Ec 517
Spring 2019
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Assignment 6 (revised)
(Learning with delays)

Question 3 may require more time than the first two for an answer and is a test of the understanding of the basic model of delay.

Consider the canonical model of delay with the state of nature is $\theta \in \{0, 1\}$, two agents in state 1 and one agent in state 0 and payoff of investment in period t equal to $\delta^{t-1}(E[\theta] - c)$. The initial belief of any agent, μ satisfies the inequality $0 < \mu - c < \delta\mu(1 - c)$. We now assume that in the first period and in the first period only, any agent is prevented from investing with some probability π. That "shock" is exogenous and 'shocks" are independent between agents and periods. We know that if $\pi = 0$, the game lasts at most 2 periods. The purpose of the present exercise is to analyze this property when $\pi > 0$.

1. Show through an intuitive argument that for some δ and some π, if in an equilibrium there is no investment after the first period, agents may still invest in period 2.
2. Assume that λ is the probability of investment of an unconstrained agent in the first period. Determine algebraically the belief $\mu^{-}(\lambda; \pi)$ of any agent at the end of the first period if there is no investment in that period. Find a necessary condition on the parameters of the model such that there an equilibrium with strictly positive probability of investment in the *second* period (a property that is different from that of the standard model with $\pi = 0$).
3. Show there exists $\bar{\pi}$ which depends on δ such that if $\pi > \bar{\pi}$, there exist values of μ and c such that in an equilibrium $0 < \lambda < 1$. Analyze that equilibrium.