Ec717
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Problem 1

1. Assume that an agent undertakes a project which succeeds (fails) with probability ω $(1-\omega)$ where ω is drawn from a uniform distribution on $(0,1)$.

   (a) Determine the *ex post* distribution of ω for the agent after the failure of the project.

   (b) Assume that the project is repeated and fails n consecutive times. The outcomes are independent with the same probability ω. Determine an algebraic expression for the density of ω of this agent. Discuss intuitively the property of this density.

2. In the model in Section 5.1 of Notes 1, assume that $c(q) = q^\gamma$ with $\gamma > 0$.

   (a) Analyze qualitatively the convergence of social learning, depending on γ.

   (b) (Optional). Analyze the rate of convergence.

3. (a) Consider the model in Section 3 of Notes 1. Assume that any agent t observes only the action of agent x_{t-1} (instead of the history h_t of all past actions). Analyze the convergence of social learning.

   (b) Consider now the model in Section 4.1. Answer the previous question.

4. There is a state of nature ω that is fixed by the realization a uniform prior density on $(0,1)$. There are N agents, N large who know the distribution but cannot observe ω directly.. Each agent t has a *private* signal $s_t \in \{0,1\}$ such that $P(s_t = \omega|\omega) = q > 1/2$. Agent t takes the action x_t in period t to maximize the payoff $E[-(x_t - \omega)^2)]$.

   (a) Each agent observe the history $h_t = \{x_1, ..., , x_{t-1}\}$ of past actions. Is the history of actions is informationally equivalent to the history of signals $\{s_1, ..., , s_{t-1}\}$? Analyze the convergence of beliefs.

   (b) Each agent observes t only the actions of the two previous agents, $\{x_{t-2}, x_{t-1}\}$. (Agent 2 observes x_1.) Answer the previous question.

   (c) Each agent t observes only the previous action x_{t-1}.Answer the previous question. (You are not asked to solve the model).