EXERCISE 1.1. (Simple probability computation, searching for a wreck)

An airplane carrying "two blackboxes" crashes into the sea. It is estimated that each box survives (emits a detectable signal) with probability s. After the crash, a detector is passed over the area of the crash. (We assume that we are sure that the wreck is in the area). Previous tests have shown that if a box survives, its signal is captured by the detector with probability q.

1. Determine algebraically he probability p_D that the detector gets a signal. What is the numerical value of p_D for $s = 0.8$ and $q = 0.9$?

2. Assume that there are two distinct spots, A and B, where the wreck could be. Each has a *prior* probability of 1/2. A detector is flown over the areas. Because of conditions on the sea floor, it is estimated that if the wreck is in A, the detector finds it with probability 0.9 while if the wreck is in B, the probability of detection is only 0.5. The search actually produces no detection. What are the *ex post* probabilities for finding the wreck in A and B?

EXERCISE 1.2. (non symmetric binary signal)

There are two states of nature, θ_0 and θ_1 and a binary signal such that $P(s = \theta_i|\theta_i) = q_i$. Note that q_1 and q_0 are not equal.

1. Let $q_1 = 3/4$ and $q_0 = 1/4$. Does the signal provide information? In general what is the condition for the signal to be informative?

2. Find the condition on q_1 and q_0 such that $s = 1$ is good news about the state θ_1.

EXERCISE 1.3. (Bayes' rule with a continuum of states)

Assume that an agent undertakes a project which succeeds with probability θ, (fails with probability $1 - \theta$), where θ is drawn from a uniform distribution on $(0, 1)$.

1. Determine the *ex post* distribution of θ for the agent after the failure of the project.

2. Assume that the project is repeated and fails n consecutive times. The outcomes are independent with the same probability θ. Determine an algebraic expression for the density of θ of this agent. Discuss intuitively the property of this density.

EXERCISE 2.1.

BHW suggest that the submission process of publications may be subject to herding. Explain how herding may arise and some good papers may not be published.

EXERCISE 2.2. (Probability of a wrong cascade)

Consider the $2 \times 2 \times 2$ model that we have seen in class (2 states 1 and 0, 2 actions and symmetric binary signal), where μ_1 is the prior probability of the state 1, $c \in (0, 1)$ the cost of investment, and q the precision of the binary signal. There is a large number of agents who make a decision in a fixed sequence and who observe the actions of past agents. Assume that $\mu_1 < c$ and that the difference $c - \mu_1$ is small. Let $x_t \in \{0, \}$ the action of agent t. We assume that the true state (unknown by agents) is $\theta = 0$.

1.

2. Represent on a diagram with time (horizontal axis) and the probability of state 1 in the public information (vertical axis), different examples of trajectories of the public belief that end in a cascade with investment, which is a "wrong" cascade (since the state is 0). We want to compute the probability of all these wrong cascades.

3. What is the probability that a cascade begins immediately after $x_1 = 1$. What do agents do in that cascade?

4. Call $\mathcal{I}$ the outcome that a cascade begins in some period in which all agents take action 1. Show that the probability of $\mathcal{I}$ before the decision of the first agent, call it π_0 is the same as before the decision of the third agent after a history of actions $x_1 = 1$, $x_2 = 0$.

5. Let π_1 the probability of $\mathcal{I}$ after the history $x_1 = 1$. Determine π_1 as a function of π_0.

6. What is the probability that a cascade with investment begins after $x_1 = 0$?

7. Using the previous questions, find another relation between π_0 and π_1.

8. Determine the probability π_0 of a wrong cascade.

EXERCISE 2.3. (The model of Banerjee, 1992)

Assume that the state of nature is a real number θ in the interval $(0, 1)$, with a uniform distribution. There is a countable set of agents, with private signals equal to θ with probability $\beta > 0$, and to a number uniformly distributed on the interval