Ec 517
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Problem 8

(The white van problem)

`This problem is not been checked recently. You may improve on it.`

The problem is derived from real facts around Washington D.C. when a sniper killed 10 people in different days and the public looked, erroneously, for a suspect travelling in a white van.

Consider the following story. A burglary has been committed in place A or B. The true place defines the state of the world $\theta \in \{A, B\}$. The police will investigate the more likely spot and it has an informant. The informant can look only in one place, either in place A or place B. If the informant looks in a place $i \in \{A, B\}$, his observation is like a signal s_i that takes the value 1 or 0. More specifically, he either sees something suspect ($s_i = 1$) or they see nothing ($s_i = 0$). A signal is informative because it is more likely that he sees something suspect place i if in fact the true state is i. The setting is assumed to be completely symmetric and for $i = A, B$,

$$\begin{aligned}
\Pr(s_i = 1|\theta = i) &= p \\
\Pr(s_i = 0|\theta = i) &= 1 - p \\
\Pr(s_i = 1|\theta \neq i) &= q \\
\Pr(s_i = 0|\theta \neq i) &= 1 - q
\end{aligned}$$

1. It is assumed that $p > q$. Justify this assumption.
2. It assume that $p + q < 1$. Does the condition hold if $p \leq \frac{1}{2}$? Provide in words, an interpretation of this condition..
3. The police an informer that can go either to place A or to place B and to get signal s_A or signal s_B. The informer makes a report that is truthful. After the report, the police goes to the most likely place (it can go only to one place), and then catches the thief if the place is the correct one. The prior public belief, is given: $\mu = \Pr(\theta = A) > 1/2$.
4. Assume for all the next questions that the police can order the informer to get to place A or to place B. Show that that the police will only benefit from a signal which can alter their action. Show that this can happen only if $s_A = 0$ or $s_B = 1$.

5. Show that the conditions for the previous cases are, respectively

$$\left(\frac{1-p}{1-q}\right) < 1, \qquad \left(\frac{\mu}{1-\mu}\right)\left(\frac{q}{p}\right) < 1.$$

6. Using the assumption $p + q < 1$, show that a positive sighting causes a larger movement in the belief than not seeing anything.

7. Define the following cut-off values $\overline{\mu}_A$ and $\overline{\mu}_B$ by

$$\left(\frac{\overline{\mu}_A}{1-\overline{\mu}_A}\right)\left(\frac{1-p}{1-q}\right) = 1, \qquad \left(\frac{\overline{\mu}_B}{1-\overline{\mu}_B}\right)\left(\frac{q}{p}\right) = 1,$$

   and, using previous assumptions, verify that

$$\frac{1}{2} < \overline{\mu}_A < \overline{\mu}_B < 1.$$

8. Suppose that $\mu > \overline{\mu}_B$. Show that wherever the informant goes, his report has no value for the police.

9. Assume that $\bar{\mu}_A < \mu < \bar{\mu}_B$. Where should the police send the informant?