Ec 517
Spring 2017
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First mid-term exam (February 23)

1. There are two urns that look identical from the outside. The first contains two black balls and one white balls. It is called urn 1. The second contains two white balls and one black ball. It is called urn 2. One urn is put on a table. If it is urn i ($i = 1$ or 2), then we say that the state of nature is i. The urn 1 is put on the table with probability 2/3 and the urn 2 with probability 1/3. One ball is drawn from the urn on the table. It is a white ball. What is the probability of state 1? (The main part of your answer is your argument. The numerical answer is not the most important part your answer).

2. (a) What is the definition of a cascade in the standard model of social learning? (You will specify your standard model).
   (b) What is the difference between a cascade and a herd? You may illustrate with a short description of a model.
   (c) What is the definition of a martingale?

3. Consider a standard model of social learning in which there are two possible states of nature, say 1 and 0. There is an infinite sequence of agents and each agent t has a symmetric binary private signal s_t such that $P(s_t = \theta) = q > 1/2$. Each action is 0 or 1. The payoff of action x is $(E[\theta] - c)x$ where the expectation is conditional on the information of the agent and $1/2 < c < q$. Let μ_t be the public belief from the observation of $\{x_1, \dots, x_{t-1}\}$.
   (a) Does the belief μ_t converge when the number of observations tends to infinity? If yes, find all possible limits of μ_t.
   (b) Assume now that in each round there is a probability π (known to everyone) that the agent who takes the action is not rational: in this case, the agent invests with probability 1/2. Answer the previous question.

4. There is a standard model of social learning, 2 states $\theta = 0$ or 1, action 0 or 1, private SBS with precision $q > 1/2$, payoff of action 1 equal to $E[\theta] - c$. It is assumed that $1/2 < c < q$. The prior belief in state 1 is 1/2.
   (a) Assume that the signal of the first agent is $s_1 = 0$. What are is the action of agent 2?
   (b) A social planner would like to know the signal of agent 2 and can affect the cost of investment of agent 2 by a tax or a subsidy. (Don't worry about the use of the tax or the financing of the subsidy). What should the social planner do?

5. Consider the following story. A burglary has been committed in place A or B. The true place defines the state of the world $\theta \in \{A, B\}$. The police will investigate the more likely spot and it has an informant. The informant can look only in one place, either in place A or place B. If the informant looks in a place $i \in \{A, B\}$, his observation is like a signal s_i that takes the value 1 or 0. More specifically, he either sees something suspect ($s_i = 1$) or they see nothing ($s_i = 0$). The setting is assumed to be completely symmetric and for $i = A, B$,

$$\begin{aligned} \Pr(s_i = 1|\theta = i) &= p \\ \Pr(s_i = 0|\theta = i) &= 1 - p \\ \Pr(s_i = 1|\theta \neq i) &= q \\ \Pr(s_i = 0|\theta \neq i) &= 1 - q \end{aligned}$$

(a) It is assumed that $1/2 > p > q$. Provide, in words, a justification for each of these two inequalities.

(b) If the informer goes to place i and reports s_i, how does this report affect the belief (probability of state i) of the police?

(c) The police an informer that can go either to place A or to place B and to get signal s_A or signal s_B. The informer makes a report that is truthful. After the report, the police goes to the most likely place (it can go only to one place), and then catches the thief if the place is the correct one. The prior public belief, is given: $\mu = \Pr(\theta = A) > 1/2$.

(d) Assume that for all the next questions that the police can order the informer to get to place A or to place B. Show that that the police will only benefit from a signal which can alter their action. Show that a *necessary* condition for this when the agent is sent to A is that $s_A = 0$, and when the agent is sent to B, the condition is $s_B = 1$. Are these conditions sufficient. The answer should have no algebra.

(e) Show that the necessary and sufficient conditions for the reports from A and B to have an impact are, respectively

$$\left(\frac{1-p}{1-q}\right)\left(\frac{\mu}{1-\mu}\right) < 1, \qquad \left(\frac{1-\mu}{\mu}\right)\left(\frac{p}{q}\right) > 1.$$

(f) Define the following cut-off values $\underline{\mu}$ and $\overline{\mu}$ by

$$\left(\frac{\underline{\mu}}{1-\underline{\mu}}\right)\left(\frac{1-p}{1-q}\right) = 1, \qquad \left(\frac{\overline{\mu}}{1-\overline{\mu}}\right)\left(\frac{q}{p}\right) = 1,$$

and, using previous assumptions, verify that

$$\frac{1}{2} < \underline{\mu} < \overline{\mu} < 1.$$

(You are reminded that if $p < 1/2$, then $q(1-q) < p(1-p)$).

(g) Suppose that $\mu > \bar{\mu}$, should the policy use an informant? Explain in words.

(h) Assume that $\underline{\mu} < \mu < \bar{\mu}$. Where should the police send the informant?