Ec 717
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Problem 5

Social learning with partial memory

The purpose of the exercise is to put your mind on an issue that is analyzed in the following paper.

- Kabos, Eszter and Margaret Meyer. "A Welfare Analysis of a Steady-State Model of Observational Learning,,' (presented at a workshop in December 2023).

There are two states $\omega = in\{L, H\}$ with equal prior, and a countable number of agents. Each agent has a signal s on the state with the density $f_\omega(s)$. At this stage, assume that $f_H(s) = 2s$, $f_L(s) = 2(1-s)$. Define the power of the signal as p such that

$$p = \frac{1}{2} + |s - \frac{1}{2}|.$$

An agent's action is H or L and the payoff is strictly positive if and only if the agent's action matches the state (which is revealed after all actions). Time is descrte, $t \in \{-\infty, ..., -1, 0, 1, ...\infty\}$ and one agent acts per period, and only once.

An agent's action is *misrecorded* with probability $\pi < 1/2$ in which event the report is the opposite of the true action.

So far, this is the same as the standard BHW model (with an observation error). The key assumption now is that agents do not observe the history of past reports (subject to error). There may be sampling, observations of a limited number of reports, even just the past report. No specific assumption is made.

One considers the following strategy for an agent's action:

- If the state indicated by the private signal is opposite to that indicated by the observed data, then for some $\tilde{p} \geq 1/2$,
  - follow the observed data if the private signal is sufficiently weak, $p \in (1/2, \tilde{p})$.
  - follow the private signal if it is sufficiently strong: $p \geq \tilde{p}$.

A *steady state* is described by the stationary distributions of the two Markov processes, one for each ω. An *equilibrium sate state (ESS)* is defined as a steady sate in which each agent's optimal strategy matches the conjectured common cutoff strategy of all predecessors.

1. Determine

$$\begin{aligned} A(\tilde{p}) &= P(d_t \text{ correct } | d_{t-1} \text{ correct } ; \tilde{p}), \\ B(\tilde{p}) &= P(d_t \text{ incorrect } | d_{t-1} \text{ incorrect } ; \tilde{p}). \end{aligned} \tag{1}$$

   Represent the graphs of these two functions.

2. Write the two transition matrices, one for each state, with d_{t-1} in the rows and d_t in the columns.

$$\begin{array}{c} \\ \text{no} \\ \\ \text{yes} \end{array} \begin{array}{c} \begin{array}{cc} \text{no} & \text{yes} \end{array} \\ \begin{pmatrix} \cdot & \cdot \\ & \\ \cdot & \cdot \end{pmatrix} \end{array}$$

3. What is the stationary value of a correct action in the steady state?

4. Determine the likelihood ratio between L and H for an agent who observes "no" and the LR between H and L after the observation "yes". Show that the two signals have the same strength (same LR). Represent the graph in a figure.

5. Determine the value of $\tilde{p}$ in the equilibrium steady state.