Ec 365
Spring 2026

Exercise on seignorage with metallic money
due Monday 2/9, in class

If you know how to solve this exercise, you understand most of what there is to know about seignorage with metallic money.

Recall that the face value of a coin is the exchange value exchange of the coin (also the mark on the coin that indicates the exchange value—such as a quarter for U.S. coins). The intrinsic value is the value of the metals in the coin.

Let us assume an economy with one type of coin, which is made of silver and copper. Silver is valuable, copper is free. The degree of fineness of a coin is *defined* as the ratio between the weight of the precious metal (silver) and the weight of the coin.

The main assumption (which applies to any economy), is the demand for money is determined by the demand for transactions. In this exercise, it is assumed that these transactions (which correspond to real economic activity) are constant. As a comment, this assumption is a good one in the medium to long-term but not in the short-term. More elements also come into play such as expectations about future prices, but this is not a course in monetary economics and for us in this exercise, the constant demand for the real quantity of money is a good simplification to focus on one issue: the demand for the **real** quantity of money M/P is constant, and equal to, say, k. As stated in class this is an equilibrium equation. M is the supply and P is the price of goods (relative to money) that adjusts itself so that the demand is equal to the supply. (Like the price of any market).

In the year 200, there is only one type of coin in circulation, called “denarius”. Each coin, with a face value of one denarius, weighs 4 grams and contains half silver and half copper (measured by weight). The price of copper with respect to silver is so low (recall the data in class) that it is assumed to be free. The degree of fineness (the fraction of silver in the coin) is 1/2. By assumption, at the beginning of the year 200, prices are stable and the market price of four grams of silver is one denarius.

Assume that the quantity of coins in circulation is 100 million “denarius”. Each coin has a face value of one denarius and weights 4 grams.

Coins are produced by shops (the mints), which for simplicity, have zero production cost. They are private enterprises which operate under regulation: they buy the silver on the market, at the market price, produce coins according to instructions of the emperor and turn the profits to him. They do not make a loss: if the instructions generate a loss, they just withdraw from the production of coins. Silver has a constant price with respect to other goods: if the price level increases by, say, 10 percent, the price of silver also increases by 10 percent.

In the year 200, the price of silver is equal to 1 (under proper normalization). The emperor orders the mints to produce 100 million more coins of the same degree of fineness (1/2). At the end of the year, the new coins are in circulation and the price level has stabilized.

1. What is the price of silver at the end of the year? Compare the face value and the intrinsic value of the coins at the end of the year.
2. Show that the mints can produce the coins as requested while buying the silver in the market. (More difficult: discuss the profit of the mints).
3. In the year 220, the emperor orders the mints to produce 200 million additional coins, each with a degree of fineness of only 25 percent. Show that the quantity of coins *in circulation* does not change. (Remember Gresham's Law). What happens to the "old" coins (with a degree of fineness of ½)?
4. How does this policy affect the price level (the price of silver)?
5. What is the amount of revenues that the emperor can obtain from this policy?
6. Suppose that later, the ruler instructs the mint to produce more coins with the degree of fineness of 1/4. Is this policy feasible? What will be the effect on the price level?