Ec 517
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Assignment 7

The exercise is a first attempt to analyze an information problem in a team of two agents, to be specific, the cockpit of an aircraft. The situation is difficult, the two pilots are busy dealing with problems or emergencies when the "stall" alarm goes on, an indication of the wrong combination of angle and speed, which requires a specific corrective response. Neither pilot takes the alarm into account and neither even talks about it. One possible explanation–perhaps–is that both pilots were busy with other issues and each pilot may have thought that if the other pilot thought the alarm was so important, he would deal with it. Since there was no response, and no talk about it, each pilot may have thought after a while that the other pilot did not response because it was a false alarm (always a possibility). Both pilots ended in an informational cascade where the signal was ignored.

We assume first one period. There are two agents, 1 and 2, and each chooses one of two actions, called here action 0 or 1. The payoff of action 0 for agent i is c_i which is the realization of a random variable with a uniform distribution on the interval $(1-\gamma, 1)$. c_1 and c_2 are independent. That action represents "dealing with current problems". The value of this activity, for each agent, is represented by c_i which increases with the importance of the problem to be dealt with. The action 1 is to respond to the alarm. The payoff of this action depends is normalized to 1 if there is a true problem that triggers has triggered the alarm and to 0 if there is no problem. We therefore define the state $\theta \in \{0, 1\}$, and the payoff of action 1 is equal to θ. The two agents have the same prior belief (probability) of the state 1, which is equal to μ. Each agent has a binary individual signal s_i such that $P(s_i = 0|\theta = 0) = 1 - \alpha$ and $P(s_i = 1|\theta = 1) = 1 - \beta$. The parameters α and β are between 0 and 1/2, and they measure the probabilities of the alarm going off wrongly or not going off when it should.

1. In all questions except the last one, agent i does not observe the parameter c_{3-i} of the other agent. Assume in this question that $\mu > 1 - \gamma$ and that there is no individual signal. Determine the optimal action of each agent.

2. Each agent now has an independent binary signal as described above. Determine the strategy of each agent for each of the two possible values of the signal, 0 and 1. Denote by $\bar{c}$ and $\underline{c}$ the values of c_i such as an agent, i,

is indifferent between the two actions when his signal is equal to 1 and 0, respectively. Show that $\underline{c} < \bar{c}$.

3. What is the condition on μ such that What is the probability π_1 that *both* agents take action 0, conditional on $\theta = 1$.

4. Show that there is $\underline{\mu}$ such that if $\mu < \underline{\mu}$, an agent with a signal 0 always chooses action 0 and $\bar{\mu}$ such that if $\mu < \bar{\mu}$, and agent with signal 1 always chooses action 0. The values of $\underline{\mu}$ and $\bar{\mu}$ are the highest ones that satisfy this property.

5. Assume that at the at the end of the first period, an outside observer sees the action of *one agent* and that agent has taken action 0. Determines the belief μ_1 (probability of state 1) of the outside observer. Show that $\mu_1 < \mu$ and provide an interpretation in words.

6. There is now an infinite number of periods. The state does not change between periods and each agent receives only *one* binary signal on the state. In each period, each agent takes action 0 or action 1 as before. The payoff of action 1 is the same in all periods but in any period t, the payoff of action 0 is c_{it} for agent i where c_{it} is drawn from the uniform distribution on $(1-\gamma, 1)$. Assume that $\mu < \bar{\mu}$ and that the difference $\bar{\mu} - \mu$ is arbitrarily small. Show that if both agents take action 0 in period 1, an informational cascade begins at the end of that period and that they never take action 1.

7. Discuss informally what may happen if $\bar{\mu} - \mu > 0$ and is not small.

8. Keeping the same assumption on μ, suppose that in the first period, c_1 and $c2$ are close to 1 and that at the end of the first period, each agent i observes the parameter c_{3-i} of the other agent. Does an informational cascade start at the end of the period? On the basis of the result, make some recommendation on whether agents should communicate their parameters (assuming they can).