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INFORMATION REVELATION AND STRATEGIC DELAY IN A MODEL OF INVESTMENT

BY CHRISTOPHE CHAMLEY AND DOUGLAS GALE¹

We model investment as an N -player game with a pure informational externality. Each player's payoff depends only on his own action and the state of nature. However, because a player's action reveals his private information, players wait to see what other players will do. Equilibrium is inefficient because delay is costly and information is imperfectly revealed. We characterize the unique symmetric perfect Bayesian equilibrium and study the robustness of delay, which turns out to be sensitive to the reaction speed and the number of players. We establish the following results. (i) When the period length is very short, the game ends very quickly and there is a form of herding or informational cascade which results in a collapse of investment. (ii) As the period length increases, the possibility of herding disappears. (iii) As the number of players increases, the rate of investment and the information flow are eventually independent of the number of players; adding more players simply increases the number who delay. (iv) In the limit, the time-profile of investment is extreme, a period of low investment followed either by an investment surge or a collapse.

KEYWORDS: Investment, delay, informational externality.

1. INTRODUCTION

IN THE DEPTHS OF A RECESSION, it often seems as if everyone is waiting for some signal before making the decisions that, collectively, will mean the recession is at an end. In effect, everyone is waiting to see what everyone else is going to do. This might be the case, for example, if individual agents thought they could learn about the state of nature by observing the decisions of other agents. The problem is that not all agents can do this; someone has to go first. The attempt to be the last to decide leads to strategic behavior, as individuals compete for the best place in the decision-making queue.

In this paper we investigate a strategic model of investment in which individual agents time their decisions to take advantage of the information revealed by the others. The model is characterized by a pure informational externality: an agent's payoff depends on his own decision and the state of nature, but not on the actions of others. Agents take account of others' actions only because of the information revealed by them.

Because of the strategic nature of the timing decision, equilibrium must be inefficient. If agents expected all the information to be revealed quickly, they would strictly prefer to wait, no one would be willing to make the first move, and nothing would be revealed. The main lesson is that the market failure, which must be present in any equilibrium, takes one of two forms. Either there

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is *delay*, in which some information is revealed, but so slowly that impatient agents will invest without the benefit of information; or there is an investment *collapse*, which prevents information from being revealed at all. Delay and collapse are not mutually exclusive phenomena, since failure to invest can be thought of as infinite delay. Typically, both delay and collapse occur with positive probability in equilibrium.

When investment collapses, there is a positive probability that agents wrongly believe that the aggregate state is bad. They base this belief on the failure of other agents to invest and not on their private information. Models of this kind of *herd behavior* have been proposed by Banerjee (1992) and Bikhchandani et al. (1992). They show that it is rational for individuals to ignore their own information, after observing a finite number of other decisions, and imitate the "herd." However, in these models the timing of decisions is assumed to be exogenous. We assume that timing is endogenous and our results are sensitive to this assumption. For example, the extent of delay and information revelation depend crucially on the speed with which agents can react to the information revealed by the first movers. When the period length is extremely short, there is an analogue of the informational cascades of Bikhchandani et al. (1992) and the herd behavior of Banerjee (1992). As the period length increases, the probability of informational collapse declines to zero.

Another interesting property of equilibrium is a bias in the direction of underinvestment. For example, when the number of players becomes large the equilibrium path takes an extreme form, a period of negligible investment followed either by an investment surge or a collapse. However, while the surge never occurs unless investment is justified, collapse may occur even though investment is socially optimal.

In order to focus on the main properties of this process, we use a stylized model, which is related to a growing literature. Cukierman (1980) emphasized the role of the option value of delay for a discrete decision of one agent, when information is released exogenously through time. A survey of related work is provided by Pindyck (1991). Zeira (1987) and Demers (1991) show how this option value is operationally equivalent to a concave adjustment cost, as far as the determination of the level of investment is concerned. This framework has been extended to models with multiple agents whose investment decisions reveal information about the size of the market (Zeira (1990), Rob (1991)) or about the cost of production (Caplin and Leahy (1991)). These models are characterized by *strategic substitutability*: entry reduces the equilibrium price. Delay, in the form of gradual entry, is necessary in equilibrium so that firms can recover their fixed costs.

The timing of decisions in the presence of *strategic complementarities* has been studied in the macroeconomics literature under the assumption of complete information (Shleifer (1986), Chatterjee and Cooper (1989), Chatterjee, Cooper, and Ravikumar (1990), Krugman (1991), and Matsuyama (1991)). Gale (1992) contains a game-theoretic analysis of delay in the presence of strategic complementarities under complete information. This work is complementary to

the present study, but the structure of the game and the results are quite different. For example, under certain circumstances all equilibria of the game studied in Gale (1992) are approximately efficient, whereas none are in the game studied here.

Caplin and Leahy (1992) study a model in which agents receive an exogenous stream of private information over time. Agents delay the decision to abandon their projects because most of the costs are sunk, so their private information is not revealed. Unlike the present model, where the dynamics are driven by strategic behavior and the revelation of information is endogenous, the dynamics of the Caplin-Leahy model are effectively driven by the exogenous inflow of private information.

Delay is also found in models of technology adoption or the adoption of a standard (Farrell and Saloner (1985), Farrell (1987), Farrell and Saloner (1988), Bolton and Farrell (1990), Farrell (1993)). These games have a very different structure from our model. In particular, there is a first-mover advantage which has no counterpart here. In mixed-strategy equilibria, delay can occur because the advantage of being a first mover is balanced by the risk of making incompatible decisions.

In the papers cited, delay occurs for a variety of reasons, because of exogenous information flows, because of strategic complementarities or strategic substitutabilities, or because timing is exogenous and sometimes the source of delay is not clear. In this paper, we study a model characterized by pure informational externality, in which the timing of decisions and revelation of information are both endogenous, in order to clarify the strategic determinants of delay. We should also mention the paper by Mariotti (1992), which presents an example very much in the spirit of this paper.

The rest of the paper is organized as follows. Section 2 analyzes the basic decision problem and the informational value of delay. Section 3 describes the extensive form game in which agents choose the date at which to exercise their investment options. Section 4 characterizes the symmetric equilibria of this game. Section 5 studies the sensitivity of our results to the period length. Section 6 studies the behavior of the model as the number of agents increases and characterizes the limiting behavior of the equilibrium. Section 7 discusses some extensions.

2. THE VALUE OF DELAY

Imagine a situation in which there are N economic agents, a random number n of whom have an opportunity to undertake a profitable but risky investment. Whether a particular agent has an investment opportunity or *real option*, as it is sometimes called, is private information: it is only when the agent exercises his option that his private information is revealed.

Agents are ex ante identical. In particular, each player has the same probability of having an investment option. Formally, agent i 's type is a random variable t_i , where $t_i = 1$ if he has an investment option and $t_i = 0$ if he does not. The

random variables $(t_1, \dots, t_N)$ are *exchangeable*, that is, if π is any permutation of $\{1, \dots, N\}$ then

$$\text{Prob}[(t_1, \dots, t_N)] = \text{Prob}[(t_{\pi(1)}, \dots, t_{\pi(N)})].$$

Formally, this is equivalent to the following two-stage experiment: first choose the number of investment options n with probability $g_0(n)$ and then allocate them randomly among the N agents. Conditional on n , the probability that player i will receive an option is n/N , so the unconditional probability is $\sum_{n=0}^N g_0(n)n/N$. Given that agent i has received an option, the posterior probability of n is $g(n) \equiv g_0(n)n(\sum_{n'=0}^N g_0(n')n')^{-1}$, from Bayes' rule.

We assume that all investment options are identical and indivisible. The returns to exercising the options are uncertain, but correlated with the agents' information. Exchangeability implies that a sufficient statistic for the sum of the agents' information is the number of options n . Then the expected return to investment is a function $v(n)$ of the agents' information. The larger the number of options, the better the prospects for investment, so we assume that $v(n)$ is an increasing function. We also assume that the value of investment $V \equiv \sum_n g(n)v(n)$ is positive, but $v(n)$ can be negative for some values of n . To avoid certain pathological cases, we assume that $v(n) \neq 0$ for all n .

To fix ideas, consider the following concrete example. Suppose there is a finite number of states $s = 1, \dots, S$ and the true value of an investment option is v_s in state s , where $v_s < v_{s+1}$. In state s , each agent receives an option with probability π_s , where $\pi_s < \pi_{s+1}$. Then the probability of n options in state s is

$$\text{Prob}[n|s] = \frac{N!}{n!(N-n)!} (\pi_s)^n (1 - \pi_s)^{N-n}$$

and the probability of n options is

$$g(n) = \sum_s \text{Prob}[n|s] f(s) = \frac{N!}{n!(N-n)!} \sum_s (\pi_s)^n (1 - \pi_s)^{N-n} f(s),$$

where $f(s)$ is the prior probability of state s . The posterior probability of state s , conditional on n options, is therefore

$$\text{Prob}[s|n] = \frac{(\pi_s)^n (1 - \pi_s)^{N-n} f(s)}{\sum_{s'} (\pi_{s'})^n (1 - \pi_{s'})^{N-n} f(s')}.$$

Then the value of investment, conditional on n options, is

$$v(n) \equiv \sum_s \text{Prob}[s|n] v_s,$$

which is clearly increasing in n . The reader can easily construct other examples for himself.

Now consider a situation in which each agent independently chooses to exercise his investment option with probability $0 < \lambda < 1$. Those who do not have an option do nothing, of course. An agent who invests reveals that he has

an option; an agent who does not invest may or may not have one. Suppose there were an outside observer who also had an investment option. Because of the symmetry of the investors' behavior, the information revealed depends only on the total number who invest. The larger the number who invest, the larger the likely number of players with options. So the observer can increase his payoff by conditioning his decision on the number who invest.

If k is the number of agents who decide to invest, then the value of investment to the observer is

$$V(k) \equiv \sum_n g(n|k)v(n),$$

where $g(n|k)$ is the probability of n conditional on k .

LEMMA 1: *Whenever the conditional distribution $g(\cdot|k)$ is well defined, it is increasing in k in the sense of first-order stochastic dominance, if $0 < \lambda \leq 1$ and $g(\cdot)$ is nondegenerate. If $\lambda = 0$ or $g(\cdot)$ is degenerate, then $g(\cdot|k) = g(\cdot)$ whenever $g(\cdot|k)$ is defined.*

PROOF: The proposition is trivially true if λ equals 0 or 1 or the distribution $g(n)$ is degenerate, so suppose that $g(n)$ is nondegenerate and $0 < \lambda < 1$. Using Bayes' theorem,

$$g(n|k) = \frac{b(k; n, \lambda)g(n)}{\sum_{n'} b(k; n', \lambda)g(n')},$$

where the binomial coefficient $b(k; n, \lambda)$ is the probability of k successes in n independent, Bernoulli trials, with probability of success λ in each trial. This expression is valid whenever the denominator is positive.

If n and $n + m$ are in the support of $g(\cdot)$, then for any $n > k$,

$$\begin{aligned} \frac{g(n|k)}{g(n+m|k)} &= \frac{b(k; n, \lambda)g(n)}{b(k; n+m, \lambda)g(n+m)} \\ &= \frac{(n+m-k)!n!g(n)}{(n-k)!(n+m)!(1-\lambda)^m g(n+m)} \end{aligned}$$

which is decreasing in k , i.e.,

$$\frac{g(n|k)}{g(n+m|k)} > \frac{g(n|k+1)}{g(n+m|k+1)}.$$

Now compare $g(\cdot|k)$ and $g(\cdot|k+1)$. For $n = 1, \dots, k-1$, $g(n|k) = g(n|k+1) = 0$ and for $n = k$, $g(n|k) > 0 = g(n|k+1)$ if n is the support of $g(\cdot)$. Further-

more, for any $n > k$ in the support of $g(\cdot)$, $g(n|k) \leq g(n|k+1)$ implies that

$$\frac{g(n+m|k)}{g(n+m|k+1)} < \frac{g(n|k)}{g(n|k+1)} \leq 1.$$

Therefore, the two density functions $g(\cdot|k)$ and $g(\cdot|k+1)$ have the single-crossing property: $g(n|k+1) \leq g(n|k)$ if $n \leq n_0$ and $g(n|k+1) > g(n|k)$ if $n > n_0$. As is well known, this implies the conclusion of the proposition. *Q.E.D.*

An immediate corollary is that $V(k)$ is increasing in k , unless it is defined for only one value of k , which only occurs when $g(n)$ is degenerate or λ equals 0.

The outside observer's maximum payoff is $W(\lambda) \equiv \sum_k p(k) \text{Max}\{V(k), 0\}$, where $p(k) = \sum_n b(k; n, \lambda) g(n)$ is the probability of k investments. Contrast this with the situation of an agent who makes his decision before observing the other investors' decisions. It is optimal for him to invest if and only if $V \equiv \sum_n v(n) g(n) \geq 0$. Note that $V = \sum_k p(k) V(k)$, so from the definition of $W(\lambda)$ it is obvious that $W(\lambda) \geq V$. Further, it follows that $W(\lambda) > V$ if and only if $V(k)$ is negative for some values of k . The difference, $W(\lambda) - V$, is the *option value* of waiting to see the investors' decisions before making an investment decision. The option value is positive if and only if the observer's investment decision depends in a nontrivial way on the value of k .

We can also see that when there is a positive option value, the observer will invest only if at least one other agent has invested, i.e., $k \geq 1$. To see this, recall that $V(k)$ must be negative for at least one value of k . Since $V(k)$ is increasing, this implies $V(0) < 0$. We sum up what we have learned:

PROPOSITION 1: *If $g(n)$ is nondegenerate and $0 < \lambda < 1$, then $V(k)$ is increasing; otherwise $V(k)$ is a constant function. If $W(\lambda) > V \geq 0$ there exists $0 < k^* \leq N$ such that $V(k) < 0$ if $k < k^*$ and $V(k) \geq 0$ if $k \geq k^*$. In particular, the observer will invest only if at least one other agent has invested.*

In a precise sense, λ measures the amount of information revealed by the number of agents investing. For example, if $\lambda = 0$ then no information is revealed; if $\lambda = 1$ then the true value of n is revealed. As λ increases from 0 to 1, the amount of information revealed increases.

Consider two different probabilities of investment $\lambda' < \lambda$. Let X and X' denote the random number of agents who invest when the probabilities are λ and λ' , respectively. We can think of the random variable X' as being generated by the following experiment. First, let n agents randomize with probability λ ; then take the k successful investors from the first group and have them randomize with probability λ'/λ . The result is a series of n Bernoulli trials with probability of success in each trial equal to λ' . What this shows is that X' is equal to X plus "noise." In other words, X is more informative than X' in the sense of Blackwell. It is immediate from Blackwell's theorem (Cremer (1983)), that $W(\lambda') \leq W(\lambda)$ whenever $\lambda' \leq \lambda$. This can be strengthened to a strict inequality if the option value is positive.

PROPOSITION 2: $W(\lambda)$ is increasing in λ , whenever $W(\lambda) > V$.

PROOF: From Proposition 1, for some number $k > 0$, the expected return to investment is positive when $X = k$ and negative when $X = 0$. Now when $X' = 0$ both $X = k$ and $X = 0$ might have occurred with positive probability. So the value of observing X is strictly higher than the value of observing X' . *Q.E.D.*

3. A GAME OF TIMING

We are now ready to model the more complex situation in which each of the N agents is able to choose either to invest now or to delay his decision in order to observe the others' actions. This environment is very different from the decision problem discussed in Section 2, where only the outside observer could make use of the information revealed by the agents' decisions. However, with some adjustments, we can apply the results of Section 2 to the analysis of the more complex environment.

We begin by defining an extensive form game. There are N players, n of whom have an investment option. The option can be exercised at any one of a countable number of dates $t = 1, 2, \dots, \infty$. All players discount the future using a common discount factor $0 < \delta < 1$. Thus, if there are n players with options and a player invests at date t , his payoff is $\delta^{t-1}v(n)$. If a player never invests, his payoff is 0. The players without options remain passive throughout the game.

The assumptions introduced in Section 2 are retained throughout the rest of the paper.

Equilibrium

A player's type is private information, but his actions are publicly observed. Denote the i th player's action at date t by x_{it} , where $x_{it} = 1$ if player i invests at date t and $x_{it} = 0$ if he does not. Let $x_t \equiv (x_{1t}, \dots, x_{Nt})$ denote the *outcome* at date t , that is, the vector of decisions made by the players. The *history* of the game at t is a sequence of outcomes $h_t = (x_1, \dots, x_{t-1})$. The set of possible histories at t is denoted by H_t , with the restriction that in any possible history $h_t \in H_t$ a player is allowed to invest at most once. Let $H_1 = \{\emptyset\}$ denote the initial history and $H = \bigcup_{t=1}^{\infty} H_t$, the set of all histories.

In the sequel, we look only at *symmetric* equilibria, in which a player's actions depend only on his type and on the publicly observed history of the game. In a symmetric equilibrium, it is only the *number* who invest at each date that matters, so without essential loss of generality we can write a history $h \in H_t$ as a sequence $(k_1, \dots, k_{t-1})$, where k_s is the number of players who invested at date s . Since players without options are passive, we only need to describe strategies and beliefs for the players with options. For any history h , let $\lambda(h)$ denote the probability that a player who has not yet exercised his option does so after observing the history h . Then a *behavioral strategy* is a function $\lambda: H \rightarrow [0, 1]$.

Since a player only knows his own type, this is a game of incomplete information. Players' beliefs about other agents' options must be specified as part of the equilibrium. A *probability assessment* for a player is a function $\mu: H \times N \rightarrow [0, 1]$ with the interpretation that $\mu(n|h)$ is the probability that n players have options, conditional on the history h .

A *perfect Bayesian equilibrium* (PBE) consists of a strategy λ and a probability assessment μ such that (i) each player's strategy is a best response at every information set and (ii) the probability assessments are consistent with Bayes' rule at every information set that is reached with positive probability.

4. CHARACTERIZATION OF SYMMETRIC PBE

The *equilibrium path* E is the set of histories that occur with positive probability in equilibrium. In this section, we characterize E for a fixed but arbitrary symmetric PBE (λ, μ) .

We begin by establishing an important property of symmetric PBE, which we call the *one-step property* (OSP), but first some additional notation is needed. In Section 2, we let $V(k)$ denote the payoff from immediate investment after k other players had invested and let $W(\lambda)$ denote the undiscounted payoff from waiting one period before deciding whether to invest. In a natural extension of this notation, let $V(h)$ denote the payoff from immediate investment at the information set h and let $W(\xi, h)$ denote the *undiscounted* payoff from waiting one period at the information set h and then making an irrevocable decision, when other players invest with probability ξ . Let (h, k) denote the information set reached if k players invest after history h . In this notation, we should now write $V(\emptyset)$ for V , $V(\emptyset, k)$ for $V(k)$, $W(\lambda, \emptyset)$ for $W(\lambda)$, and so on. It is also useful to let $W^*(h)$ be the equilibrium payoff from waiting at the information set h .

Suppose a player waits at h and makes a once-for-all decision either to invest at (h, k) or never to invest. His payoff at (h, k) will be $\text{Max}\{V(h, k), 0\}$, so the payoff from waiting will be $\sum_k p(k|h) \text{Max}\{V(h, k), 0\}$, where $p(k|h)$ is the distinguished player's probability assessment that k others will invest at h . We say that the *OSP is satisfied at h* if

$$W^*(h) = W(\lambda(h), h) \equiv \sum_k p(k|h) \text{Max}\{V(h, k), 0\},$$

in other words, it is optimal for a player who waits at h to make a once-for-all decision next period. Note that the OSP does not claim that a player who waits at h must make a once-for-all decision at the next date; only that it would be optimal for him to do so.

The next proposition shows that the OSP is satisfied at every information set that is reached with positive probability. The key observation in the proof of the proposition is that $V(h, k) > 0$ implies $\lambda(h, k) > 0$. Either it is strictly optimal to invest at (h, k) or else $V(h, k) \leq 0$ and no one chooses to invest. In the latter case, because no information is revealed, this becomes an absorbing state and no one ever invests.

PROPOSITION 3: For any fixed but arbitrary symmetric PBE (λ, μ) , the OSP holds at any information set $h \in E$.

PROOF: If $\lambda(h, k) > 0$, it is obviously optimal for the player to invest at the information set (h, k) so we need only consider the case where $\lambda(h, k) = 0$.

(i) If $V(h, k) > 0$ and $\lambda(h, k) = 0$, the player must plan to invest at some future information set h' . Let $h' = (h, k, 0, \dots, 0)$ be the first information set following (h, k) at which $\lambda(h') > 0$. Since the probability of investment is 0 between (h, k) and h' , the information available at h' is the same as at (h, k) and $V(h') = V(h, k) > 0$. Because of discounting, a player who invests at h' would have been better off investing at (h, k) , contradicting $\lambda(h, k) = 0$.

(ii) Suppose $V(h, k) < 0$. If $h' = (h, k, 0, \dots, 0)$ and no information has been revealed since (h, k) then $V(h') = V(h, k) < 0$ and $\lambda(h') = 0$. This proves by induction that it is never optimal to invest at any information set $h' \in E$ following (h, k) .

(iii) In the intermediate case, $V(h, k) = 0$, it is also true that no one ever invests. To see this, let $h' = (h, k, 0, \dots, 0)$ be the first information set following (h, k) at which $\lambda(h') > 0$. Since no information has been revealed since (h, k) , $V(h') = V(h, k) = 0$. There must be some uncertainty about the true value of n , since $v(n) \neq 0$ for all n . Then $\lambda(h') > 0$ and $V(h') = 0$ imply there is a positive option value and players would strictly prefer to wait, a contradiction. *Q.E.D.*

The OSP is important because it allows us to use the analysis from Section 2 to characterize the equilibrium path. Using the OSP, we can immediately calculate

$$\begin{aligned}
 W(\xi, h) &= \sum_{k=0}^{N-1} p(k|h, \xi) \text{Max}\{V(h, k), 0\} \\
 &= \sum_{k=0}^{N-1} p(k|h, \xi) \text{Max}\left\{\sum_{n=0}^N p(n|h, k, \xi) v(n), 0\right\} \\
 &= \sum_{k=0}^{N-1} p(k|h, \xi) \text{Max}\left\{\sum_{n=0}^N \frac{p(n, k|h, \xi)}{p(k|h, \xi)} v(n), 0\right\} \\
 &= \sum_{k=0}^{N-1} \text{Max}\left\{\sum_{n=0}^N p(n, k|h, \xi) v(n), 0\right\} \\
 &= \sum_{k=0}^{N-1} \text{Max}\left\{\sum_{n=0}^N b(k; n - K(h) - 1, \xi) \mu(n|h) v(n), 0\right\},
 \end{aligned}$$

where $p(k, n|h, \xi)$ is the distinguished player's probability assessment of k investments and n options, given the history h , and $K(h)$ is the number of

players who have invested previously. From the results in Section 2, we know that $W(\xi, h)$ is continuous and increasing in ξ whenever $W(\xi, h) > V(h) > 0$.

Now we can use these facts to characterize the equilibrium path.

PROPOSITION 4: *Let (λ, μ) be a fixed but arbitrary symmetric PBE. After any history $h \in E$, one of the following mutually exclusive situations occurs:*

- (a) $V(h) \leq 0$ and $\lambda(h) = 0$.
- (b) $V(h) \geq \delta \sum_n \mu(n|h) \text{Max}\{v(n), 0\} > 0$ and $\lambda(h) = 1$.
- (c) Neither (a) nor (b) apply. In that case, $0 < \lambda(h) < 1$ is the unique value such that $V(h) = \delta W(\lambda(h), h) = \delta W^*(h)$.

PROOF: It is clear that one of three mutually exclusive situations arises in any symmetric PBE after any history $h \in E$:

(a) $V(h) \leq 0$. In this case, the claim follows by the argument given in the proof of Proposition 3.

(b) $V(h) \geq \delta \sum_n \mu(n|h) \text{Max}\{v(n), 0\} > 0$. From the equilibrium conditions, if $\lambda(h) < 1$ Proposition 3 implies $\delta W(\lambda(h), h) \geq V(h)$. Then Proposition 2 implies that $W(1, h) > W(\lambda(h), h)$, contradicting $\delta W(1, h) \leq V(h)$.

(c) Neither (a) nor (b) apply. In this case, $\delta W(1, h) > V(h) > \delta W(0, h)$, since $W(0, h) = V(h) > 0$. Continuity implies there exists a solution to $\delta W(\xi, h) = V(h)$ and the solution is unique since $W(\cdot, h)$ is increasing whenever $W(\xi, h) > V(h)$.

Q.E.D.

Proposition 4 characterizes the equilibrium path by dividing it into three cases. For sufficiently optimistic beliefs (case (a)), all players immediately invest and the game ends. For sufficiently pessimistic beliefs (case (b)), no one is willing to invest and, since no information is revealed, this is also an absorbing state. For intermediate beliefs (case (c)), players are indifferent between investing and waiting and randomize between the two. The proposition can be used to define the equilibrium path recursively. Given any history $h \in E$, define the probability assessment $\mu(h)$ using Bayes' theorem and then calculate the value of $V(h)$. Use the conditions of Proposition 4 to define $\lambda(h)$ uniquely.

THEOREM 1: *There exists a unique symmetric PBE outcome.*

5. THE EFFECT OF PERIOD LENGTH ON DELAY

We say that a symmetric PBE *exhibits delay* if players who have a positive payoff from investment choose not to invest, i.e., if $V(h) > 0$ and $\lambda(h) < 1$, for some history h that is reached with positive probability.

Suppose that a player could obtain complete information by delaying his investment one period after the start of the game. Then his equilibrium payoff at the second date must be at least $W(1, \emptyset) \equiv \sum_N g(n) \text{Max}\{v(n), 0\}$, where $g(n)$ is the probability assessment of an active player at the first date. Let δ_1 be

defined implicitly by

$$V(\emptyset) = \delta_1 W(1, \emptyset)$$

where $V(\emptyset) \equiv \sum_n g(n)v(n)$ is the value of investment to a player at the first date.

PROPOSITION 5: *Any symmetric PBE must exhibit delay if $\delta > \delta_1$. No equilibrium will exhibit delay if $\delta < \delta_1$.*

PROOF: The proof is immediate from Proposition 4 and the observation that $V(\emptyset) < \delta W(1, \emptyset)$ if and only if $\delta > \delta_1$. Q.E.D.

The length of a time period measures the reaction lag between the release of information at one date and its incorporation in decisions at the next. This lag may be substantial if, for example, it takes “time to build” and decision makers are responding to physical evidence of investment decisions. Letting τ denote the length of a time period and ρ the rate of time preference, the discount factor δ is equal to $e^{-\rho\tau}$. Let τ_1 be the period length corresponding to δ_1 . Then Proposition 5 says that there will be delay if $0 < \tau < \tau_1$ but not if $\tau > \tau_1$, so lengthening the period can reduce delay. However as the period length τ becomes very short, the delay also becomes very short, as the next proposition shows.

PROPOSITION 6: *In any symmetric PBE all investment ends after at most N periods. Thus, as $\tau \rightarrow 0$, the length of the game converges to zero.*

PROOF: Propositions 1 and 4 tell us that, in order to keep investment going, at least one player must invest at each date. So the game lasts at most N periods.

Obviously, the finiteness of the number of players is crucial in this proof. A different argument is needed when the number of players is unbounded (see Proposition 9 in Section 6).

Proposition 6 identifies conditions under which the game ends quickly. Players make their decision either to invest or else never to invest within an arbitrarily short time after the start of the game. Since the decision never to invest is equivalent to infinite delay, it is not strictly correct to say that Proposition 6 implies there is no delay. Rather it is the process of investment and information revelation which ends almost instantaneously when the period length is vanishingly short.

Although the game ends quickly, the outcome is not efficient. In fact, the information revealed must be imperfect or the first movers would be unwilling to invest. The imperfection takes the form of a collapse of investment: with positive probability players stop investing, even though the true return to investment is positive. Thus, when the period length is vanishingly small, the model produces something like the herd behavior of Banerjee (1992) or the informational cascades of Bikchandani et al. (1992). As the period length

increases toward τ_1 , the probability of herd behavior and informational cascades decreases toward zero. Endogenous timing clearly matters; it determines whether we get an “informational collapse” or not.

6. DELAY WITH LARGE NUMBERS OF PLAYERS

In this section, we study the asymptotic properties of PBE as the number of players becomes unboundedly large. There are two reasons for studying this case. First, certain results become sharper when the number of players is large (see the discussion in Section 7). Secondly, the large numbers case clarifies the equilibrium relationship between the rate of investment and the rate at which information is released. The rate of investment is determined by the condition that the time-cost of delay is just balanced by the option value, independently of the number of players.

The simplest way to increase the number of players, *ceteris paribus*, is to replicate the game. The *r-fold replica* of the game is denoted by Γ^r and consists of rN players. The number of players with investment options is nr , where n is distributed according to $g(n)$. The random variables $\{t_i\}_{i=1}^{nr}$ are assumed to be exchangeable. In other respects, the game is the same as before.

For any game Γ^r , we can describe a symmetric PBE by an ordered pair (λ, μ) . However, as the number of players changes, the set of histories changes and hence the set of beliefs and strategies also changes. To avoid this nuisance, we introduce a description of the equilibrium path which is independent of the number of players. Recall that the game effectively ends at any information set where $\lambda(h) \in \{0, 1\}$. To analyze the properties of equilibrium, we only need to pay attention to the *active phase*, that is, the set of histories in which $0 < \lambda(h) < 1$.

Let (λ, μ) be a fixed but arbitrary symmetric PBE for the game Γ^r . We know from the analysis in Section 4 that for any information set h in the active phase of the equilibrium, the *state* of the game can be characterized by the number of players K who have already invested and the beliefs $m = \mu(h)$ of the remaining players. The evolution of the game in the active phase is described by two functions:

$$\begin{aligned}\lambda(h) &= F(\mu(h); r, \delta, K), \\ \mu(h, k) &= B(k; \lambda(h), \mu(h), r, K),\end{aligned}$$

where k is the number of players who invest at the information set and $B(\cdot)$ represents the posterior probability assessment after the observation of k . What matters for the analysis that follows is the function F . From the analysis of Sections 2 and 4, we know that F is well-defined (in the active phase), continuous, and solves the equation

$$(F) \quad \sum_n m(n)v(n) = \delta \sum_k \text{Max} \left\{ \sum_n b(k; m - K, \lambda)m(n)v(n), 0 \right\},$$

where m is a probability assessment and we abuse notation by writing λ for a player's probability of investment, rather than a strategy.

Different stages of the equilibrium path are characterized by different values of the probability assessments m , corresponding roughly to the three cases in Proposition 4. Let Δ^N denote the $N-1$ dimensional simplex and let $n^* \equiv \sup\{n | v(n) < 0\}$. Define the sets

$$U = \{m \in \Delta^N | m(n) = 0, \forall n \leq n^*\},$$

$$L = \{m \in \Delta^N | m \cdot v \leq 0\},$$

and

$$M = \Delta^N \setminus (U \cup L).$$

If $m \in L$, no players will invest. If $m \in U$, then a player will invest immediately, since there is no gain from acquiring additional information. If $m \in M$, $V(m) > 0$ and $F(m; r, \delta, K) > 0$ (see the proof of Proposition 3). However, for a given discount factor δ , when $m \in M$ approaches the boundary between M and U , the option value of waiting tends to 0 and it becomes strictly optimal for a player to invest, in which case $F(m; r, \delta, K) = 1$ and the game stops. To ensure the game remains in the active phase, we restrict attention to a subset M_ε of M which excludes a neighborhood of the boundary between M and U :

$$M_\varepsilon = \left\{ m \in M \mid \sum_{n \leq n^*} m(n) \geq \varepsilon \right\},$$

for any fixed $\varepsilon > 0$. For any $\varepsilon > 0$ there exists $\eta(\varepsilon) > 0$ such that

$$\delta W(1, m; r, \delta, K) = \delta \sum_{n > n^*} m(n)v(n) > \sum_n m(n)v(n) = V(m),$$

for any $\delta > 1 - \eta(\varepsilon)$ and $m \in M_\varepsilon$. This implies that $F(m; r, \delta, K) < 1$ and we have proved the following result.

LEMMA 2: *For any $\varepsilon > 0$, there exists $\eta(\varepsilon) > 0$ such that for all $m \in M_\varepsilon$, for all (K, r) and for all $\delta > 1 - \eta(\varepsilon)$, $F(m; r, \delta, K)$ is well defined and $0 < F(m; r, \delta, K) < 1$.*

The amount of information revealed in equilibrium is related to the expected number of investors at each date. This is roughly measured by the rate of investment, which we identify with $r\lambda$. If $r\lambda$ becomes too large, too much information is revealed and players would rather wait than invest. If $r\lambda$ becomes too small, not enough information is revealed and they would rather invest than wait. Equilibrium requires that the rate of investment be bounded above and bounded away from 0, as long as $m \in M_\varepsilon$.

PROPOSITION 7: *For any fixed but arbitrary K , there exist numbers $0 < \beta < \bar{\beta} < \infty$ such that $r\lambda \in [\beta, \bar{\beta}]$ whenever $(\lambda, m, r, \delta) \in [0, 1] \times \Delta^N \times \mathbb{N} \times [0, 1]$, $\bar{\lambda} = F(m; r, \delta, K)$, $m \in M_\varepsilon$, and $\delta > 1 - \eta(\varepsilon)$.*

The proof is contained in the Appendix.

Using Proposition 7, we can go further and characterize the behavior of investment in the limit, when the number of players is infinite. Let δ, m , and K be fixed but arbitrary and let $\lambda = F(m; r, \delta, K)$ for each value of r . Proposition 7 shows that if $m \in M_\varepsilon$ and $\delta > 1 - \eta(\varepsilon)$ then $r\lambda$ is bounded away from 0 and ∞ as $r \rightarrow \infty$. Along some subsequence, $r\lambda \rightarrow \beta$, say. We call β the *propensity to invest*. In the limit, the number of players investing at this information set (m, K) is given by the Poisson approximation to the binomial distribution with parameter β (Billingsley (1986, pp. 312–314)). Then the probability that k players invest at the information set (m, K) is given by

$$b(k; \beta_n) = e^{-\beta_n} \frac{(\beta_n)^k}{k!},$$

where $\beta_n \equiv \beta n/N$. For any finite value of β , $b(\cdot; \beta_n)$ is a well defined probability distribution, even though the number of players is unbounded in the limit. It shows that the number of players investing at any date is almost surely finite.

Using the Poisson approximation, the equilibrium condition in the region $m \in M_\varepsilon$ can now be written

$$V(m) = \delta W^\infty(\beta, m),$$

where

$$W^\infty(\beta, m) \equiv \sum_{k=0}^{\infty} \text{Max} \left\{ \sum_{n=0}^N m(n) v(n) e^{-\beta_n} \frac{(\beta_n)^k}{k!}, 0 \right\}.$$

This equation can be analyzed in the same way as the finite equilibrium condition.

PROPOSITION 8: *For any value of $m \in M_\varepsilon$, there is a unique solution $\beta = F^\infty(m)$ to the equation (F^∞) .*

The proof is in the Appendix.

This allows us to strengthen the conclusion of Proposition 7. For any convergent subsequence $\{r\lambda\}$ the limiting value of $r\lambda$ is $\beta = F^\infty(m)$, so the original sequence $\{r\lambda\}$ is convergent and has β as its limit. As $r \rightarrow \infty$, the limiting distribution of investment and the information flow are both governed by the equilibrium condition that the time-cost of delay equal the option value:

$$(1 - \delta)V(m) = \delta\{W^\infty(\beta, m) - V(m)\}.$$

The solution of this equation is independent of both K and r . Intuitively, this is because as long as $m \in M_\varepsilon$, the rate of investment is negligible relative to the size of the economy, so we are effectively sampling with replacement.

To sum up, we have established the following properties for the limiting case when the number of players becomes very large. (i) The rate of investment is determined by the condition that the option value equal the time-cost of delay. (ii) In particular, the rate of investment is independent of the number of

players. (iii) Therefore, since increasing the number of players has no effect on the rate of investment, it must increase the number who wait by an equal amount. (iv) Finally, since the rate of investment is finite, the time-profile of investment has an extreme form: initially, there is a period with negligible investment ($m \in M_e$), followed either by an investment surge ($m \in U$) or an investment collapse ($m \in L$).

We can also extend Proposition 6 to the case when the number of players is very large. Since the proof of Proposition 6 depends crucially on having a finite number of players, it is interesting to see whether it continues to hold when the number of players is unbounded. It does, but a different argument is required.

PROPOSITION 9: *For any $\varepsilon > 0$ there exists an $\eta > 0$ such that if $\delta > 1 - \eta$, then with probability $1 - \varepsilon$, $\mu(t) \notin M$ for some $t \leq \varepsilon$, where $\mu(t)$ is the equilibrium probability assessment at time t .*

The proof is contained in the Appendix.

7. DISCUSSION

7.1. Welfare

In equilibrium, there is no gain from information revelation. Because the equilibrium is symmetric, every player's payoff is equal to V , the expected return to investment when there is no public information.

In the first best, n would be revealed at the first date and all agents would immediately make the optimal investment decision. A simple policy to overcome the informational externality consists of paying a lump sum transfer S to any player who invests in the first period. The payment S is chosen so that the payoff to investment in period 1 is just slightly less than the value of delay *under the assumption that the true value of n is revealed in the first period*. That is, S is chosen so that

$$\sum_n v(n)g(n) + S = \sum_n \text{Max}\{v(n), 0\}g(n) - \varepsilon.$$

By the usual arguments, we can show that the equilibrium probability of investment will result in a large amount of information being revealed in the first period. The details are left to the reader. When r is very large, the proportion of players investing will be small, so the total subsidy to investment is relatively small. As ε becomes vanishingly small and the period length τ becomes very short, the first best is approached. With finite values of r and τ there will always remain some inefficiency.

One can think of this as a kind of countercyclical fiscal policy. It provides some rationale for temporary, pump-priming measures. In fact, it is essential for the investment subsidy to be temporary; otherwise it would not increase the marginal cost of delay.

7.2. *Asymmetric Equilibria*

We have only studied symmetric equilibria, which seem the natural candidates when coordination is an issue. Asymmetric equilibria may have different properties. The following example illustrates the possibilities. Suppose that the payoff function $v(n)$ is defined by

$$v(n) = \begin{cases} 1 & \text{if } n = N, \\ -1 & \text{if } n < N. \end{cases}$$

For simplicity, suppose that the players' types are i.i.d. and the probability of getting an option is $0 < p < 1$. Consider the following *asymmetric* equilibrium. Player i makes a decision at date i : if he gets an option and all preceding players have invested, he invests; otherwise, he does not. To show that this strategy defines a PBE, it is only necessary to check that no player wishes to invest before his allotted time. Player i 's equilibrium payoff given that $i - 1$ players have already invested is

$$\pi_i = \delta^{i-1} [p^{N-i} - (1-p)^{N-i}].$$

Suppose that $i - 1$ players have invested. If player $i + 1$ invests at $t = i$, he receives the same payoff as player i . If player $i + 1$ waits, then with probability p player i invests and player $i + 1$ receives π_{i+1} ; with probability $1 - p$ player i does not invest and player $i + 1$ gets 0. Then we need to show that $\pi_i \leq p\pi_{i+1}$, which will be satisfied for all $i = 1, \dots, N - 1$ if and only if it is true for $i = N - 1$, i.e.,

$$2p - 1 \leq \delta p.$$

Let N get large, holding constant the probability that there are N optimists, i.e., holding p^N constant. Then we must let τ approach 0 in order to satisfy the inequality above. However, it can be shown by a rather tedious calculation that $N\tau$ is bounded away from 0. In other words, there is nonnegligible delay, even when the period length converges to 0, if we can increase the number of players at the same time.

Clearly, the example is special—we allow the probabilities to change as $N \rightarrow \infty$ —but it is important to think about whether symmetric equilibria are special too.

The results in Section 2 are all independent of symmetry, in the limited sense that analogues exist for the general case. The OSP does not hold for asymmetric equilibria, which makes them much more difficult to analyze, but analogues of the most important results of Section 5 continue to hold, since they depend only on results from Section 2. The asymptotic analysis of equilibrium as the number of players becomes unboundedly large does make use of symmetry in an essential way. Although we have a strong belief that the example of delay given above is nonrepresentative, the behavior of asymmetric equilibria when there are large numbers of players remains an open question.

7.3. Underinvestment

There is an asymmetry in the form of equilibrium, which comes out very clearly when the number of players is large and $\delta = 1$. In this case, all players make a decision either to invest or never to invest very soon after the start of the game. However, the rate of investment remains negligible until there is almost no option value from waiting. In other words, most players do not invest until they are almost certain the return to investment is positive. On the other hand, equilibrium is not efficient so there must be a nonnegligible probability of error. This error occurs only in the negative direction: the probability that a large number of players fails to invest when the return is positive is bounded away from zero. So the investment process seems to have a constitutional weakness in the direction of underinvestment.

7.4. Noise

We have assumed that all players have perfect information, that is, they know precisely who has exercised his investment option. In many contexts, it seems appropriate to introduce noise. Would that change the results in a significant way here? Not necessarily. The easiest way to introduce noise is to assume that there is a fixed probability $0 < \theta < 1$ that investment by player i is observed by the other players. Then the analysis of Section 2 tells us that the equilibrium conditions will be satisfied if the investment strategy $\lambda(h)$ of the original equilibrium is replaced by $\lambda'(h) = \lambda(h)/\theta$. In other words, noise increases the probability of investment, but otherwise leaves the equilibrium unchanged. In particular, the amount of information revealed remains the same.

Other, possibly more realistic, methods of introducing noise will be harder to analyze, but it appears that they will have a qualitatively similar effect.

7.5. Continuous Actions

Gul and Lundholm (1992) study a model in which agents delay, even though they make decisions in continuous time, the analogue of letting $\tau \rightarrow 0$. The reason appears to be that their agents choose actions from an interval. Why does this make a difference? In our model, a positive option value is necessary for delay. The option value is positive only if an agent's decision depends in a nontrivial way on the information revealed. In any period, investment must stop for some values of k . In other words, the amount of information revealed in each period must be large in order to change an agent's decision from 0 to 1.

When an agent chooses a continuous decision variable, on the other hand, he can respond to a small amount of new information by making a small change in the variable. Even small amounts of information have a positive option value, so it is possible to keep the process going with only a small amount of information being released each period. The process can continue like this for a long time.

The comparison of the two models suggests that indivisibilities may be important for investment collapse, at least in this context.

In the context of a dynamic, competitive economy, Vives (1993) also studies information revelation when agents make choices from a connected set, the commodity space, and shows that all information is eventually revealed.

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APPENDIX

A.1. Proof of Proposition 7

The proof is by contradiction and proceeds in two steps.

Step 1: Suppose first that along some subsequence, which we take to be the original sequence, $r\lambda \rightarrow 0$. In this case, too little information is revealed and players would rather invest immediately. Without essential loss of generality, we can assume that we are at the first date and $K = 0$. The probability that no one invests is $(1 - \lambda)^{nr}$ which converges to 1 as $r \rightarrow \infty$. To see this, note that

$$\log(1 - \lambda)^{nr} \approx nr(-\lambda) \rightarrow 0.$$

From Proposition 1, if no one invests today it will not be optimal to invest tomorrow, so the payoff from delay is less than or equal to $(1 - (1 - \lambda)^{nr})\text{Max}_n v(n) \rightarrow 0 < V(m)$, a contradiction.

Step 2: Now suppose that along some subsequence, which we take to be the original sequence, $\lambda r \rightarrow \infty$. In this case, too much information is revealed and players would rather wait. Consider the strategy of waiting one period and investing if and only if the number of investors at the first date is at least $k + 1$, where k is a parameter to be determined as a function of λ and r . Let $S(nr, \lambda)$ denote the sum of nr independent Bernoulli random variables that take the value 1 with probability λ and take the value 0 otherwise. Where there is no risk of ambiguity, we shall indicate this random variable by S^{nr} . The payoff to this investment strategy is

$$\delta \sum_n \text{Prob}[S^{nr} > k] m(n) v(n).$$

Chebyshev's inequality tells us that

$$\text{Prob}[|X - \mathcal{E}X| \geq k] \leq \text{Var}[X]/k^2,$$

so

$$\begin{aligned} & \delta \sum_n \text{Prob}[S^{nr} > k] m(n) v(n) \\ & \geq \delta \sum_{n \leq n^*} \text{Prob}[|S^{nr} - \mathcal{E}S^{nr}| > k - \mathcal{E}S^{nr}] m(n) v(n) \\ & \quad + \delta \sum_{n > n^*} (1 - \text{Prob}[|S^{nr} - \mathcal{E}S^{nr}| > \mathcal{E}S^{nr} - k]) m(n) v(n) \\ & \geq \delta \sum_{n \leq n^*} \frac{\text{Var}[S^{nr}]}{(\mathcal{E}S^{nr} - k)^2} m(n) v(n) + \delta \sum_{n > n^*} \left(1 - \frac{\text{Var}[S^{nr}]}{(\mathcal{E}S^{nr} - k)^2}\right) m(n) v(n). \end{aligned}$$

Put $k = \lambda r(n^* + 1/2)$. Then

$$\begin{aligned} \frac{\text{Var}[S^{nr}]}{(\mathcal{E}S^{nr} - k)^2} &= \frac{\lambda(1-\lambda)m}{(\lambda m - \lambda r(n^* + 1/2))^2} \\ &= \frac{\lambda(1-\lambda)m}{(\lambda r)^2(n - n^* - 1/2)^2} \\ &= \frac{\lambda(1-\lambda)n}{\lambda^2 r(n - n^* - 1/2)^2} \rightarrow 0 \end{aligned}$$

since $\lambda r \rightarrow \infty$ by assumption. Therefore,

$$\lim_{\delta} \delta \sum_n \text{Prob}[S^{nr} > k] m(n) v(n) \rightarrow \delta \sum_{n > n^*} m(n) v(n) > \sum_n m(n) v(n),$$

so again we have a contradiction.

Q.E.D.

A.2. Proof of Proposition 8

We begin with a result which is really a corollary of Proposition 7.

LEMMA 3: *For any fixed but arbitrary symmetric PBE, and for any $\varepsilon > 0$, there exists a number T , which is independent of the period length, such that $\text{Prob}[\mu(t) \in M_\varepsilon : \forall t \leq T] < \varepsilon$.*

PROOF: First note that λr is bounded uniformly in r and δ . Then in each period, the probability that no one invests is bounded away from 0. So each period the probability of entering L is bounded from 0 and the result is immediate. *Q.E.D.*

By the usual arguments (see the proof of Proposition 7), $W^*(0, m) = V(m)$ and

$$\lim_{\beta \rightarrow \infty} W^\infty(\beta, m) = \sum_{n=0}^N m(n) \text{Max}\{v(n), 0\} > V(m)$$

so

$$\delta W^\infty(0, m) < V(m) < \delta W^\infty(\infty, m)$$

for all $m \in M_\varepsilon$ and $\delta > 1 - \eta(\varepsilon)$. Then the continuity of $W^\infty(\cdot, m)$ implies the existence of a solution $\beta = F^\infty(m)$ to the equation $V(m) = \delta W^\infty(\beta, m)$.

To prove that β is uniquely defined, write

$$W^\infty(\beta, m) = \sum_{k \geq k^*} \left\{ \sum_{n=0}^N m(n) v(n) e^{-\beta_n} \frac{(\beta_n)^k}{k!} \right\},$$

where k^* is the smallest value of k such that the expression in braces is positive, and differentiate this expression with respect to β to get

$$\begin{aligned} \frac{dW^\infty(\beta, m)}{d\beta} &= \sum_{k \geq k^*} \sum_{n=0}^N m(n) v(n) \left\{ e^{-\beta_n} \frac{(\beta_n)^k}{k!} \frac{-n}{N} + k e^{-\beta_n} \frac{(\beta_n)^{k-1}}{k!} \frac{n}{N} \right\} \\ &= \sum_{n=0}^N m(n) v(n) \sum_{k \geq k^*} \{ -b(\beta_n, k) + b(\beta_n, k-1) \} \frac{n}{N} \\ &= \sum_{n=0}^N m(n) v(n) b(\beta_n, k^* - 1) n / N \\ &= \frac{k^*}{\beta} \sum_{n=0}^N m(n) v(n) b(\beta_n, k^*) > 0. \end{aligned}$$

This proves that the solution $\beta = F^\infty(m)$ is unique and a calculation similar to the one used previously shows that F^∞ is increasing on the set M_ε . Q.E.D.

A.3. Proof of Proposition 9

With probability at least $1 - \varepsilon$, $\mu(t)$ escapes from M_ε within T periods. From the lemma, within the first T periods of the game there are three, mutually exclusive possibilities:

- (i) $\mu(t)$ reaches L , in which case the game ends;
- (ii) $\mu(t)$ remains in the set M_ε which can only happen with probability ε ;
- (iii) $\mu(t)$ comes within ε of U , i.e., $\mu(t) \notin L \cup M_\varepsilon$, at least once.

In the first case, delay is bounded by T . In the second, delay can occur, but this case can only happen with probability ε . In the last case, delay is small. To see this, suppose that $m = \mu(t)$ lies within distance ε of U . By investing immediately, the player can get at least $\sum_{n > n^*} m(n)v(n) - \varepsilon(v(N) - v(0))$. Suppose that his equilibrium strategy involves delay. Let σ be the Markov stopping time to the end of the game, given that the current state is $(\mu(t), K)$, i.e., σ is the real-time delay between reaching $(\mu(t), K)$ and the last investment by any player. Since it is always optimal for him to delay until the last player invests, his payoff from delay cannot be more than

$$\mathcal{E}[e^{-\rho\sigma}] \sum_{n > n^*} m(n)v(n),$$

where ρ is the instantaneous rate of time preference. Equilibrium requires that

$$\mathcal{E}[e^{-\rho\sigma}] \sum_{n > n^*} m(n)v(n) \geq \sum_{n > n^*} m(n)v(n) - \varepsilon v(N) + \varepsilon v(0).$$

For any $\gamma > 0$ and all ε sufficiently small, this inequality implies that $\text{Prob}[\sigma < \gamma] > 1 - \gamma$.

Now we can establish a limit on delay. Fix some small number $\gamma > 0$. For any $\varepsilon > 0$ we know there is a number of periods T such that either the game ends in T periods with probability at least $1 - \gamma$ or $\mu(t)$ comes within ε of U at least once within the first T periods. But if $\mu(t)$ is ε -close to U , the delay is less than γ with probability $1 - \gamma$, for ε sufficiently small, so the game ends within time $\tau T + \gamma$ with probability $(1 - \gamma)^2$, as required. Q.E.D.

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