Chapter 1

Learning to coordinate

— C’est une révolte?

— Non Sire, une révolution.

In an economy with strategic complementarities, when the structure of individual payoffs evolves randomly and slowly and agents have perfect information, multiple equilibria may occur in some phases. When each agent has only imperfect information about other agents, a strong hysteresis effect takes place: the mass of agents taking one action hovers near its value in the previous period most of the time and jumps to a different level with a small probability. Applications may be found in macroeconomics and discontinuities of social opinions. In a special case of distributions, the equilibrium is unique and strongly rationalizable.

In the winter of 1989, despite of the simmering of future events in the Soviet Union, Kissinger delivered another Cold-War rhetoric in a speech to US governors (Halberstam, 1991). In the Spring and the Summer, the simmering led to ebullition with growing demonstrations in East Germany. “On October 6, 1989, the East German Socialist Unity Party (SED) celebrated the fortieth anniversary of the German Democratic Republic (GDR) with official parades and orchestrated pro-regime demonstrations. Erich Honecker, the general secretary of the SED, pro claimed the founding of the republic a ‘historical necessity’ and a ‘turning point in the history of the German people.’”¹ Twelve days later, Honecker resigned. On November 9 the Berlin Wall fell.

“Western observers were initially stunned at the speed of the economic and political collapse of the East German regime. With hindsight, however, the regime’s economic collapse seems to have been inevitable, given its outdated and obsolete industrial structure and the depleted state of its environment.”

Events that had been hard to imagine in the sphere of public information acquired an aspect of obvious inevitability. Later, “springs” of various colors, orange or green, would bring surprises in the Ukraine and in Arab countries. The subsequent fading of the flowers do not foreclose the possibility of similar surprises in the future. This chapter focuses on the mechanism by which such events can take rational agents by surprise and on the contrast between the low expectations \textit{ex ante} and a feeling of obvious determination \textit{ex post}.

Sudden and unexpected changes in political regimes, economic activity, financial crises, share a fundamental underlying property. The payoff of individuals' actions, (\textit{e.g.}, street demonstration, investment) increases with other agents taking the same action. The collective behavior generates strategic complementarities.

In the previous chapter, the coordination game with strategic complementarities took place in one period. All individuals were thinking simultaneously without learning from the past. The process of equilibrium selection between a high and a low level of aggregate activity rested on the agents’ imperfect information about others’ payoffs and the possibility that the fundamentals of the economy took “extreme values” where one action (\textit{e.g.}, investment or no investment) was optimal independently of others’ actions. In the one-period setting, all individuals were thinking by induction without the possibility of learning. Learning from the observation of others’ actions is the central issue in this chapter.

\section*{1.1 Contexts and issues}

How do business cycles, demonstrations toward a revolution and conventional discourse (what the French call “wooden speak”) share the property that the payoffs of individual action is augmented by the number of other individuals taking the same action? Three different contexts are first presented to justify a canonical model.

\textit{Business cycles}\n
The profitability of individual investment increases with the level of activity in the economy, which itself increases with the level of individual investments. This feature has been represented in models with imperfect competition by Blanchard and Kiyotaki (1987), Schleifer

\footnote{Lohmann (1994, 43).}
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In such models, more aggregate investment increases the productivity of the economy and the demand curve of each firm shifts upwards, thus generating more profits which in turn stimulate more investment by each individual firm. The strategic complementarity between individuals’ investments, if sufficiently strong, generates multiple Nash equilibria that are indicative of business cycles.

A canonical model is a simple analytical representation that focuses on a particular effect and is abstracted from the clutter of non essential features of the reality. Assume that there is a large number of agents, that is a continuum with a mass normalized to 1. There is one period and each agent has to make a 1 or 0 decision, for example whether to make a fixed size investment or not. Each agent is characterized by his own cost of investment, $c$, that is taken from a distribution with a density $f$, as represented by the graph $(f)$ in the lower panel of Figure 1.1. The cumulative distribution for that density is represented by the curve $(F)$ in the upper panel of the figure.

The positive impact on the payoff of anyone’s investment by the aggregate investment, $X$, is represented here by an increasing function of $X$. Without much loss of generality, we can assume that this function is linear. We are thus led to the payoff function for investment $x$ by an agent with cost $c$:

$$ w(x, X, c) = \begin{cases} X - c, & \text{if } x = 1, \\ 0, & \text{if } x = 0. \end{cases} \quad (1.1) $$

Suppose that all agents follow the monotone strategy to invest when their cost is less some cutoff $\bar{c}$. The value $\bar{c}$ defines the strategy. By definition of the c.d.f., the gross payoff of any investing agent is $F(\bar{c}) - c$. For an agent with a cost less than $F(\bar{c})$, investment has a positive payoff. We can thus define a reaction function. A strategy is defined as investing when the cost is less than $c$. (It is monotone). When others have the strategy $\bar{c}$, the optimal response is $F(\bar{c})$. A Nash equilibrium strategy $c^*$ is a fixed point of the cumulative distribution function: $F(c^*) = c^*$. In Figure 1.1, a fixed point is represented by an intersection of the graph of $F$ with the $45^\circ$ line. Here, there are three such points. The middle point can be discarded by a loose argument on stability. The points $L$ and $H$ represent low and high levels of aggregate activity.

Financial crises and speculative attack

Consider a bank for which the probability of bankruptcy increases with the quantity of deposits withdrawals, $X$. Using a previous argument, assume that this function is linear. Let $c$ the cost for depositors to withdraw their deposits, and for example invest them in some non linear gross payoff functions can be transformed into a linear payoff by changing the distribution of the costs $c$. 

projects of lower return. One can normalize the costs and the gross payoff from avoiding the capital loss in case of the bank failure such that the payoff for withdrawing \((x = 1)\), and not withdrawing \((x = 0)\) are given by

\[
w(x, X, c) = \begin{cases} 
X - c, & \text{if } x = 1, \\
a(1 - X), & \text{if } x = 0, 
\end{cases}
\tag{1.2}
\]

where \(a\) measures the payoff if the bank does not go bankrupt, an event with probability \(1 - X\). The payoff difference between the two actions is \((1 + a)X - c - a\). It has the same form as (1.1). A speculative attack against a central bank that manages a regime of fixed exchange rate is the formally same as an attack against a commercial bank that manages a fixed exchange rate between its deposits and the legal currency.

*The Leipzig demonstrations*

The fall of the Berlin wall was preceded by a wave of increasing demonstrations in Leipzig,
beginning in September 1989.\(^4\) Suppose that the individual benefit from participating in a demonstration (action \(x = 1\)) increases with the size of the demonstration, \(X\), and depends on the individual cost \(c\) that increases with approval for the regime. Such a payoff can be represented by the same function as (1.1). The framework that is now presented provides an analytical representation of the sudden change of beliefs from the unpredictable to the “inevitable” of the Western observers at that time.

**Social changes and revolutions**

Why do sudden changes of opinions or revolutions which were not anticipated with high probability seem anything but surprising in hindsight? This question was asked by Kuran (1995). The gap between the *ex ante* and the *ex post* views is especially striking when no important exogenous event occurs (*e.g.*, the fall of the communist regimes)\(^5\).

These social changes depend essentially on the distribution of individuals’ payoffs, on which each agent has only partial information. According to Kuran, “historians have systematically overestimated what revolutionary actors could have known”. If a revolution were to be fully anticipated, it would probably run a different course. The July 14th entry in the diary of Louis XVI was “today, nothing”\(^6\). Before a social change, individuals who favor the change do not have perfect information on the preferences of others *ex ante*, but they are surprised to find themselves in agreement with so many *ex post*, and this common view in hindsight creates a sense of determinism.

Following Kuran (1988), (1995), suppose that individuals decide in each period between two actions or “expressed opinions” as revealed by some behavior: action 1 is to speak against a given political regime, while action 0 is to speak in favor. Each individual is characterized by a preference variable \(c\) which is distributed on the interval \([0, 1]\) with a cumulative distribution function \(F(c)\). The preference for the regime increases with \(c\). There is a continuum of individuals with a total mass equal to one. For an individual with parameter \(c\), the payoff of his action \(x\) (which is either 0 or 1), is a function which is (i) decreasing in the “distance” between his action and his preference, (ii) increasing in the mass of individuals who choose the same action. For example, in talking to someone, the probability to find a person speaking against the regime increases with the mass \(X\) of people speaking against the regime. Assume that speaking against the regime yields a

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\(^4\)Other waves of demonstrations took place after the fall of the wall, See Lohmann (1994).

\(^5\)For a common view before the fall, read the speeches of H. Kissinger in Halberstam (1991).

\(^6\)However, the entry may mean “no hunting”. The quote at the beginning of the chapter is from a conversation between Louis XVI and the duke of La Rochefoucault-Liancourt. In the numerous stages of the French revolution, the actors did not seem to have anticipated well the subsequent stages, especially when they manipulated the crowds.
payoff $X - c$. Likewise, speaking for the regime has a payoff $1 - X - (1 - c)$. The difference between speaking against and speaking for is thus

$$u(c) = X - c - (1 - X) + (1 - c) = 2(X - c). \quad (1.3)$$

It has the same form as the previous utility for demonstrating.

The model of “Private Truths and Public Lies” of Kuran is thus a special case of the canonical model with strategic complementarities. For a suitable distribution of individual preferences, the model has multiple equilibria under perfect information. Kuran follows the ad hoc rule of selection and assumes that a regime stays in power as long as the structure of preferences allows it. When this structure evolves such that the regime is no longer a feasible equilibrium, society jumps to the other equilibrium regime. But it is obvious that for the analysis of sudden changes of beliefs, such an ad hoc rule in a static model, with perfect information, is not appropriate. The previous discussion points to a dynamic approach and an explicit formulation of expectations in a setting of imperfect information and learning. These features have a central place in the dynamic models of this chapter.

In such a model, we will see that until the very end of the old regime, the public information is that a large fraction of the population supports the old regime, whereas the actual distribution could support a revolution. When the regime changes, beliefs change in two ways: first, the perceived distribution of preferences shifts abruptly towards the new regime; second, the precision of this perception is much more accurate. The high confidence in the information immediately after the revolution may provide all individuals with the impression that the revolution was deterministic.

### 1.2 Analysis in a canonical model

Following the previous discussion, the canonical model is defined by the continuum of heterogeneous agents, each characterized by his cost of “investment” $c$, and the payoff function in (1.1). As with many analyses of strategic complementarities, we begin with the case of perfect information, both on the structure of the economy, and on the strategies of all agents. We then move on to imperfect information.

#### 1.2.1 Perfect information

Suppose that in some period, the actual distribution of costs is represented by the c.d.f. $F_1$ as in Figure 1.5. Under perfect information about $F_1$, there are two equilibria in monotone strategies (to act when the cost is less than some cutoff), $L_1$ and $H_1$. In a setting of
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$F_1$, $F_2$, and $F_3$ are the realizations of the c.d.f. $F$ for three periods. The c.d.f. evolves slowly between consecutive periods and agents coordinate on the equilibrium strategy that is closest equilibrium strategy in the previous period. Between the second and the third period, the strategy jumps to a higher equilibrium level.

Figure 1.2: Evolutions of a cdf (or the reaction function)

perfect information within one period, there is no criterion for choosing a high or a lower cutoff point for an equilibrium strategy. And recall that either equilibrium requires that all agents have no uncertainty on the strategies of others and coordinate on the same strategy.

**Hysteresis as a device for equilibrium selection**

Suppose that agents coordinate on the equilibrium $L_1$ and that the game is reproduced in another period with a structure of costs (the density function of the costs) that is slightly different. In the figure, the new c.d.f. is represented by the curve $(F_2)$. When agents know in period 2 that the equilibrium $L_1$ has been achieved in the first period, it is reasonable to assume that with the two possible equilibria $L_2$ and $H_2$, they choose $L_2$ which is closest to $L_1$. This selection device may be loosely defined as inertia or hysteresis. Between the two periods, a small change in the structure of the economy generates a small change in the aggregate activity.

Suppose now that the structure of economy moves a little more from $(F_2)$ to $(F_3)$: the low equilibrium vanishes and $H_3$, at a high level, is the unique equilibrium. A small change in the structure of the economy generates a large jump of the aggregate activity. After the jump, further small changes of the structure do not generate another jump. For example,
if the c.d.f. returns to \((F_1)\), the level of activity stays high at \(H_1\) and does not jump down to \(L_1\). In this setting with perfect information and the selection through inertia, when the structure evolves by small steps, the aggregate activity in any period is strongly dependent on its level in the previous period. There is \textit{hysteresis}. The aggregate activity evolves by small steps during extended regimes that are separated by large jumps.

The assumptions about perfect information on the structure and the strategies, together with the \textit{ad hoc} criterion of inertia are somewhat problematic in a setting with a large number of diverse agents. We will now see that the pattern of hysteresis and regime switches will be robust when agents have imperfect information and learn from past level of aggregate activity.

### 1.2.2 Coordination with imperfect information

The distribution of the costs is not directly observable. It is perceived by agents through probability distributions that are agent specific. These distributions are updated after the observations of aggregate activity. The distribution of costs evolves randomly by small steps from period to period. Nature does not make jumps.\(^7\) In each period, agents play a one period game under imperfect information with a payoff equal to the expected value of the payoff in (1.1). For practicality, the pool of agents is new in each period.\(^8\) As in a one-period setting with no learning from the actions of others (in a global game), imperfect information on the structure of fundamentals will enable us to solve the problem of strategic coordination.

#### Learning from activity with strategic complementarities and the tail property

Consider again the point \(L\) in Figure 1.1. That point is compatible with the functions \((F)\) and \((F_1)\). For the first, there is another equilibrium (under perfect information) with a higher activity, at the point \(H\). For the second, \((F_1)\), there is no such equilibrium. The level of activity at the point \(L\) is determined by the mass of agents with a cost lower than \(c_L\), in the left tail of the density function \(f\). When the cost \(c_L\) is low, the left tail of the distribution should, in a realistic model, provide little information on the rest of the distribution. We call this the \textit{tail property}.

The tail property is important when agents learn about the structure of an economy with strategic complementarities. In such a setting, the strategic complementarity operates like a critical mass. Either few agents take action because that critical mass is not reached and

\(^7\)\textit{Natura non facit saltus} (Leibnitz).

\(^8\)If agents live more than one period, the evolution of their cost provides additional information on the evolution of the distribution of costs, and the inference problem becomes very complex.
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the payoffs of action are low, except for these few agents, or that critical mass of active agent is reached and that is why a large mass is acting, except for the few that have a cost much higher than the average. In the present setting where agents are differentiated by their own cost of action, the strategic complementarity imposes that in an equilibrium the cutoff point of the cost for investment is in the tail of the distribution, to the left or to the right. In that case, agents learn little from the observation of others. We can thus expect that under strategic complementarity when the structure of the economy evolves randomly by small steps, the level of activity hovers around successive plateaus where little is learn, which are separated by abrupt changes that generate a large amount of information.

Modeling the essential property of leaning under strategic complementarity

In the construction of a canonical model we have to keep it simple and yet to embody the complexity of the possible states of nature that individuals face under imperfect information. For tractability, one has to choose a family of possible distributions that are indexed by some parameters. However, the reduction of the states of the world to a few parameters may also trivialize the inference problem. If for example, the distribution of the costs is normal with mean $m$ and variance $\sigma^2$, two observations of the mass in a tail of the distribution, no matter how far from the mean, are sufficient to identify perfectly the two parameters, thus providing perfect information on the distribution. That is obviously not a plausible representation.

One example of a family of distribution that keeps the tail property is presented in the Appendix, following Chamley (1999). It has the shape of a square hat with the central part moving randomly left or right. The observation of the mass in tail on the side of the “hat” provides on information on the position of the central part as long as that central part does not “bite” on the tail. The properties of the model can be investigated analytically. In particular, provided that the variance of the distribution is within some bounds, there is a unique equilibrium that is strongly rationalizable. The model is effectively a global game model. At this stage, this may be the only global game model with an infinite number of periods where the fundamental evolves random in small steps.\(^9\)

The tail property can also be modeled by the combination of a simple family of cost distributions, normal distributions with fixed variance, and the observation of aggregate activity subject to a noise with a fixed variance. In this setting, when the cutoff for taking action is very low, the mean of the fundamental distribution has a small impact on the

\[^9\] Other models with multiple periods either assume that the fundamental is subject to unbounded shocks between periods, thus generating a sequence of one period global game models (Carlsson and Van Damme \(*\*\*\*)*, or have a global game with a unique equilibrium only in the first period, after which equilibria are multiple (Angeletos and Hellwig \(*\*\*)).
mass of agents taking action, and that impact is drowned by the noise. In this case, the
observation of aggregate activity provides little information on the mean of the distribution.
When the cutoff point is near the center of the distribution, small variations of the mean have an impact that dwarfs the noise and the observation of aggregate activity is highly informative.

**Observing the activity of others through noise**

We assume that the population is the sum of two groups. In the first, the distribution of costs is normal $N(\theta_t, \sigma^2_\theta)$, where $\sigma^2_\theta$ is a publicly known constant, and $\theta_t$ follows a random walk that will be discussed below. The second is the sum of a fixed mass $a$ that always invest, and a population with a uniform distribution of costs with density $\beta$ on the interval $(b, B)$. At the end of any period $t$, agents observe the variable $Y_t$ defined by

$$Y_t = a + \beta(b + c^*_t) + F(c^*_t; \theta_t) + \eta_t, \quad \text{with} \quad \eta_t \sim N(0, \sigma^2_\eta).$$  \hspace{1cm} (1.4)

The noise $\eta_t$ may arise from imperfect data collection or from the activity of “noise agents” who act independently of the level of the aggregate activity.

Since individuals follow the strategy to invest when their cost is lower than $c^*_t$, that value is publicly known and the observation of aggregate activity is informationally equivalent to the observation of

$$Z_t = F(c^*_t; \theta_t) + \eta_t. \hspace{1cm} (1.5)$$

As discussed above, when $|c^* - \theta|$ is large, $F(c^*; \theta)$ does not depend much on $\theta$ and it is near 0 or 1. In that case, the noise $\eta$ dwarfs the impact of $\theta$ on $F(c^*; \theta)$, and the observation of $Y$ conveys little information on $\theta$. Learning is significant only if $|c^* - \theta|$ is relatively small, i.e., when the associated density function $f(c^*; \theta)$ is sufficiently high. But the strength of the strategic complementarity is positively related to $f(c^*; \theta)$ (which is identical to the slope of the reaction function under perfect information). We thus verify that *learning and strategic complementarity are positively related*. Agents only learn a significant amount of information when the density of agents near a critical point is sufficiently large to push the economy to the other regime.\(^{10}\)

\textit{“Natura non facit saltus”}\(^ {11}\)

Following the discussion around Figure 1.5, there imperfect information because the structure of (the costs in) the economy evolves randomly over time. In all known cases, aggregate

\(^{10}\)This property has a strong form in the model with a rectangular distribution that is sketched in the Appendix.

\(^{11}\)Leibnitz
productivity (the inverse of the cost) does not jump but evolves only in small steps. This restriction has an important implication for multi-period models with strategic complementarity and imperfect information.\textsuperscript{12} For computation, the mean of the distribution, $\theta_t$, is assumed to take a value on the grid
\[ \Theta = \{\omega_1, \ldots, \omega_K\} \text{, with } \omega_1 = \gamma, \omega_K = \Gamma. \] (1.6)
The distance between consecutive values is equal to $\epsilon$, which can be small. Between consecutive periods, the value of $\theta$ evolves according to a symmetric random walk: it randomly either stays constant or move to a set of a small number of adjacent grid points. If $\theta$ is on a reflecting barrier ($\gamma$ or $\Gamma$), it moves away from that barrier with some probability.

### 1.3 The behavior of the canonical model

In each period, $t$, learning and decision making proceed in the following steps.

1. Let $\{\pi_{k,t-1}\}$ be the public distribution of probabilities on the grid $\Omega$ at the beginning of period $t - 1$ when agents determined the strategy $c_{t-1}^*$. This belief is updated in two steps, first using the knowledge of the strategy in the previous period, $c_{t-1}^*$, second, using the law of the random evolution of $\theta$ between period $t - 1$ and period $t$. Using the observation of the aggregate activity $Y_{t-1}$ in the previous period, which, as we have seen, is equivalent to $Z_{t-1} = F(c_{t-1}^*; \omega_k) + \eta_{t-1}$, and Bayes’ rule, the first updating leads to the distribution $\{\pi_{k,t}\}$ with
\[ \text{Log}(\hat{\pi}_{k,t}) = \text{Log}(\pi_{k,t-1}) - \frac{(Z_t - F(c_{t}^*; \omega_k))^2}{2\sigma_0^2} + \alpha, \] (1.7)

where $\alpha$ is a constant such that the sum of the probabilities $\pi_{k,t}$ is equal to one.\textsuperscript{13}

2. The second updating, from $\{\pi_{k,t}\}$ to the public belief $\{\pi_{k,t}\}$ at the beginning of period $t$, is straightforward. For example is $\theta_t$ follows a random walk with equal probabilities of $1/3$ for staying constant, or moving up or down by one step on the grid, for all points away from the boundaries,
\[ \pi_{k,t} = (\hat{\pi}_{k-1,t} + \hat{\pi}_{k,t} + \hat{\pi}_{k+1,t})/3. \] (1.8)

\textsuperscript{12}For example, it rules out multi-period global games with an aggregate parameter that is subject to unbounded random shocks (Carlsson and Van Damme ***), and for which a new global game takes place in each period.

\textsuperscript{13}In this model, agents could use the fact that $\theta_t$ takes discrete values in order to obtain more information from the observation of $Y_t$. However, this feature is spurious. The random changes of $\theta_t$ could be defined such that the distribution of $\theta_t$ has a piecewise linear density function in every period. The previous updating formula should therefore be understood as the relevant formula for the “nodes” of the density function of $\theta_t$, (at integer values of $\theta_t$). The entire distribution of $\theta_t$ could be recovered through a linear interpolation.
3. Each agent with a cost $c$ knows that $c$ is drawn from the true distribution with mean $\theta_t$. He updates the public distribution $\{\pi_{k,t}\}$ into $\{\pi_{k,t}(c)\}$ as in (?):

$$
\log(\pi_{k,t}(c)) = \log(\pi_{k,t}) - \frac{(c - \omega_k)^2}{2\sigma^2} + \alpha',
$$

where $\alpha'$ is a constant such that the sum of the probabilities is equal to one. Note that each agent “pulls” the distribution of $\theta_t$ towards his own cost $c$.

4. Each agent computes for his own cost $c$, the cumulative distribution function (CVF). By definition of the CVF, the agent assumes that all the agents with a cost not greater than his own $c$ make the investment, of equivalently that the strategy of others is $c$. Given this assumption, the agent computes the expected value of the mass of investment according to his probability estimates of $\theta_t$. The CVF is therefore defined by

$$
V_t(c) = E[F_{\theta_t}(c)|\{\pi_{k,t}(c)\}].
$$

5. The function $V_t(c)$ is increasing in $c$. In the analytical model with a rectangular density, under some parametric conditions, this function is proven to have a slope smaller than one and its graph has a unique intersection with the $45^\circ$ line. Hence there is a unique equilibrium strategy $c^*_t$ such that $V_t(c^*_t) = c^*_t$. (This equilibrium is much stronger than a Nash equilibrium because it is strongly rationalizable).

However, the model with observational noise cannot exhibit a unique equilibrium for all values of the random noise. Suppose for example that the economy is in a low state and that the distribution of costs is such that there are two equilibria under perfect information. A very high value of the noise in some period may induce a large mass of agents to act in the next period. This could reveal a large amount of information, and generate two equilibria for the next period.

The main purpose of the model in this section is not to show that there is a unique equilibrium for all realizations of $(\theta_t, \eta_t)$. It is to show that the properties of the analytical model apply for most of these realizations: under the types of uncertainty and heterogeneity which are relevant in macroeconomics or in other contexts of social behavior, the model generates a SREE for most periods. In the numerical model below, there is a SREE in each of the 600 periods which are considered.
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The numerical example

The properties of the model are illustrated for a particular realization of the random walk of \( \theta_t \) that is represented in Figure 1.3. In the region \( \theta \leq 7 \), there is only one equilibrium under perfect information with low activity. In the region \( \theta \geq 29 \), there is only one equilibrium under perfect information, with high activity. The sum of the stationary probabilities of these two events is less than \( 1/2 \). In the simulation, the values of \( \eta_t \) are set to zero but of course, unbeknownst to the agents. Note that in the first period of the simulation in Figure 1.3, \( \theta \) is in the high region with a unique regime of low activity.\(^{14}\)

Under perfect information, if \( \theta > \theta_H \) (\( \theta < \theta_H \)), the equilibrium is unique with a low (high) level of activity. In the middle band there are two equilibria with high and low activity.

Figure 1.3: The realization of the random path of \( \theta \)

The first regime switch, from high to low, takes place in period 61. The public beliefs and the CVF just before and after the switch are represented in figure 1.4. On the left panel, the vertical line indicates the true value of \( \theta_t \) and the curve is the graph of the probability distribution of \( \theta_t \) in the public information. The right panel presents the graph of the CVF.

\(^{14}\) The parameters of the model are chosen such that the random walk is symmetric with \( p = 1/3 \), and has five independent steps within each period (which is defined by the observation of the aggregate activity). There is a mass of agents equal to 2 who have negative private costs. The first sub-population has a uniform density equal to \( \beta = 0.5 \). The other parameters are \( \sigma_\theta = 1.5 \), \( \sigma_\eta = 1 \) and \( K = 35 \). The mass of the cluster is equal to 14.
On the left panel, the vertical line indicates the true value of $\theta_{60}$ and the curve represents the probability distribution of $\theta_{60}$ according to the public information at the beginning of period 60. The right panel presents the graph of the CVF.

**Figure 1.4: Public belief on $\theta$ and CVF before and after a switch**

Just before the switch, in period 60, the public belief is completely off the mark: the actual value of the fundamental $\theta_t$ is very low while the public belief puts strong probabilities on high values of the fundamental. Because the public believes that the individual costs are
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high, the CVF is low in the range (0, 20) and there is a unique equilibrium, which in this case is a SRE, with a low aggregate activity.

Just after the switch, in period 61, the public belief has completely changed while the fundamental has barely moved. The CVF has shifted up. In the equilibrium, which is also a SRE in this case, the level of activity jumps up to a new regime.
From the observation of the activity at the point $L$, the only information is that the left border of the high density (point $A$) is to the right of $L$.

Figure 1.5: A square distribution