (\alpha^{1-\epsilon} - 1)\frac{\pi_{1,1}}{\alpha^{1-\epsilon}}. \tag{3}$$

c. Using the previous results, show that there are values of the structural parameters c (defined before question 1) and ρ such that if $1 < \epsilon < 2$, there are two Nash equilibria in which no firm invests, and all firms invest, respectively.

4. Extend the model to an infinite number of periods with endogenous growth. Comment on the set of equilibria. (Don't do a formal analysis).