Ec 517

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Assignment 5

There are two projects, X and Y. The payoff of X is equal to 0 or $2 + a$, with equal prior probabilities. a is small and positive. The payoff of Y is equal to $1 - a$ or $1 + a$ with equal prior probabilities. (The projects' return are not random and they are independent). As in the BHW model, agents, in an exogenous sequence, have the choice between two projects. Each individual has a SBS s on the project X such that $P(s = 1|x = 2 + a) = q$ with $p > 1/2$. Agents have a symmetric binary signal, H or L, of precision q on the payoff of action 1. There is no signal on the project Y. By assumption a is positive and sufficiently small (whatever will be needed to answer questions).

1. Assume that the outcome of a project that is implemented cannot be observed by subsequent individuals. Show that if the first agent has a signal H, a cascade starts in period 2. Show that if the first two signals are L and L, then a cascade starts in period 3.

2. Compare the previous model with the standard BHW model with two states for a project, payoffs 1 and 0, a SBS with precision p and a prior μ on the state 1 such that $1/2 < \mu < p$. Determine the probabilities of an eventual cascade in project X and the probability of an eventual Y cascade.

3. Suppose that the first signal is H. Determine in that case, after the signal H becomes public, the long-run expected welfare of individuals, that is the average expected value of the payoff of X for agents.

4. Answer the same question after the signal L. Call that welfare U_0.

5. Using your previous answers, determine the *ex ante* long-run expected welfare of individuals.

6. Assume now that individuals can observe the payoff of project Y at the end of the first period in which it is undertaken.

   (a) Assume the first signal is L and the output of Y is high $(1 + a)$. Determine the long-run welfare conditional on that event.

(b) Assume that the first signal is L and the output of Y is low $(1 - a)$. Show that the impact of the private signals on the individual actions are the same from that point on, as in the case where the output of Y could not be observed. Using your answer in question 2 to determine the expected long-run welfare from that point on. (Be careful on the welfare in a Y cascade).

7. Using your previous answers, determine the long-run expected welfare of individuals and show that it is smaller than in the previous question.

8. Assume now the payoff of Y is observable with one period delay. Analyze the model and show that the long-run social welfare is higher than under the condition of the previous question.