Ec717
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Problem 2

These exercises illustrate the fragility of social learning. They prepare for:

> Frick, Mira, Ryota Iijima and Yuhta Ishii, (2020). "Misinterpreting Others and the Fragility of Social Learning," *Econometrica*, 88, 2281-2328.

A. Recall the Vives model. Agent t has a payoff function $-E[(\omega - x_t - \eta_t])2]$, where, to simplify here, ω has diffuse prior (infinite variance), and η_t are independent normally distributed r.v. with mean b and precision ρ_ϵ. Each agent has a signal $s_t = \omega + \epsilon_t$ with $\epsilon \sim \mathcal{N}(0, 1/\rho_\epsilon)$ as in the standard model.

Assume that agents are mistaken about the distribution of η. They believe that $b = 0$ whereas the true value is positive.

1. Assume that agents optimize without looking at the history of past actions. Let μ_t be the expected value of ω for an outside observer, an econometrician, who knows the history of past actions, $h_t = (x_1, ..., x_{t-1})$. Comment on the case b small.

2. Assume now that any agent t observe the history h_t of past actions. Analyze the evolution of μ_t. Compare with the previous question. At some point, you may use the proposition of Vives (p. 8 of Notes 1).

3. (Optional). Simulate the evolution of μ_t at a function of t for the particular realization of all noise terms equal to 0. (You may normalize other parameters to 1). Plot on a diagram the values of μ_t and ρ_t (the precision of the estimate of ω at the beginning of period t. Comment. (You may anticipate the answer in the previous question).

4. In view of your answer in question 2, what is the "flaw" in the the assumption about b, in that question? Is the exercise nevertheless informative? Comment.

B. Consider the model of social learning with two states, 0 and 1, two actions 0 and 1 with a cost of investment equal to c. The prior belief (probability of state 1) is μ_1. Any agent t has a signal $s_t = \theta + \epsilon_t$, where $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$. Assume that the true state is 1.

1. Does the belief converge? If yes, what is the limit? Does the sequence of actions eventually become a herd?

2. Assume now that in each round, with probability π, the acting agent is a "noisy" agent who does not invest, because his cost is greater than 1. Rational agents know the model and the probability π. Answer the previous question.

3. Assume that now that rational agents are not aware of the existence of noisy agents. (They think that $\pi = 0$). To make the case interesting, assume that π is very small. Answer the previous question and comment on the case where π is very small. You may give an informal answer.