Chapter 9

Endogenous uncertainty

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9.1 Aggregate activity and information

During a recession, a dominant question is the timing of the recovery. A renewal of activity by some agents provides good news to other agents but when investment decisions imply long range planning and irreversible expenditures agents may play a game of waiting for news, and that delays the recovery. In this context, more activity generates more information. However, a symmetric situation may exist in which a high level of activity in a regime of "business as usual" hides a downturn of the fundamental which becomes known only after a sufficiently large reduction of aggregate activity. In this case, information is inversely related with aggregate activity. Simple models for each case are presented in the next sections.

9.1.1 Activity and information with fixed underlying information

In this section, the state of nature and all informations, private and public, are set at the beginning of time. That is important because the analysis focuses on one mechanism, the spreading of private information to others through taking actions that are observed by others. Changes of the state of nature or of private informations will be considered later.

Information increasing with aggregate activity in a model of delay

The model of Chamley and Gale (1994), was inspired by the context of the time when the US economy played a hesitation waltz that was described by Sylvia Nasar in a NYT article with the title "The Economics of Wait and See".[1] Agents can delay an investment of fixed size that with a future payoff that depends on a state of nature. There is a cost to delay, but delaying enables an agent to get more information and should this information turn out to be bad news about the fundamentals, avoid the regret of having made an irreversible investment. In each period of an equilibrium, the cost of delay is equal to the option value of delay which is the expected value of regret.

The model makes the sensible assumption that the number of agents who can make an investment (*i.e.*, have an option to make one investment) increases with the fundamental. In this case, a higher level of activity is good news about the fundamental and it also brings more information. (If all agents have the strategy to delay, there is no information). The equality between the cost and the option value of delay acts as a brake on the recovery: there cannot be too much activity because a high level of activity would bring much information and information is bounded by the cost of delay which depends only on the current expectation of profitability. The externality of information generates a strategic substitution between the investment decisions: if other agents invest more (for example by lowering profitability requirement for investment), then I have an incentive to wait for more information and not to invest in the the current period.

The arbitrage equation between the cost and the option value of delay generates some interesting properties.

1. Good news on the profitability of investment generate a higher level of investment not through the standard q-theory (Tobin), but because they increase the cost of delay and by arbitrage, the amount of information that is generated which can be done only through a higher level of investment.

2. Activity may stop after bad news in which case no endogenous recovery is possible because information requires activity.

3. An observation noise or a large number of agents does not change the properties of the model because the level of information must match the cost of delay for individuals and that cost does not depend on an observation noise or the number of agents in the economy.

[1] NYT, May 12, 1993.

A simple version of the model of Chamley and Gale (2014)

There are two possible states of nature, $\theta \in \{0,1\}$. In state θ, there are N_θ agents each with one option to make a fixed sized investment (normalized to 1) at the fixed cost c between 0 and 1. The payoff of investment in period t is the expected value of $\delta^t(\theta - c)$, where δ is the discount factor, strictly smaller than 1. $\theta 1$ is therefore the good state and we assume that $N_1 > N_0 \geq 1$. We take the simplest model and therefore $N_0 = 1$, $N_1 = 2$. Note that all agents are identical but no one knows the number of agents who can invest and "play the game". There must be some belief however, so let μ be the belief (prior probability of state 1) of any "living" agent. We must make two assumptions in order to avoid a triviality. First $\mu - c > 0$, otherwise no individual would ever invest. Second, there must be sufficient incentive to delay. The maximum such incentive is to delay for perfect information in the next period. In this case, the expected payoff of delay is $\delta\mu(1-c)$: the discounted value of the revelation of the good state (with probability μ for the waiting person) in which case the payoff is $1-c$. If the state $\theta = 0$ is revealed no investment is made. Hence,

$$0 < \mu - c < \delta\mu(1-c) \tag{9.1}$$

Note that the second inequality must hold if δ is sufficiently high (be smaller than 1), which is the same as a sufficiently low rate of discount between periods. That is the whole model!

It is shown in CG that there is a unique symmetric equilibrium strategy which is to invest in period 1 with probability λ such that

$$\mu - c = \lambda\delta\mu(1-c) \tag{9.2}$$

For a delaying agent, there are only two possible news during the first period, good news with one investment, $x = 1$, which reveals that $N = 2$ and that the state is good; or bad news with no investment, $x = 0$. In an equilibrium, after bad news, no agent should invest and the belief should be strictly smaller than c. If an agent delays in order to invest after bad and good news, he should do so without delay and avoid the penalty of the discounting. One can therefore rewrite the previous equation such that

$$r(\mu - c) = P(x_1 = 0|\lambda,\mu)(c - P(\theta = 1|x = 0;\lambda,\mu)) \quad \text{with} \quad r \text{ such that } \delta = \frac{1}{1+r}. \tag{9.3}$$

The LHS is the cost of waiting (the foregone net income of the investment for one period) and the RHS is the expected value of regret, *i.e.* the expected value of what the agent would pay to undo the irreversible investment.

Once these properties are well understood in the simple model, one can see that there are robust and the techniques of the full model, which were required by the publication process, do not bring any new insight.

As stated above, in the model, the gross profitability of each agent's investment depends only on fundamentals that are represented by a single parameter, say θ. The externality between investments is only about information on θ. The actual payoff of an agent does not depend on the mass of other investments. (The expected payoff increases with the investment of others because of the good news about the fundamental). In the model the strategic substitutability generates a unique symmetric equilibrium and that strategic substitutability comes from the positive relation between the fundamental and the number of options in the economy. The stopping of all investment after bad news is like a "cascade" in the BHW model. As is well know, the timing of action in the BHW model is exogenous and agents have no possibility of delay. However, the Chamley and Gale model is not an extension of the BHW model with endogenous delays. Such an extension is analyzed in Chamley (2003).

Information decreasing with aggregate activity in the BHW model with delays

In the BHW model, the number of agents is fixed, say at N (large) and each agent i has a private signal s_i on θ that is binary, and for simplicity, symmetric: $P(s_i = \theta|\theta) = q > 1/2$. The set of actions and the payoffs are the same as in the previous paragraphs. In the BHW model, agent i can make a decision $x_i = 1$ or 0), only in round i. The difference now is that all the agents have an option to make one investment, which is irreversible, in any period. (There is a discount factor δ as in the previous paragraphs).

Let us call agents with a signal 1 (a "good" value) the optimists and the others the pessimists. Their beliefs are μ^+ and μ^-. For example μ^-, by Bayes' rule, is such that $\mu^-/(1-\mu^-) = ((1-q)/q)(\mu/(1-\mu))$. Suppose that the belief of pessimists is such that $\mu^- - c > 0$ and very small. Of course, optimists have a higher belief and in this case, for all agents, the payoff of investment in the first period, without delay, has a positive payoff. If all agents invest in the first period, the level of aggregate investment is N, which was perfectly predictable before the first period. Nothing is learned by waiting for this observation and no delay is indeed a equilibrium strategy.

Suppose now that pessimists choose the strategy to delay and not to investment in the first period and that all pessimists invest without delay. In this case, delaying reveals much information and it is indeed a good strategy for the pessimists. One can also suspect from the previous model of Chamley and Gale, that this information will also be useful for the optimists and that, in an equilibrium, some of them will delay. The optimists with play, at least in the first period a waiting game. The full analysis of the model is presented in Chamley (2003) and a reduced version is proposed in Exercise 9.1.

From the previous two paragraphs one can see that in this model, more information is generated by *less* aggregate activity. There is a strategic complementarity between the agents' investments. This strategic complementarity generates, under some conditions, multiple equilibria. When the private information of agents is bounded (as in the case of the previous symmetric binary signal), the outcome of the game is like in BHW. After a finite number of periods, either all agents who have not invested and still have an option to do so do not investment any more, or all the remaining agents invest without delay. In either case, there is cascade in the sense of BHW, either with no investment or with investment. Both are of course possible for each state of nature.[2]

9.1.2 Activity and information with growing underlying information

In the models that we have considered so far, the focus was on the mechanism by which private informations are publicly revealed through actions and observations. The fundamental and the private informations are set at the beginning of time. We now keep the setting by which private informations are communicated through actions (or the absence of action) and a fixed fundamental. The new element is that individuals get more information over time through exogenous channels. This gradual build up may remain hidden for a while in a regime of "business as usual". At some point, some agents decide to change their action and that triggers an avalanche of information and potentially a change of the regime of activity.

Business as usual and sudden crash in the model of Caplin and Leahy (1994)

In the model of Caplin and Leahy, the economy is first in a regime of high activity because the belief on the fundamental is high. The true value is low. In each period, agents receive private signals on the fundamental. Over time, there is a distribution of beliefs: the agents with many bad signals in their sequence of private signals are more pessimist than the other. At some point, they are so pessimistic that they stop their activity. This small downturn is observed by others and induces others who were less pessimistic to also stop their activity, and so on. There is a "sudden crash".

This story could, most likely, be represented in a model with no delay and where agent choose at the beginning of each period to be active or not. Caplin and Leahy have presented the story in a model with delay. The delay probably accentuates the sudden downturn as

[2] The Chamley-Gale model generates an asymmetry: a cascade with no investment is possible in the good state but in the bad state, there cannot be a cascade with investment. Such an asymmetry also takes place in the BHW model when an individual's investment is not discrete (*e.g.*, equal to 0 or 1) but can take any positive real value.

it enables agent to accumulate much private information before making a switch. This issue is not entirely clear at this point and you are welcome to think about it.

In the model of Caplin and Leahy (1994), agents receive private signals over time while the economy is in a level of high activity in a regime of “business as usual”. The economy is in a high activity because all agents are making fixed payments in each period towards the completion of a project. The payoff of that project is the same for all agents and depends on the state of nature. One can normalize the payoff to be 1 in state 1 and 0 in state 0. At the beginning of time, the belief of agents is sufficiently high for the undertaking of the project. Over time, agents receives signals on the state of nature and we can assume that these signals are SBS. In each period, agents can put the project on hold for one period only and resume payments in the following period. There is a cost for putting the project on hold.

The model presents an equilibrium in which all agents pursue the project during the initial business as usual phase. But the agents who get a continuous string of bad signals (the most pessimistic) put the project on hold in some period T. In that period, one can therefore observe the mass of these agents and have perfect information on the state of nature. If the state is revealed to be good, the most pessimistic resume their project and the economy goes on in a high regime. If the state is revealed to be bad, all projects stop. There is a “crash and wisdom after the fact”.

The model of Caplin and Leahy exhibits the interesting property that a *decrease* of economic activity generates *more information*. (Of course in the information generating recession yields two possible news, good or bad, and after good news, aggregate activity resumes in a high regime. Bad news lead to a crash. In both cases, the lower level of activity in the recession reduces the uncertainty. The model is simpler to explain in the symmetric case where “business as usual” is no activity and the change of regime is not a crash but an explosion of activity. In that version, public information increases with a positive change of aggregate activity.

Build-up of private informations

The population of agents is fixed and forms a continuum of mass normalized to 1. Each agent has one option to make an investment[3] of a fixed size in any period $t \geq 1$, with payoff

[3] Caplin and Leahy consider the symmetric problem of agents who have an option to stop an investment process. The two models are isomorphic. The model with options to invest generates a boom, with positive probability; the model with options to stop the flow of investment expenditures generates a crash. The model with a crash was chosen for its effect on the reader, but it is technically more complicated. The model with an option to invest enables us to make a direct comparison with the model of Chamley and Gale (1994).

$E[\theta] - c$, at the time of the investment ($\theta \in \{0, 1\}$ and $0 < c < 1$). The payoff at time t is discounted to the first period by δ^{t-1}. Each agent receives a new SBS about θ in *every* period with precision q which is independent of other variables.

Equilibrium with deterministic strategies

We look for a symmetric Bayesian subgame perfect equilibrium (PBE) with deterministic strategies. (If necessary, the parameters of the model will be adjusted to obtain deterministic strategies). We will show later that the players who move first are the super-optimists (they have an unbroken string of good signals). We can focus on these players. From Bayes' rule, their LLR between the good and the bad states, λ_t, increases linearly with the number of periods:

$$\lambda_t = \lambda_0 + at, \quad \text{with} \quad a = Log(q/(1-q)).$$

Let λ^* be the minimum belief (measured by the LLR) for a positive payoff of investment: $\lambda^* = \gamma$ which is defined by $\gamma = Log(c/(1-c)$.

Define by λ_1^{**} the belief of an agent who is indifferent between investing and delaying with the revelation of the true state one period later. An exercise shows[4] that

$$\lambda_1^{**} = \lambda^* + Log(\frac{1}{1-\delta}),$$

which is strictly higher than λ^* because of the information obtained after one period.

A necessary condition for investment by a super-optimist in period t is $\lambda_t \geq \lambda^*$. This condition is not sufficient however: if all the super-optimists invest in period t, they reveal perfectly the state (there is a continuum of them). If the state is revealed at the end of period t, they prefer not to delay only if $\lambda_t \geq \lambda_1^{**}$. The evolution of the belief of the super-optimists is represented in Figure 1.

Let T be the smallest t such that $\lambda_t > \lambda_1^{**}$:

$$T = Min\{t \text{ integer } | \lambda_0 + aT - \lambda_1^{**} > 0\}. \tag{9.4}$$

Define $\eta = \lambda_0 + aT - \lambda_1^{**}$.

If no agent has invested prior to period T, super-optimists all invest in period T. They know that their action reveals the state one period later, but by definition of T, this incentive

[4] Such an agent has the belief $\mu - c = \delta\mu(1-c)$ which is equivalent to $\mu/(1-\mu) = (c/(1-c))/(1-\delta)$.

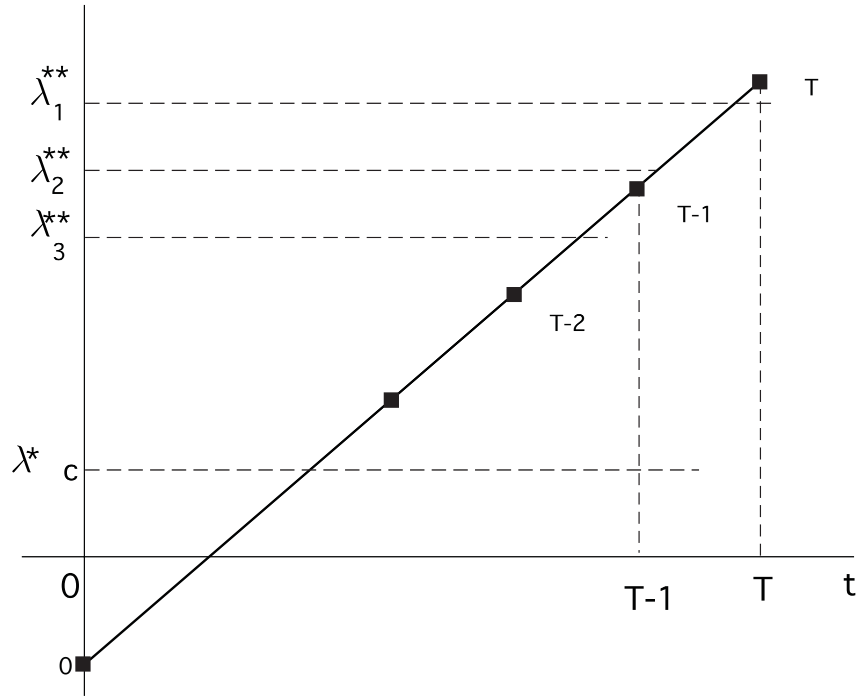

Figure 9.1: The evolution of beliefs

The LLR of the super-optimists increases linearly with time. In period T, (smallest t such that $\lambda_t > \lambda_1^{**}$), investment dominates delay with perfect information one period later. In period $T-k$, investment is dominated by delay with perfect information $k+1$ periods later.

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is not sufficient to make them wait. In period $T+1$, all the other agents either invest (if $\theta = 1$), or they don't (if $\theta = 0$). If the equilibrium strategy is to delay until period T, all agents know perfectly that full information is revealed at the end of period T. The value of T is determined by all agents in the first period.

It remains to show that the super-optimists delay until period T. Let λ_k^{**} be the value of the likelihood ratio for an agent who is indifferent between not delaying and delaying for k periods exactly and making then a decision with perfect information.

Begin with period $T-1$. Assume that the parameters of the model are such that for the super-optimists, investment in period $T-1$ is dominated by delay until period $T+1$ with perfect information in period $T+1$:

$$\lambda_{T-1} < \lambda_2^{**}, \tag{9.5}$$

where

$$\lambda_2^{**} = \lambda^* + Log(\frac{1}{1-\delta^2}) = \lambda_1^{**} - Log(1+\delta).$$

In that case, delay for two periods is not the optimal strategy: a super-optimist who will receive one more positive signal in period $T-1$ invests in that period. A sufficient condition for (9.5) is that

$$a + \eta > Log(1+\delta). \tag{9.6}$$

From Figure 1, this condition is sufficient for the following Lemma

LEMMA 9.1. *Under condition (9.6), $\lambda_{T-k} < \lambda_{k+1}^{**}$ for any $k \leq T-1$.*

We have focussed so far on the super-optimists. The optimal strategies of the other agents are simple: any agent who is not a super-optimist has at least one fewer positive signal compared to the super-optimists. Therefore his LLR in period t is not greater than $\lambda_t - a = \lambda_{t-1}$. The argument for the delay of a super-optimist in period $T-k$, with $k \geq 1$, applies *a fortiori* to any other agent.

From the previous discussion, inequality (9.6) is sufficient for the symmetric PBE where the super-optimists invest in period T. This condition is restated in terms of structural parameters in the following proposition.

Proposition

Under the definition (9.4) and the condition (9.6), there is a unique PBE, and in this PBE, all super-optimists invest in period T. All other agents invest in period $T+1$ if and only if the good state is revealed (by the mass of super-optimists) at the end of period T.

9.2 The (random) evolution of the fundamental

9.2.1 Exogenous signals

Following The state of nature follows a random walk with the normal distribution

$$\theta_{t+1} = \theta_t + \epsilon_t, \qquad \epsilon_t^\theta \sim \mathcal{N}(0, 1/\gamma_\theta). \tag{9.7}$$

If there is no information, the variance of θ_t increases linearly with time, $\gamma_{t+1} = \gamma_t + \gamma_\theta$. Note that θ_t follows a random walk with no regression to a "long-run value".

Suppose that in each period, agents observe the signal on θ, with a normal noise:

$$y_t = \theta_t + \epsilon_t^y, \qquad \eta \sim \mathcal{N}(0, 1/\gamma_y).$$

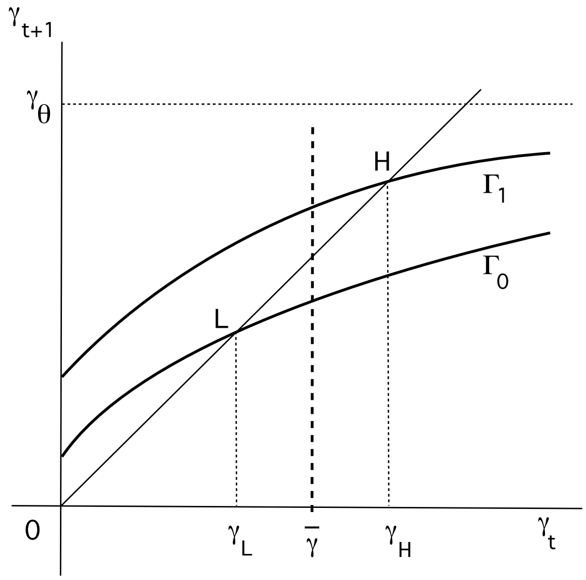

Figure 9.2: Evolution of the precisions

After the observation of the signal y, the precision (inverse of the variance) on θ is equal to

$$\gamma'_t = \gamma_t + \gamma_y.$$

From one period to the next, we have two updates: the signal y augments the precision on θ and the random change from period t to period $t+1$ reduces the precision. None of these two steps "wins" over the other. The precision cannot become infinite because of the changes of θ between consecutive periods. The precision cannot go to zero because it is bounded below by the precision of the signal y. These two mechanisms are probably at work for a wide class of processes. Here, given the normality assumption, we can write

$$\gamma_{t+1} = \frac{\gamma'_t \gamma_\epsilon}{\gamma'_t + \gamma_\epsilon} = \gamma_\epsilon \frac{\gamma_t + \gamma_y}{\gamma_\epsilon + \gamma_t + \gamma_y} = \Gamma_0(\gamma_t). \tag{9.8}$$

From (9.8), the function $\Gamma_0(\cdot)$ is increasing with $\gamma_0(0) = 0$, $\Gamma_0(\cdot) < \gamma_\epsilon$. It has therefore a unique fixed point. Its graph is represented in Figure 9.2. For given γ_θ and γ_y, when $t \to \infty$, the precision γ_t converges to the fixed point of Γ_0.

Note that if the precision of the information signal γ_y increases, the function Γ_0 increases and its graph is shifted upwards (see Figure 1), with a higher precision in the limit. That property is now exploited to extended the model to the case of endogenous uncertainty with multiple equilibria.

9.2.2 Endogenous uncertainty and traps

The issue is the following. We would like a model where information increases with economic activity and activity increases with information. That will generate a strategic complementarity and multiple equilibria with different levels of "endogenous uncertainty".

Assume that there is a mass of firm, equal to θ, that each take action 1 in any period where the uncertainty on θ is not too large, *i.e.*, when $\gamma_t > \bar{\gamma}$, for some value $\bar{\gamma}$ that will be chosen later. One could also assume a mass one of firms and an output that is equal to θ multiplied by the mass of action. If $\gamma_t < \bar{\gamma}$ no firm invests. If $\gamma_t > \bar{\gamma}$, the total level of activity is θ, but this level is observed through some noise. In this case, to the signal y one adds the signal

$$y' = \theta + \eta, \quad \text{with} \quad \eta \sim \mathcal{N}(0, 1/\gamma_\eta).$$

In Figure 9.2, we have now two regimes. For $\gamma_t < \bar{\gamma}$, the curve Γ_0 applies: γ_t converges to γ_L.

For $\gamma_t > \bar{\gamma}$, the updating formula (9.8) is trivially updated to

$$\gamma_{t+1} = \gamma_\epsilon \frac{\gamma_t + \gamma'_y + \gamma_\eta}{\gamma_\epsilon + \gamma_t + \gamma 1_y} = \Gamma'(\gamma_t), \quad \text{with} \quad \gamma'_y = \gamma_y + \gamma_\eta. \tag{9.9}$$

We have seen that an increase of the precision of γ_y shifts the curve Γ_0 upwards, here to Γ_1. For $\gamma_t > \bar{\gamma}$, we are in a high regime where γ_t converges to the higher value γ_H.

In the "low regime", agents do not produce because the uncertain is too high, and the uncertainty remains high because agents do not produce. In the regime with low uncertain, agents produce and the production provides information such that although θ_t varies from period to period, the uncertainty remains sufficiently low for the agents to keep producing.

The model and Figure 9.2 exhibit "uncertainty traps" (Fajgelbaum *et al.*, 2014). Fajgelbaum *et al.* construct a model with delays, but as shown here, the property has nothing to do with delays (which amplify the difference the low and the high regime). This property of "uncertainty traps" has already been analyzed by Pagano (1989) in the context of financial markets. An asset may be subject to price volatility and therefore be illiquid because there is only a small mass of agents who are active in that market. And there is a small mass because risk-averse agents do not enter that highly volative market. For the same structure, there is another equilibrium where the market is less volatile because, etc...

One can also assume that the fundamental is determined by a first-order auto-regressive process of with a stable long-run distribution. See Exercise 9.2.

In the previous model, the cutoff point $\bar{\gamma}$ is fixed. Regimes depend on the initial value γ_1 and are permanent. On can easily modify the payoff of agents such that the cutoff depends both on the variance and the mean of θ in the public information.

Payoff of mean and variance

Assume that agents have a constant absolute risk aversion, a and a payoff of action 1 equal to the expected value of $(1 - e^{-a\theta})/a$. When the distribution of θ is (m, σ^2), the certainty equivalent of that payoff is $(1 - e^{-az})/a$ where $z = m - a\sigma^2/2$ is the certainty equivalent of the random return θ. The net payoff of investment (taking action 1) is positive if

$$z > \frac{1}{a} Log(1 - ca).$$

Noting that $\gamma = 1/\sigma^2$, and changing the notation, an agent takes action 1 if and only if for some a and b,

$$m \geq \frac{a}{\gamma} + b. \tag{9.10}$$

Note that b does not have to be positive. The curve (Γ) marks the frontier between high and low activity. It embodies a trade-off between the mean m of θ and its precision γ. Above the frontier, the mean m is sufficiently high for a high activity and the precision increases toward $\bar{\gamma}$. Below the frontier, the mean decreases toward $\underline{\gamma}$. The vertical movements of m are driven by the evolution of θ and the learning about θ in equations.

`To be continued.`

Asymmetric regimes

One can make an argument that in a recession, the threshold c for activity gradually decreases. For example, part of the output could be capital, which does not have to be modeled here. Such a modelization adds significant complexity. It does not provide additional insight on the essential mechanism of the model.[5]

The impact of a depreciating capital stock can be modeled here by assuming that c gradually decreases with time in the low regime and increases with time in the high regime. In the previous figure, the (γ) curves shifts gradually downwards in a low regime, when the point (m, γ) is below the (Γ) curve. The reverse is true above the frontier.

[5] The model of FST-M indeed has capital and labor and also delays in taking actions. That is why it is so complicated. All these efforts are made for a reproduction of some empirical properties of a business cycle. But I that exercise seems more like playing a game. The fitting of some properties with business cycle features is proof of the skills of the authors but does not provide a scientific validation : there are many other things that go on in a business cycle, and the current mechanism may be only a part, possibly a small part, of what drives a business cycle.

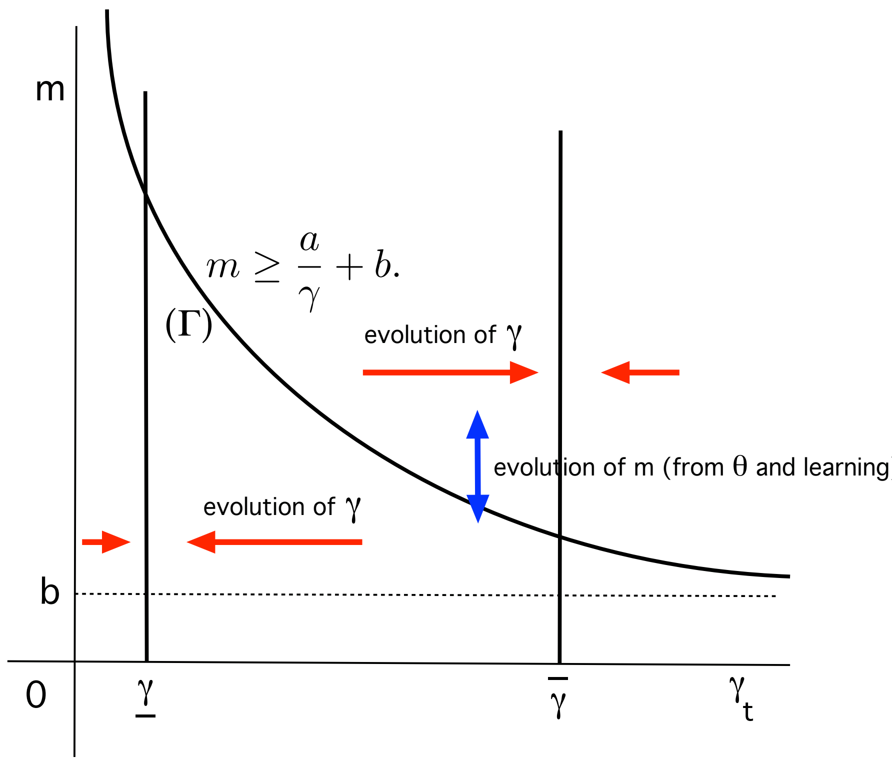

Figure 9.3: Evolution of the precisions

The timing of policy

Because of the information externality, the equilibrium is not a Pareto optimum. A policy of subsidization to lower the cost of action may be beneficial. This is not the place for a formal analysis but one can note that a subsidy is equivalent to a decrease of the parameter b, and in the Figure, a downward shift of the (Γ) frontier. In this model m and γ are public information. In a low regime, below the frontier, the policy is likely to be more effective when the (m, γ) point is near the frontier. That is probably the situation at the beginning or the end of a low regime (recession).

EXERCISE 9.1. (A BHW model with delays)

Consider now the model of delay with two agents where now **both** agents have an option to make one investment in any period at the cost c. In the state $\theta = 1$, the gross payoff of investment is 1 and it is equal to 0 in the state 0. Both agents have a symmetric binary signal s such that $P(s = \theta|\theta) = q$. (The standard symmetric binary signal). The initial public belief is equal to $\mu = 1/2$ and the discount factor between periods is $\delta < 1$ and vanishingly close to 1.

1. Assume that an agent, say agent 1, has a signal $s = 1$, delays in the first period. He expects that if the other agent invests in period 1, with no delay, if and only if his signal is 1. Determine for the agent 1 the probability that he observes an investment in the first period.

2. Determine the undiscounted value of the payoff of delay for agent 1 under the condition of the previous question.

3. Using the previous question, show that if $c > 1/2$, the strategy to invest for sure in period 1 cannot be an equilibrium strategy.

4. Using the previous questions analyze all the possible equilibria. Does the game last more than two periods?

5. In the previous section, remove the assumption that δ is vanishingly close to 1.

EXERCISE 9.2. (A simple model of learning about an AR1 fundamental)

The evolution of the fundamental is given by

$$\theta_{t+1} - \theta_t = \rho\theta_t - \bar{\theta}) + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, 1/\gamma_\epsilon), \qquad \gamma_\epsilon = (1-\rho^2)\gamma_\theta.$$

Exogenous signal on θ_t:

$$y_t = \theta_t + \epsilon_t^Y, \qquad \epsilon_t^Y \sim \mathcal{N}(0, 1/\gamma_y).$$

Endogenous agents take action iff $\gamma_t > \bar{\gamma}$, where $\bar{\gamma}$ is a parameter. There is *no delay.*

When endogenous agents take action, there is an additional signal in period t on θ_t. (All signals are at the end of the period).

$$y_t = \theta_t + \eta_t, \qquad \eta_t \sim \mathcal{N}(0, 1/\gamma_z).$$

Analyze the model.

EXERCISE 9.3. (Cycles)

Consider the model of Section 9.2.2 with a random walk for the fundamental. Introduce a cost c_t that varies with aggregate activity and through simulations, try to find parameter values that generate "reasonable" properties.

EXERCISE 9.4. (Delays and information)

Assume that the fundamental θ is such that $\theta \sim \mathcal{N}(\bar{\theta}, \sigma^2)$. There are two periods and the fundamental is invariant. There is a continuum (of mass one) of agents who each have an option to make a fixed size investment at the cost c.

1. Write and solve a model to analyze whether investment in the first period generates strategic substitutability or complementarity.

2. Extend the model by a distribution of the cost c that is determined by a density function over an interval such that there are multiple equilibria. You do not have define the density by an explicit function. A presentation relying on graphics and intuitive argument is sufficient.

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