FROM DISCRETE EVENT TO HYBRID SYSTEMS

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OUTLINE

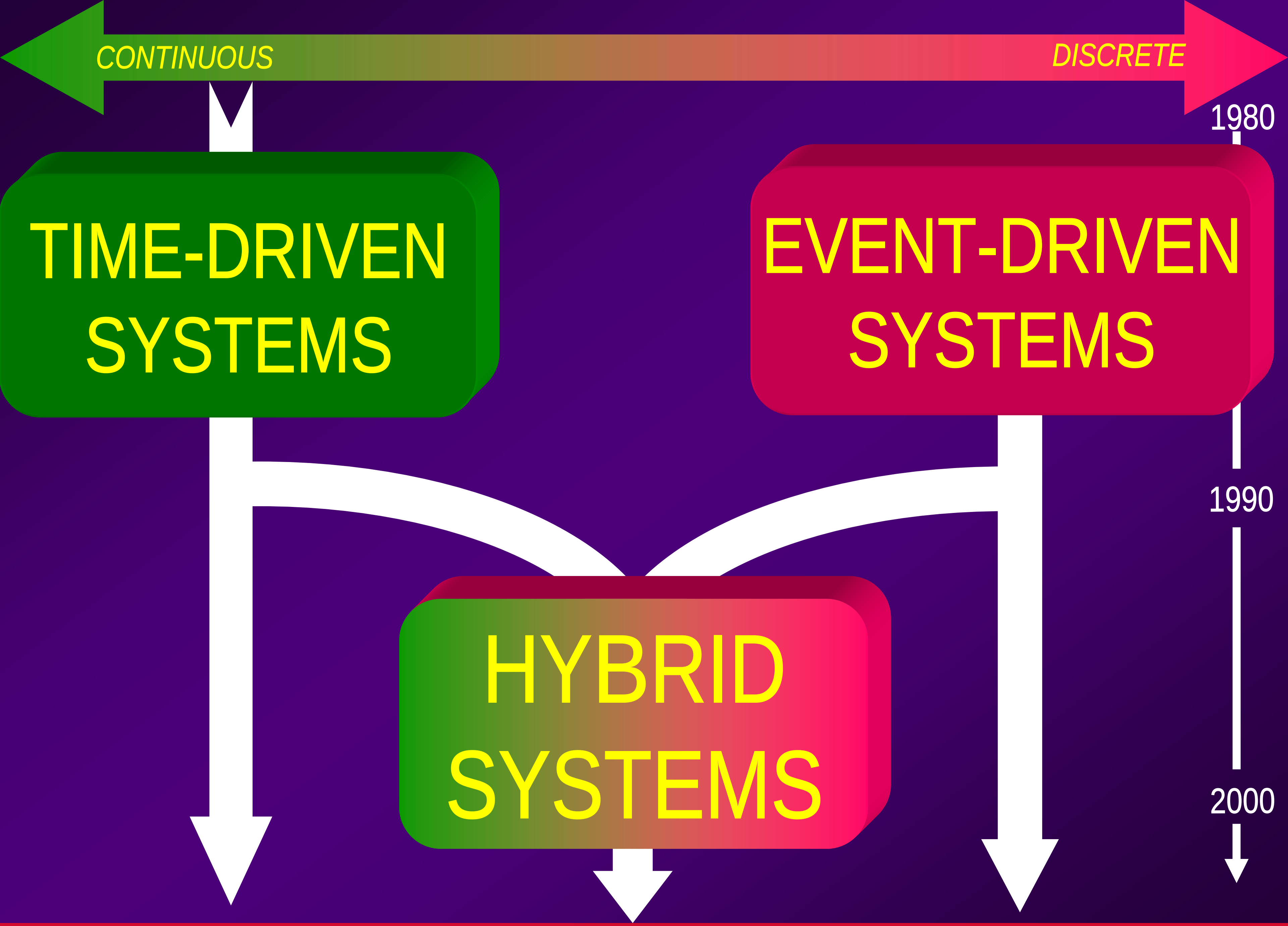
- HYBRID SYSTEMS IN COMPLEX SYSTEM…
  … *DECOMPOSITION* AND *ABSTRACTION*

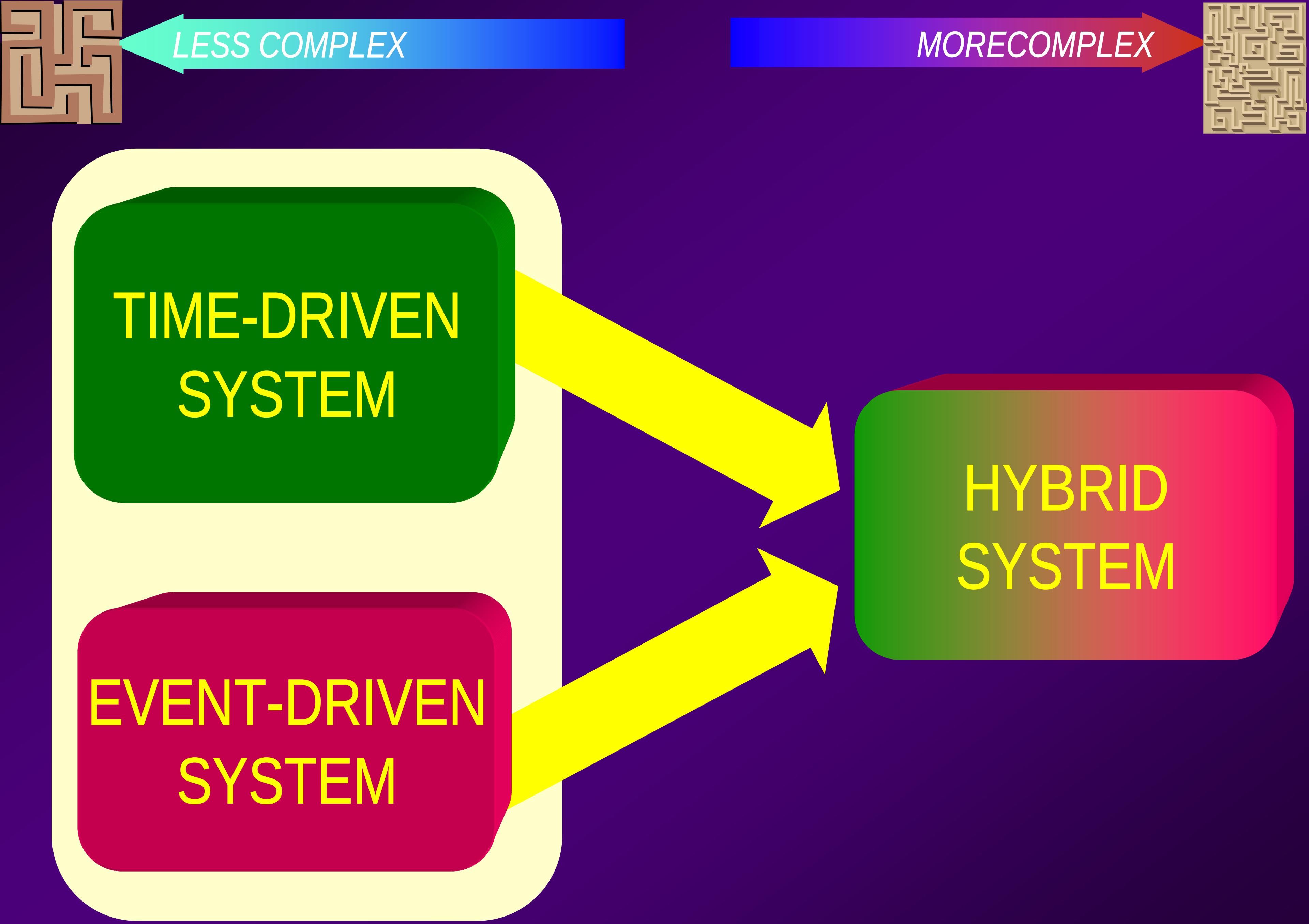
- WHAT'S A HYBRID SYSTEM…

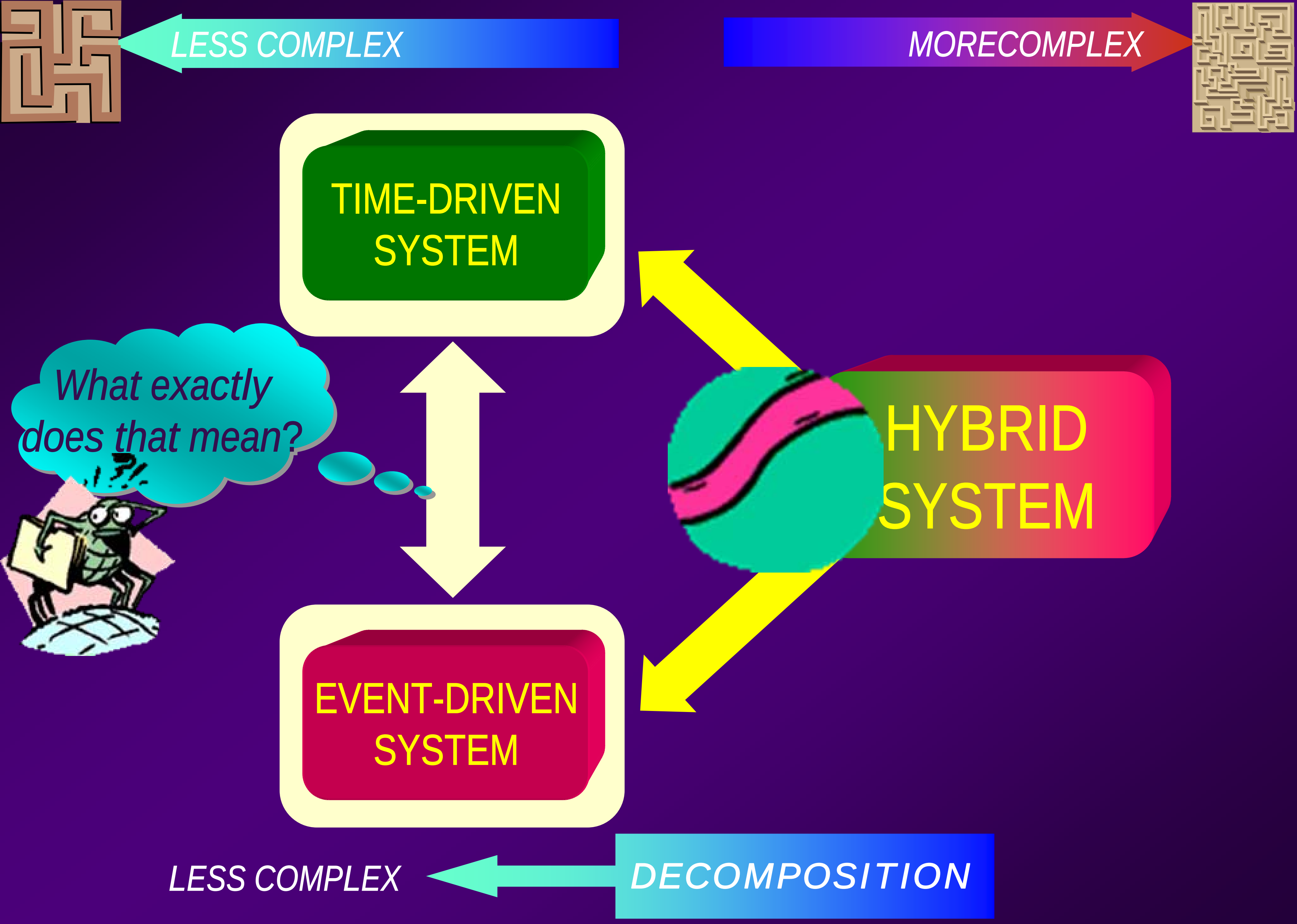
- *DECOMPOSITION*: HYBRID SYSTEM $\rightarrow$ DES
  - SOLVING OPTIMAL CONTROL PROBLEMS

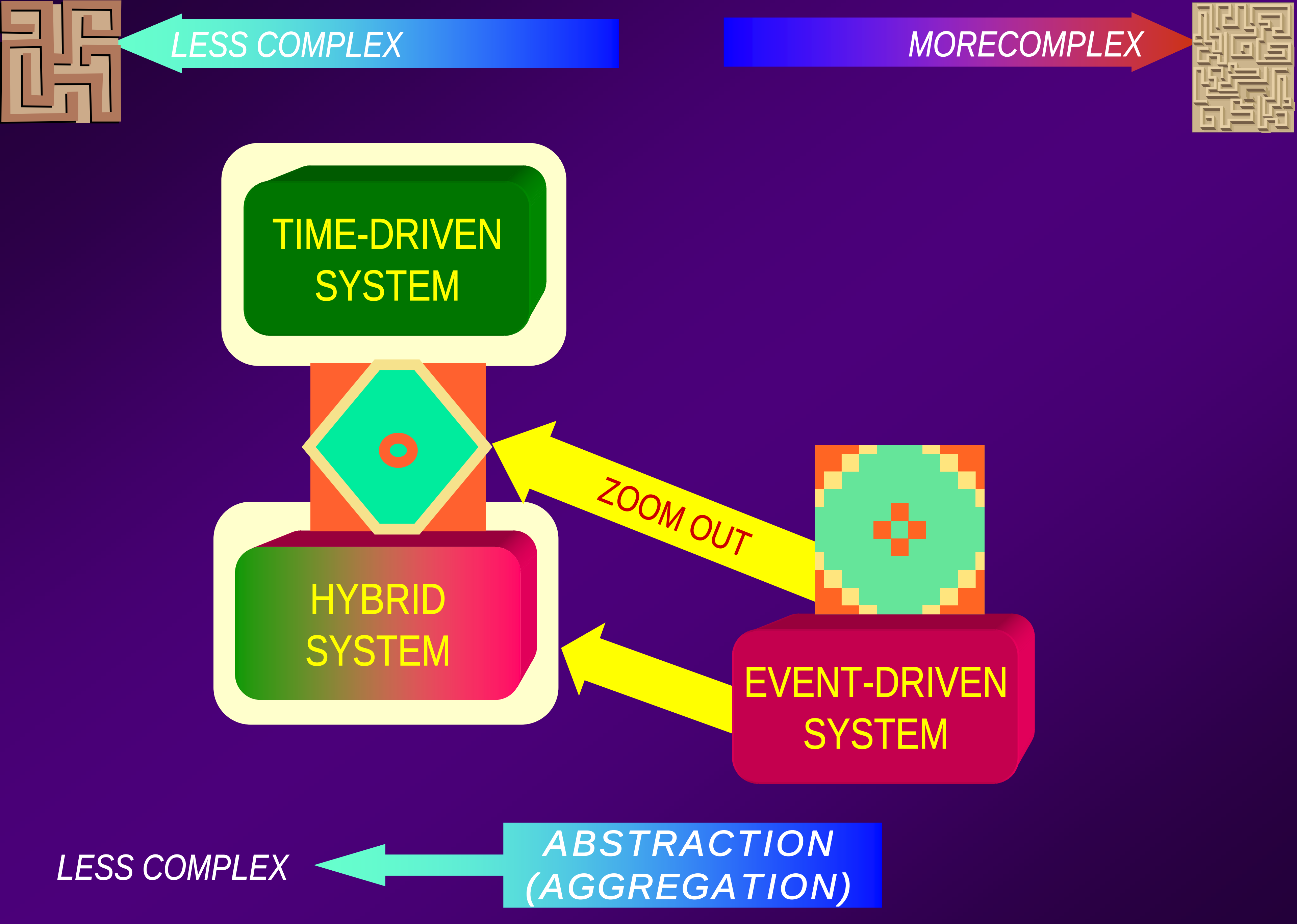
- *ABSTRACTION*: DES $\rightarrow$ HYBRID SYSTEM
  - ANALYSIS OF STOCHASTIC FLOW MODELS (SFM)

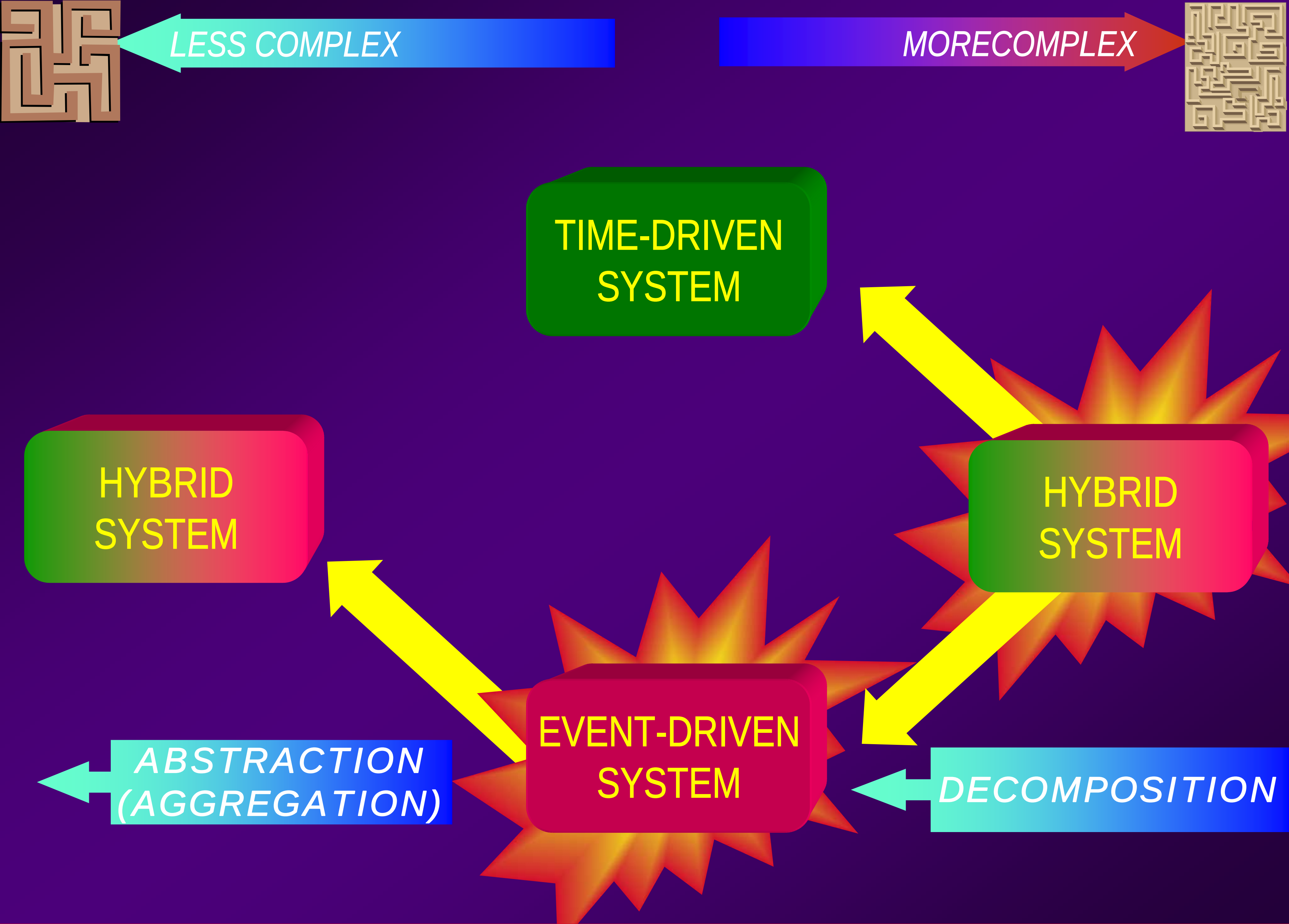
- THE FUTURE…


CONTINUOUS
DISCRETE
1980
TIME-DRIVEN SYSTEMS
EVENT-DRIVEN SYSTEMS
1990
HYBRID SYSTEMS
2000


LESS COMPLEX
MORECOMPLEX
TIME-DRIVEN SYSTEM
EVENT-DRIVEN SYSTEM
HYBRID SYSTEM

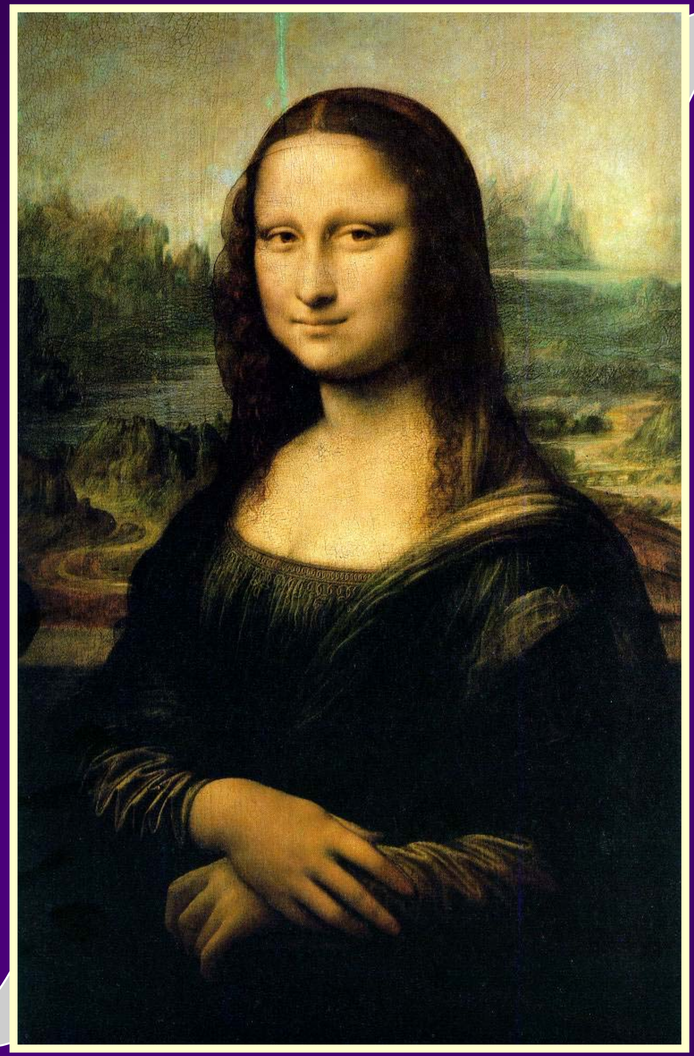

LESS COMPLEX
MORECOMPLEX
TIME-DRIVEN SYSTEM
What exactly does that mean?
HYBRID SYSTEM
EVENT-DRIVEN SYSTEM
LESS COMPLEX
DECOMPOSITION


LESS COMPLEX
MORECOMPLEX
TIME-DRIVEN SYSTEM
ZOOM OUT
HYBRID SYSTEM
EVENT-DRIVEN SYSTEM
LESS COMPLEX
ABSTRACTION (AGGREGATION)

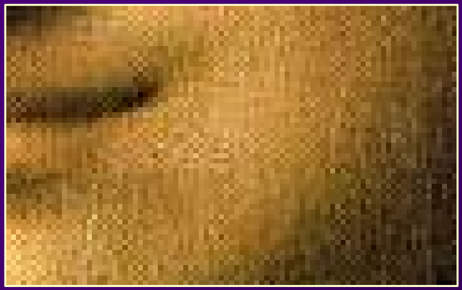

LESS COMPLEX
MORECOMPLEX
TIME-DRIVEN SYSTEM
HYBRID SYSTEM
HYBRID SYSTEM
EVENT-DRIVEN SYSTEM
ABSTRACTION (AGGREGATION)
DECOMPOSITION

WHAT IS THE RIGHT ABSTRACTION LEVEL ?

TOO FAR…
model not
detailed enough



JUST RIGHT…
good model

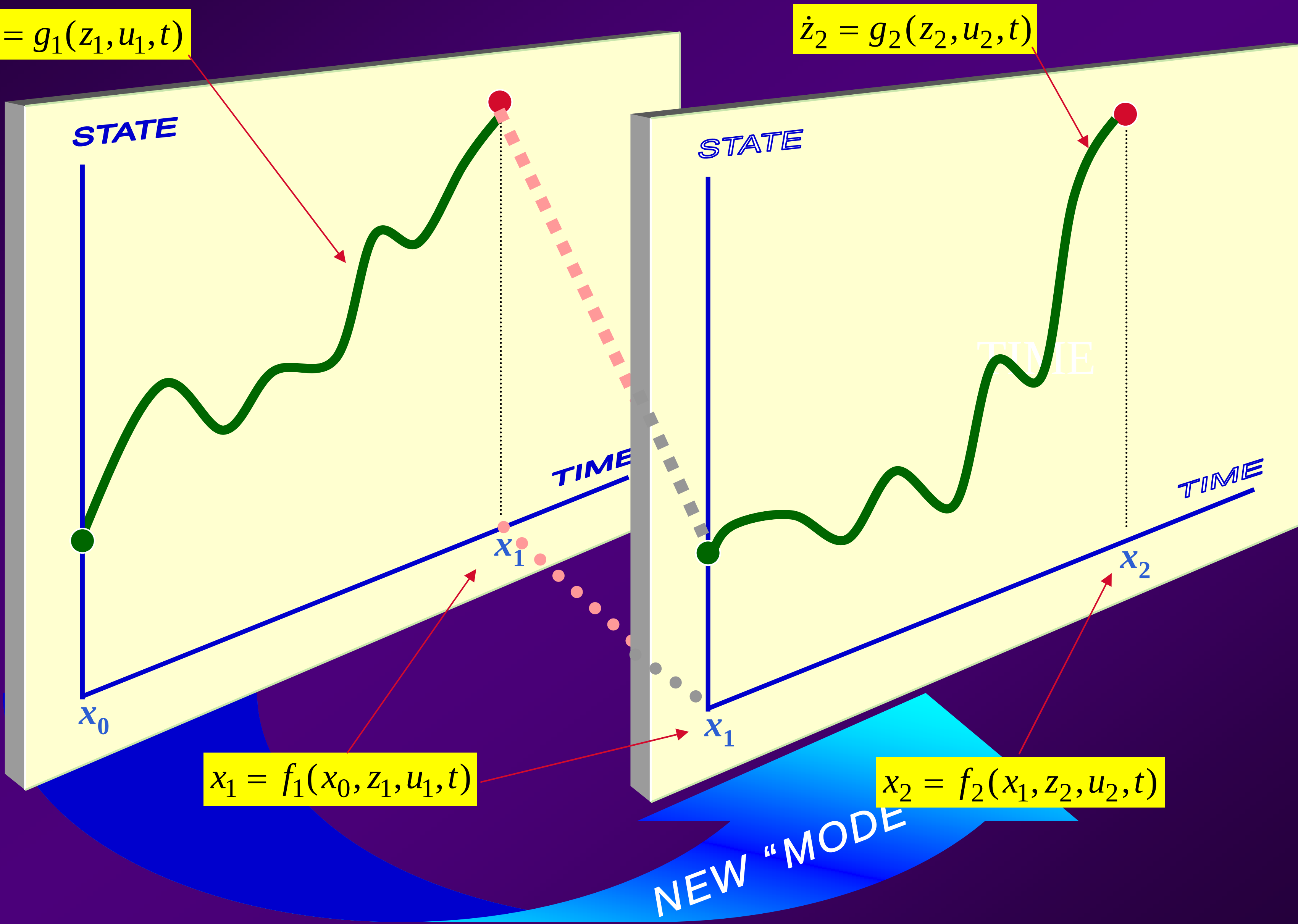


TOO CLOSE…
too much
undesirable
detail

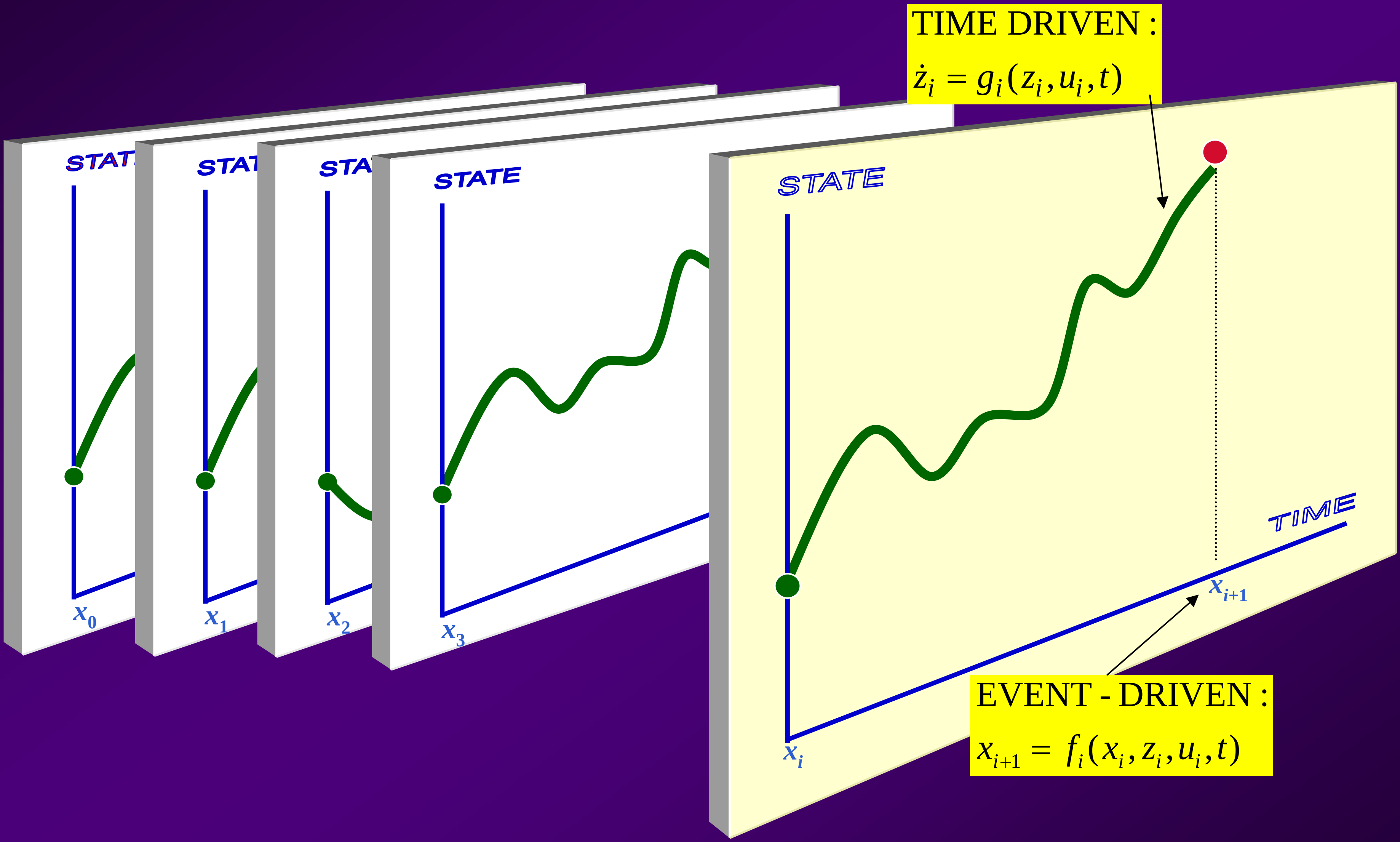
CREDIT: W.B. Gong

WHAT'S A HYBRID SYSTEM ?

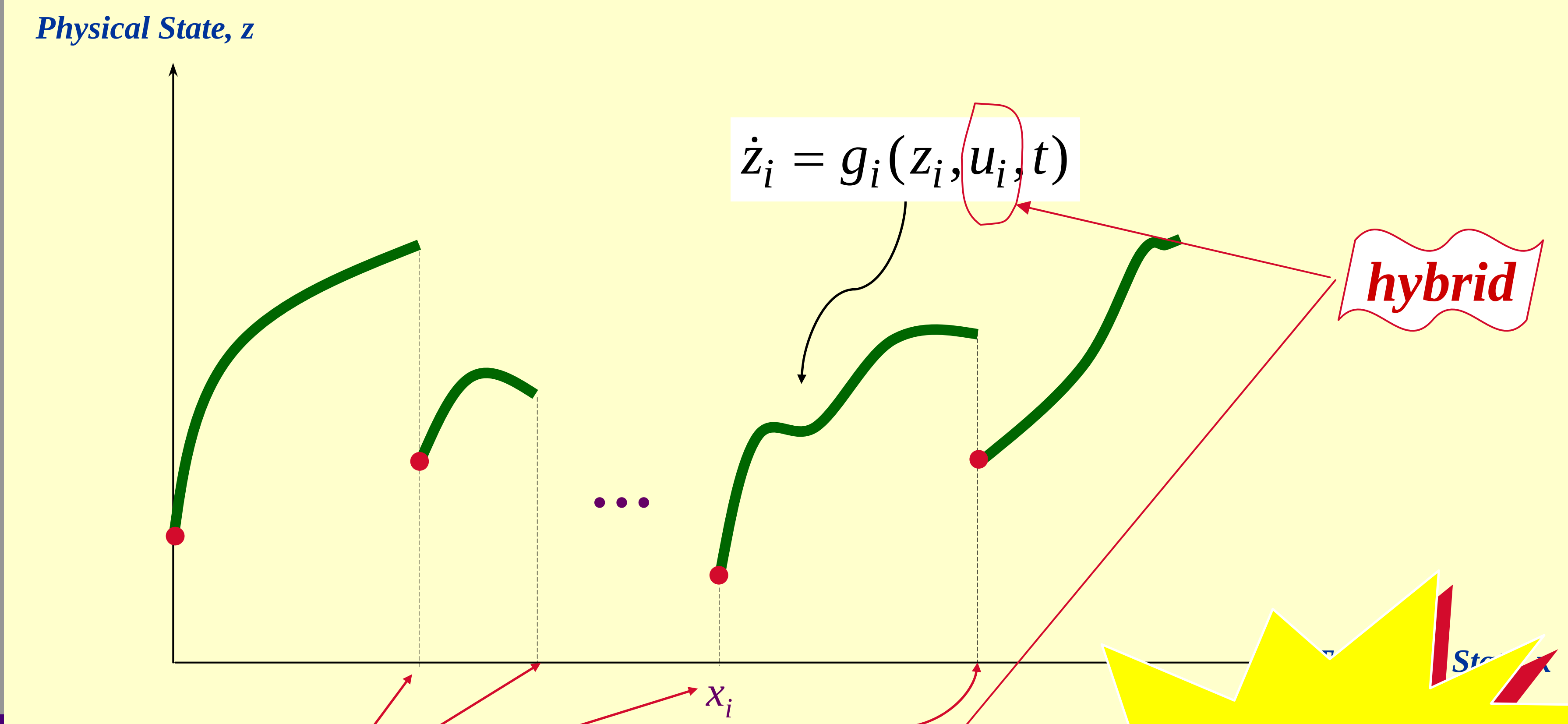
WHAT'S A HYBRID SYSTEM?



WHAT'S A HYBRID SYSTEM? *CONTINUED*



WHAT'S A HYBRID SYSTEM?



$$\dot{z}_i = g_i(z_i, u_i, t)$$

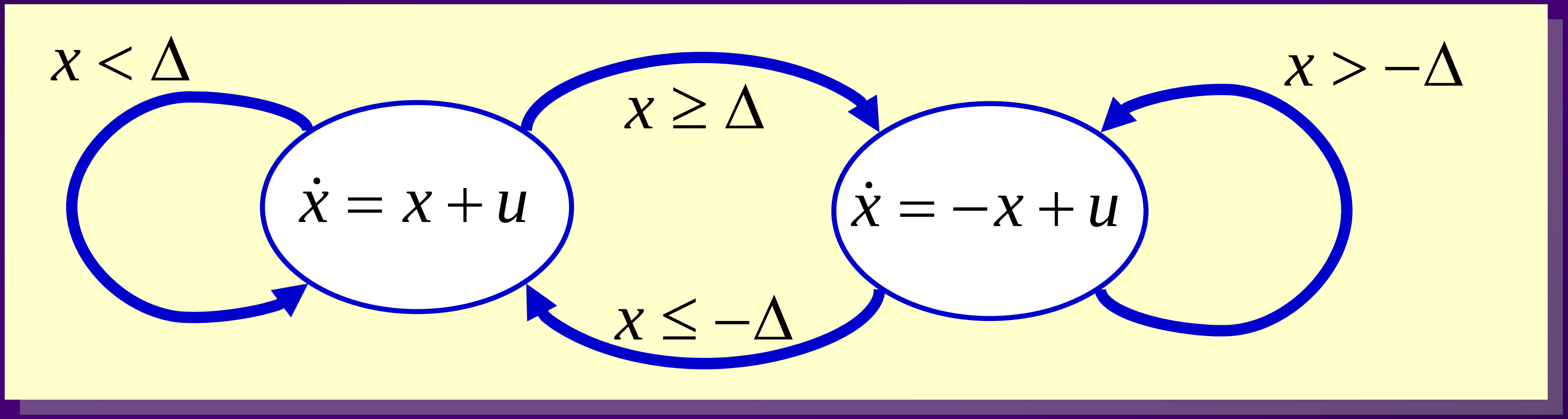
Switching Times

$$x_{i+1} = f_i(x_i, u_i, t)$$

SWITCHING TIMES HAVE THEIR OWN DYNAMICS!

HYBRID SYSTEM EXAMPLES

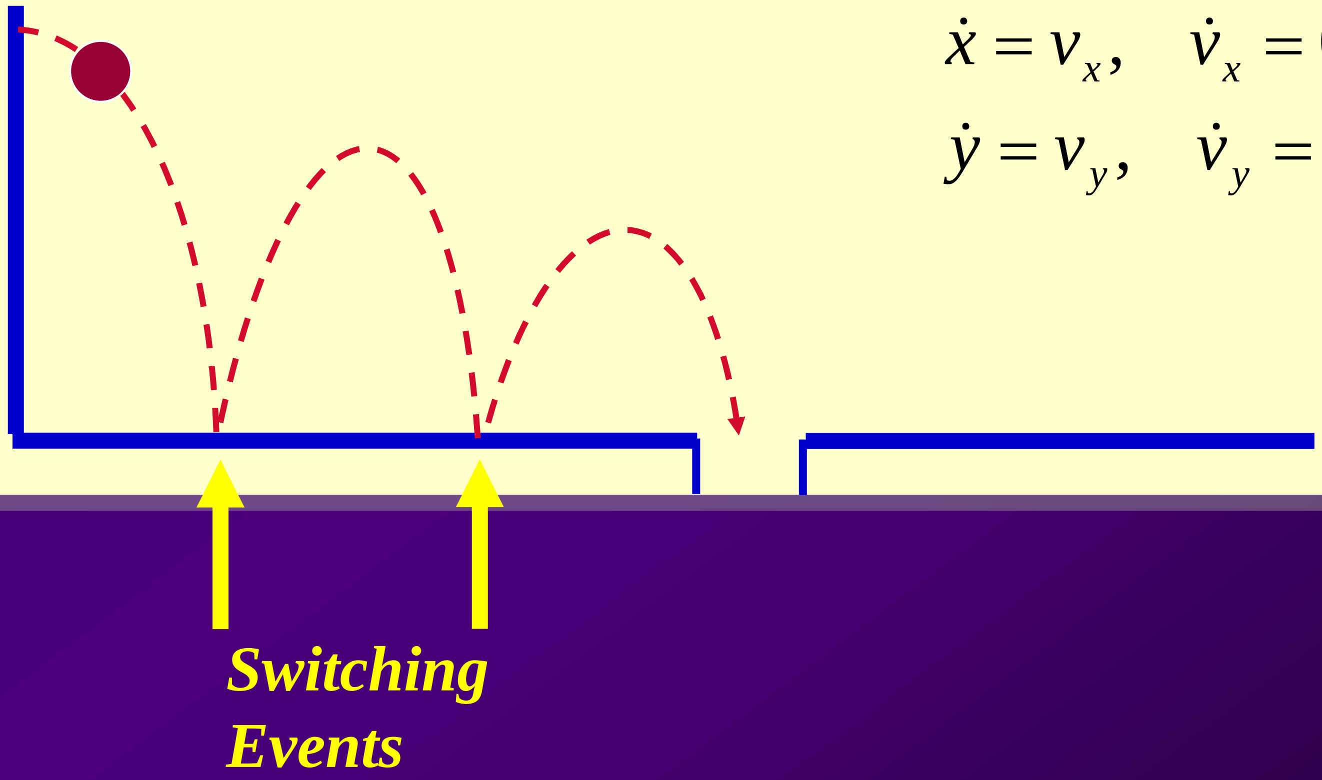
1. Autonomous Switching, e.g., *Hysteresis*



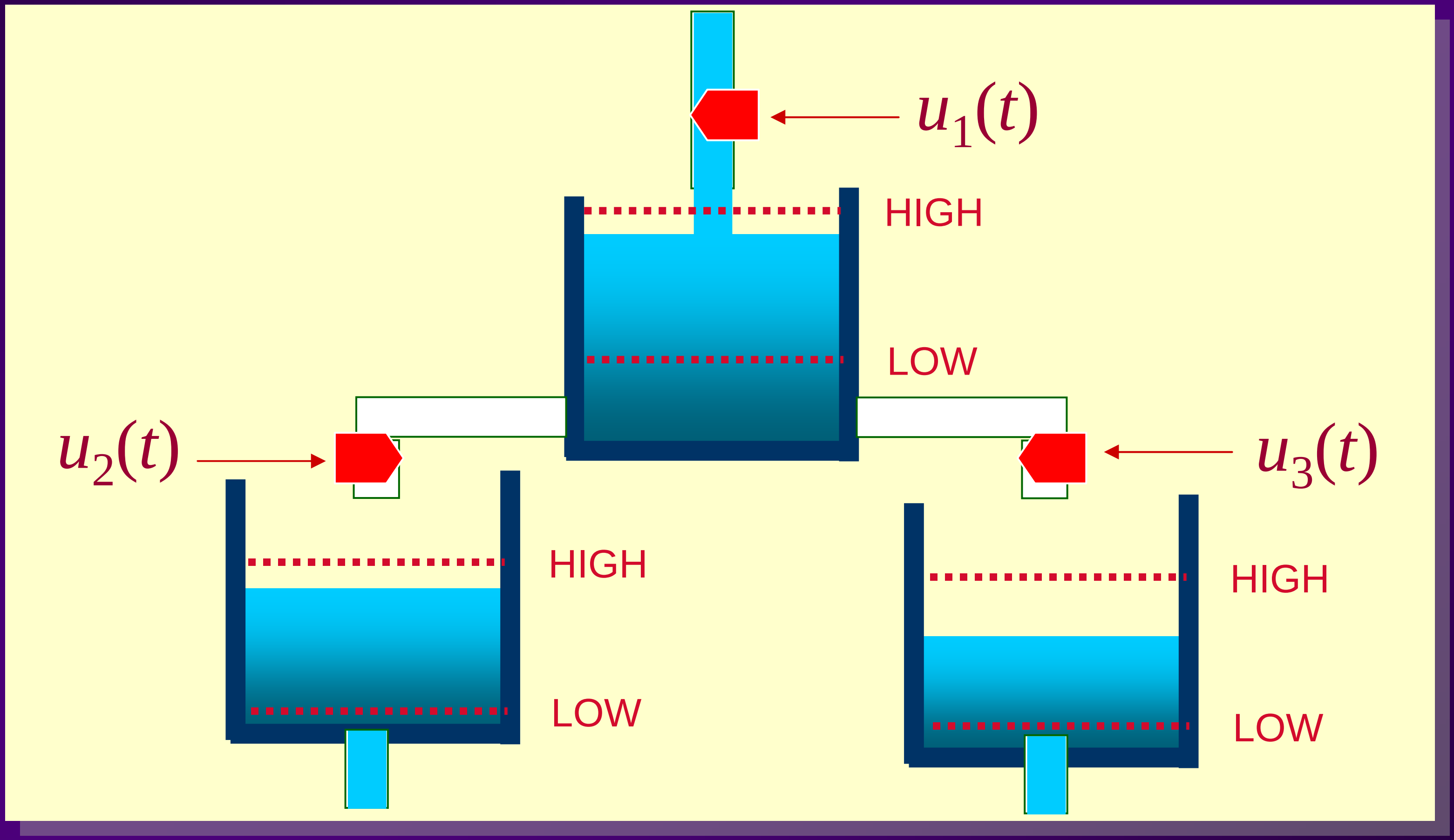
2. External Switching, e.g., *Zeno's bouncing ball*

$$\dot{x} = v_x, \quad \dot{v}_x = 0$$

$$\dot{y} = v_y, \quad \dot{v}_y = -mg$$



3. Controlled Switching, e.g., *Interconnected tanks*



HYBRID SYSTEM EXAMPLES - *MANUFACTURING*

Key questions facing manufacturing system integrators:

- How to integrate *'process control'* with *'operations control'* ?
- How to improve product Y within reasonable ***TIME*** ?

PROCESS CONTROL

- Physicists
- Material Scientists
- Chemical Engineers
- …

Time-Driven World

OPERATIONS CONTROL

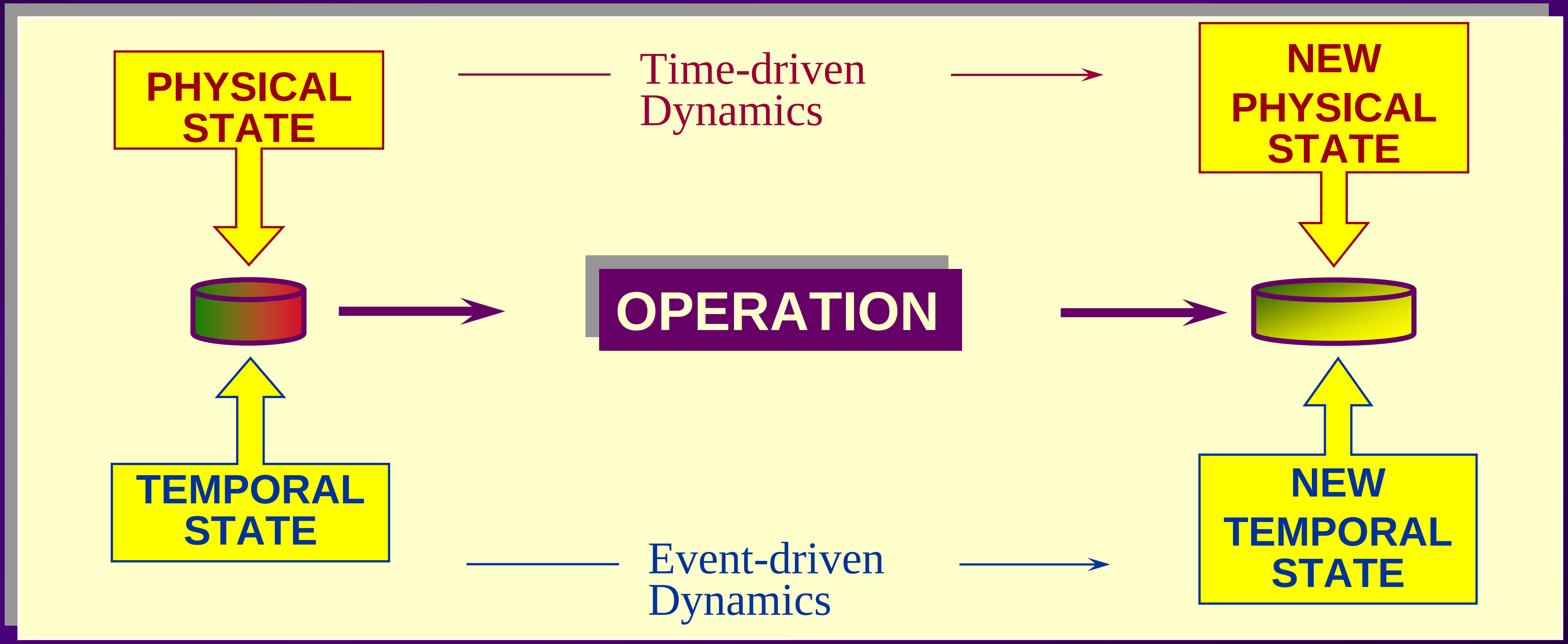
- Industrial Engineers, OR
- Schedulers
- Inventory Control
- …

Event-Driven World

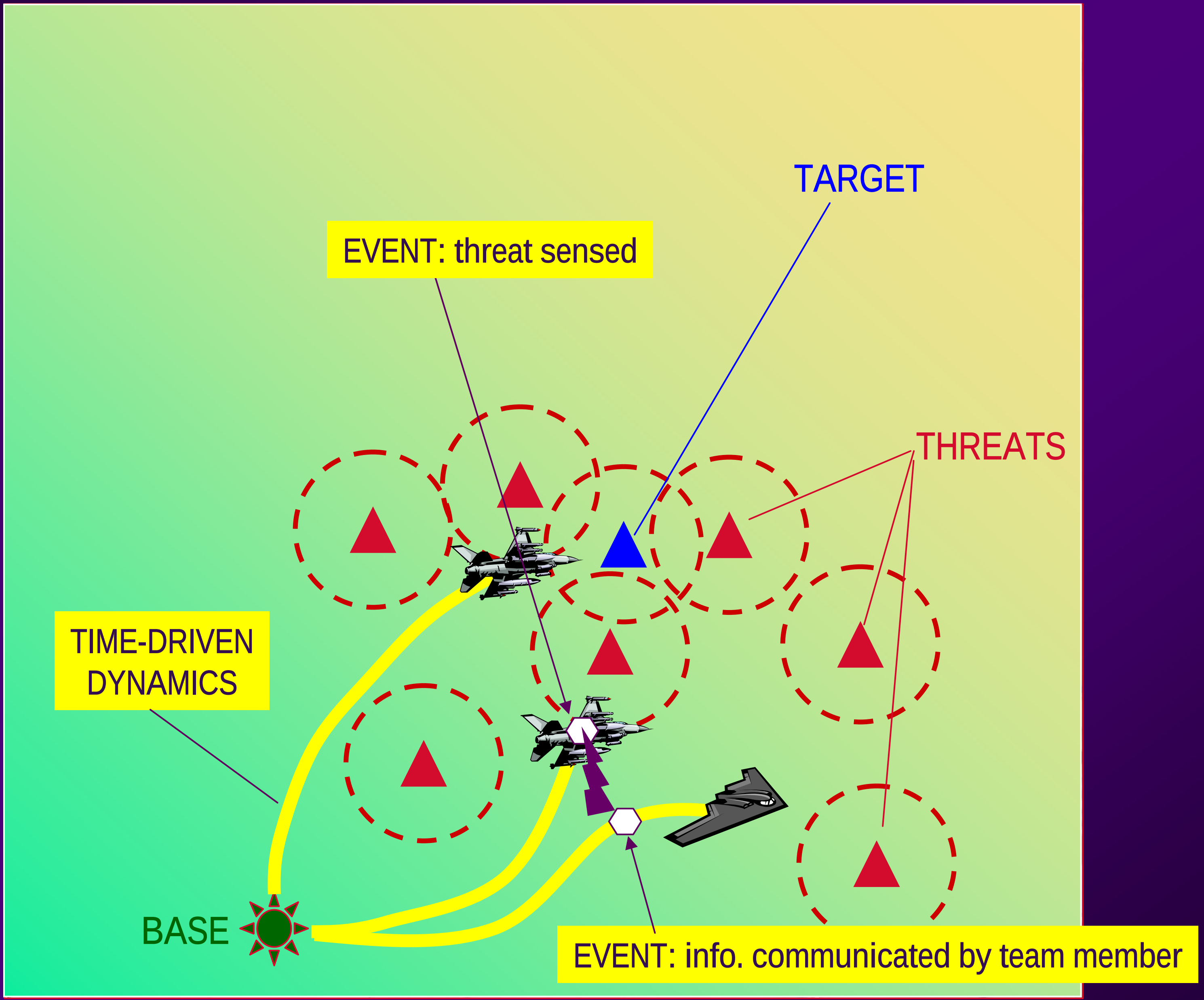
HYBRID SYSTEM EXAMPLES - *MANUFACTURING*

Throughout a manuf. process, each part is characterized by

- A **PHYSICAL** state (e.g., size, temperature, strain)
- A **TEMPORAL** state (e.g., total time in system, total time to due-date)



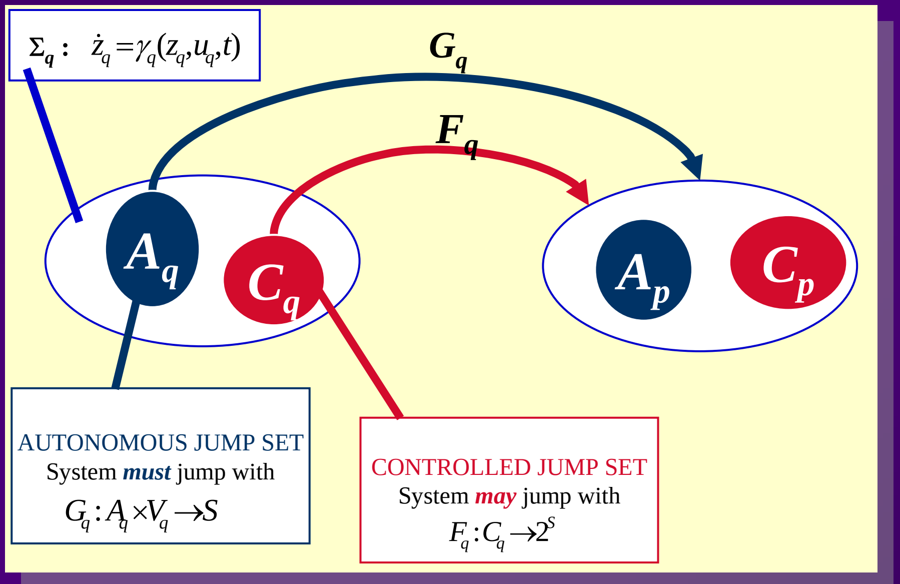
COOPERATIVE CONTROL



GENERAL MODELING FRAMEWORKS

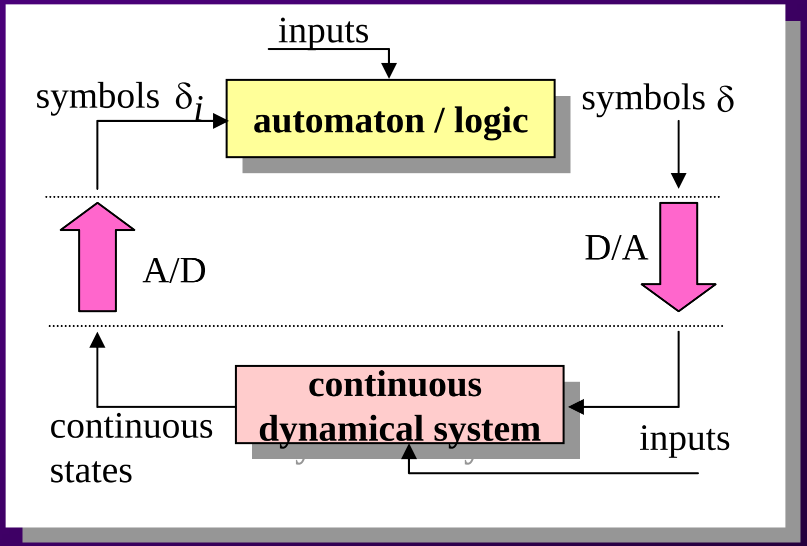
- Hybrid Automata

  [*Branicky et al., 1998*]



- Mixed Logical Dynamical Systems

  [*Bemporad and Morari, 1999*]



- etc. [*Proc. of IEEE Special Issue, 2000*]

TIME-DRIVEN AND EVENT-DRIVEN DYNAMICS

***Time-Driven* Dynamics (*STATE* = *z*):**

$$\dot{z}(t) = g(z,u,t)$$

or:

$$z_{k+1} = g_k(z_k, u_k)$$

***Event-Driven* Dynamics (*STATE* = Event Times $x_{k,i}$):**

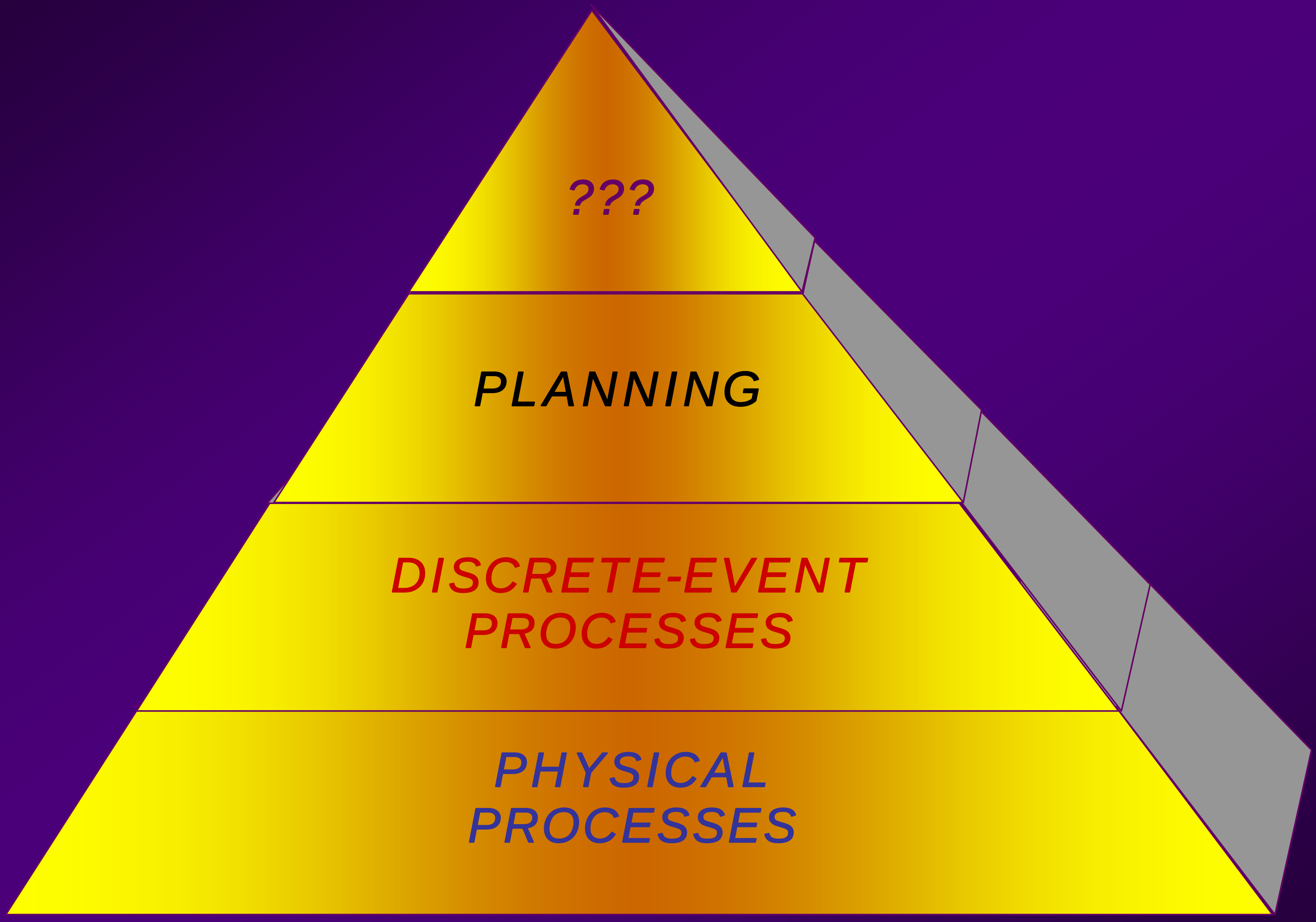
$$x_{k+1,i} = \max_{j \in \Gamma_i} \{x_{k,j} + \mathbf{a}_{k,j}\mathbf{u}_{k,j}\}$$

Event counter k = 1,2,...

Event index $i \in E = \{1,...,n\}$

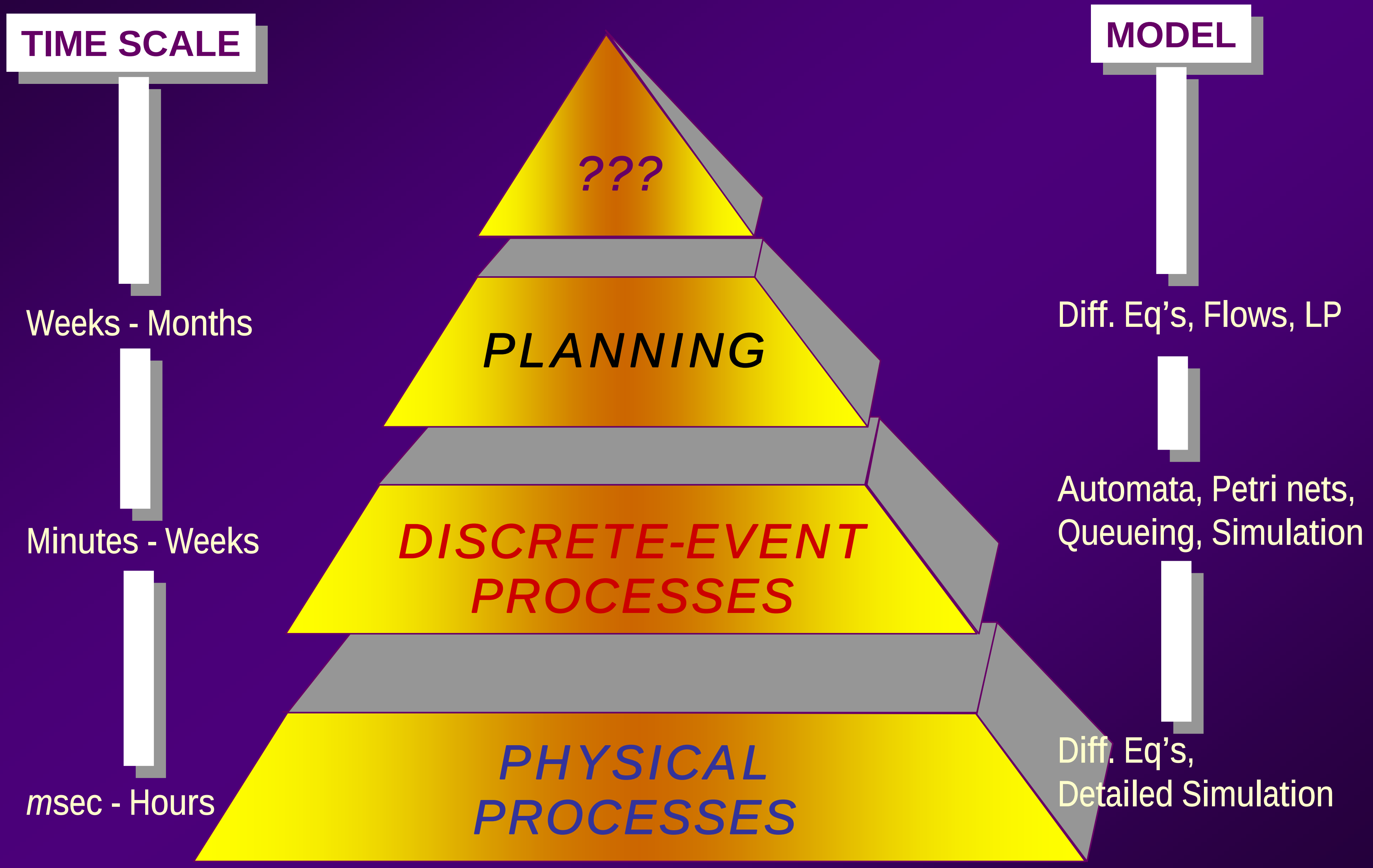
DECOMPOSITION

HIEARARCHICAL DECOMPOSITION



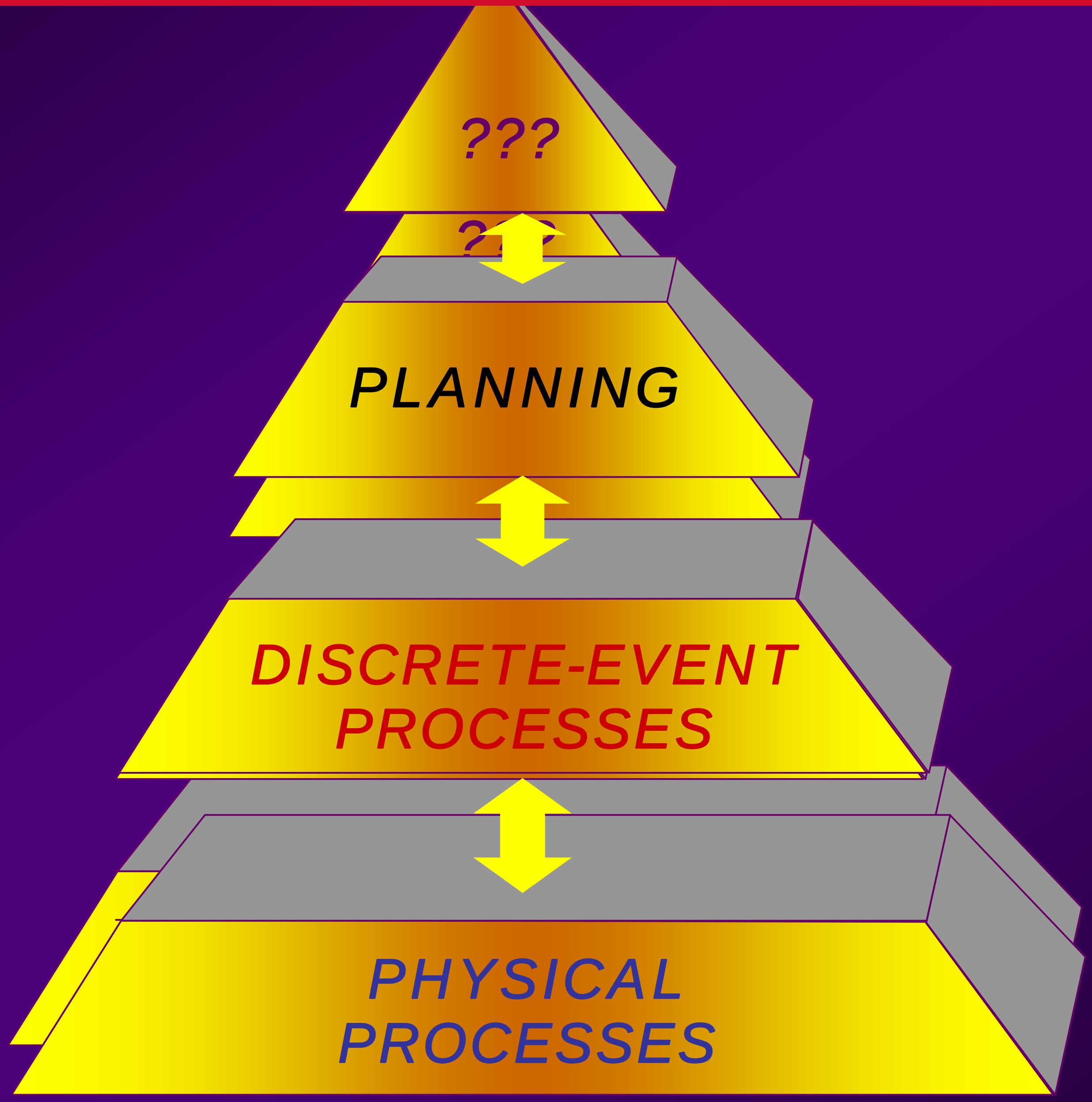
HIEARARCHICAL DECOMPOSITION

CONTINUED

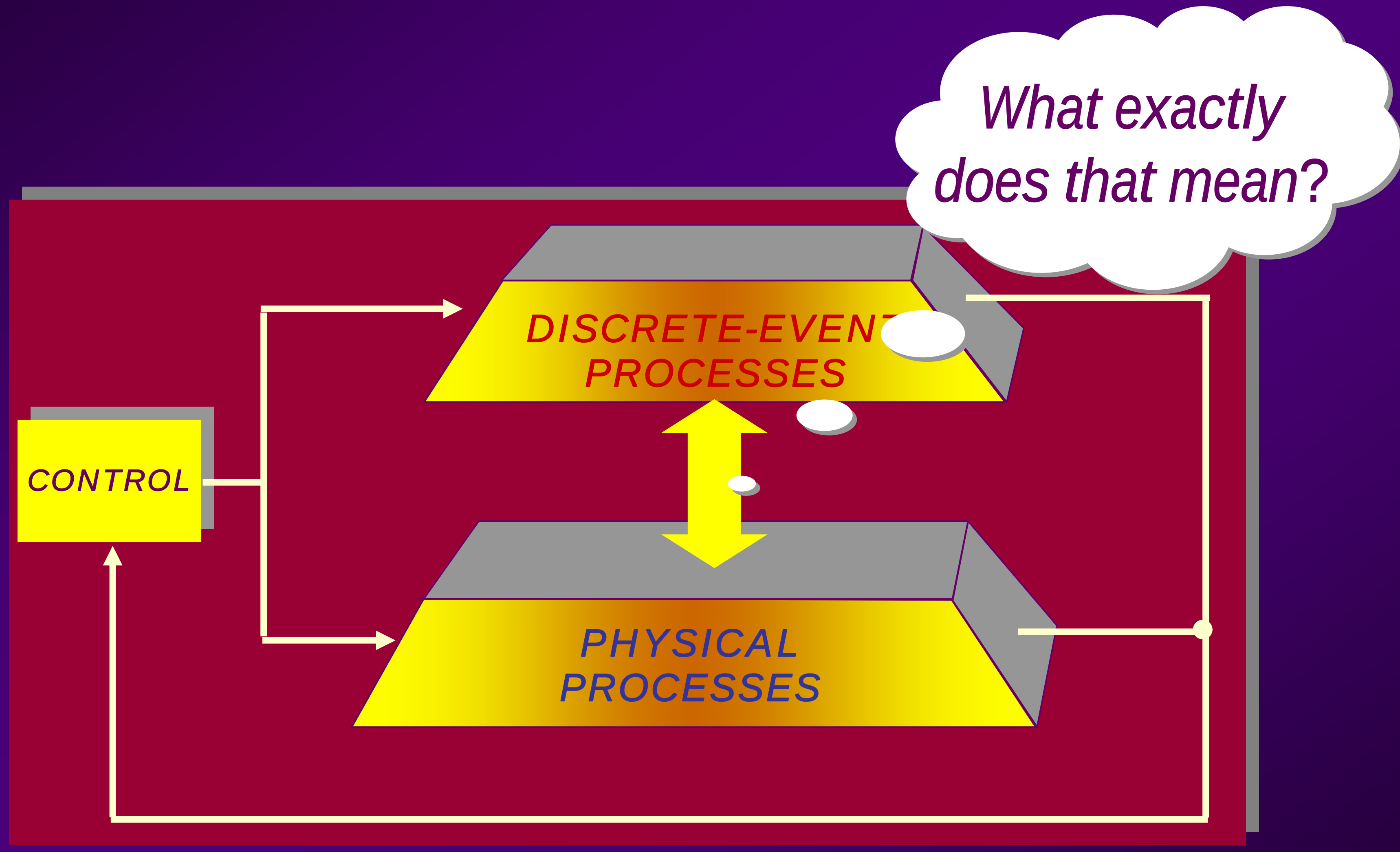


HIEARARCHICAL DECOMPOSITION

CONTINUED

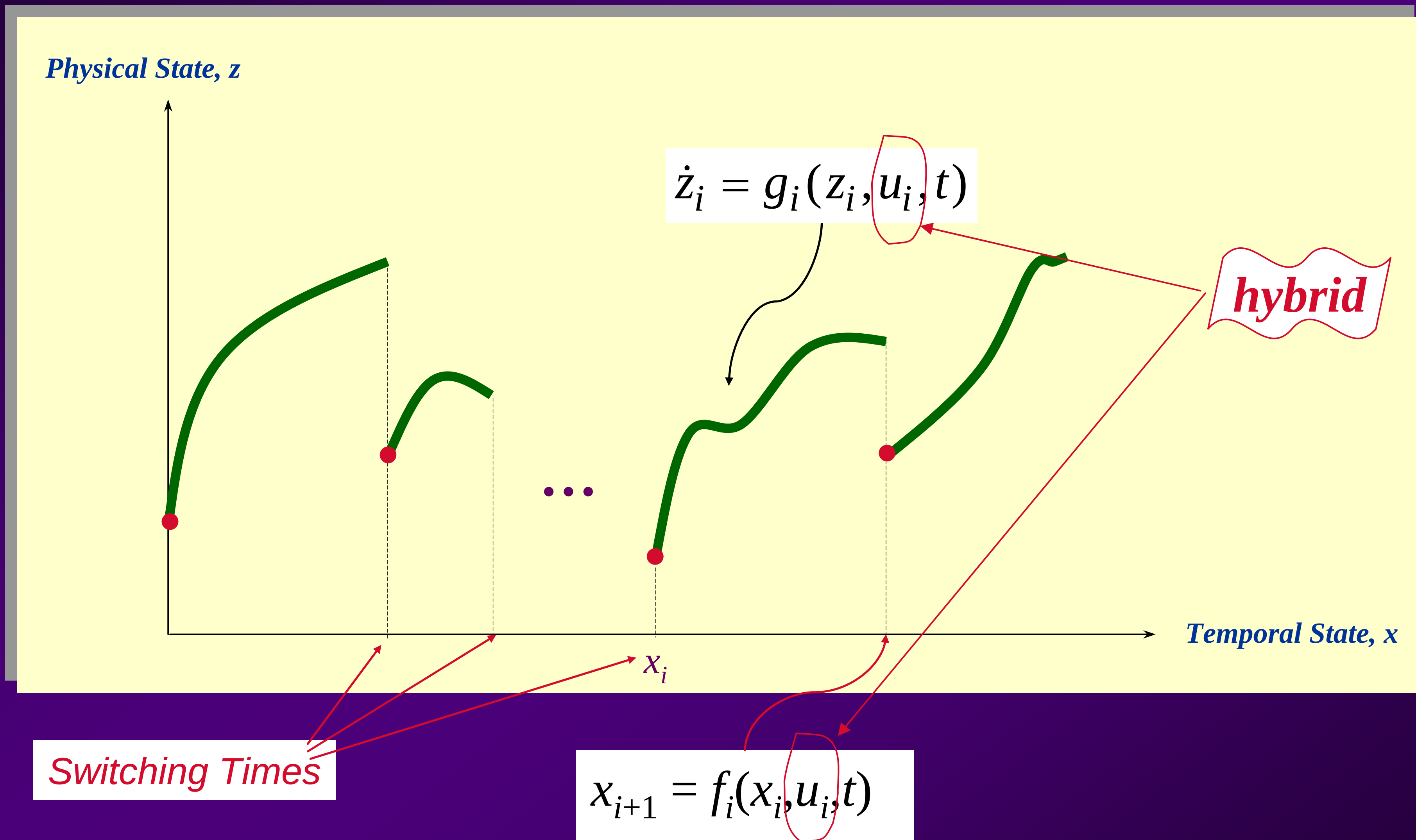


HYBRID CONTROL SYSTEM



OPTIMAL CONTROL

WHAT'S A HYBRID SYSTEM?



OPTIMAL CONTROL PROBLEMS

- Get to a desired final physical state z_N in minimum time x_N, subject to N-1 switching events

- Minimize deviations from N desired physical states: $(z_i - q_i)^2$
  and
  deviations from target desired times: $(x_i - \tau_i)^2$

In general:

$$\min_{\mathbf{u}} \sum_{i=1}^{N} \int_{x_{i-1}}^{x_i} L_i(z_i(t), u_i(t))dt \quad s.t. \begin{cases} \dot{z}_i = g_i(z_i, u_i, t) \\ x_{i+1} = f_i(x_i, u_i, t) \end{cases}$$

Temporal state

Physical state

OPTIMAL CONTROL PROBLEMS

CONTINUED

Problems we consider:

$$\min_{\mathbf{u}} \sum_{i=1}^{N} \left[\phi_i(x_i, x_{i-1}) + \psi_i(x_i) \right]$$

Cost under $\mathbf{u}_i(t)$ *over* $[x_{i-1}, x_i]$

Cost of switching time x_i

where:

$$\phi_i(x_i, x_{i-1}) = \int_{x_{i-1}}^{x_i} L_i(z_i(t), u_i(t))dt$$

Let: $s_i = x_i - x_{i-1}$

Time spent at ith operating mode

Assuming stationarity:

$$\phi_i(x_i, x_{i-1}) = \phi_i(s_i)$$

HIERARCHICAL DECOMPOSITION

$$\min_{\mathbf{u}} \sum_{i=1}^{N} \left[\phi_i(s_i) + \psi_i(x_i)\right] \quad s.t. \begin{cases} \dot{z}_i = g_i(z_i, u_i, t) \\ x_{i+1} = f_i(x_i, u_i, t) \end{cases}$$

HIGHER LEVEL PROBLEM:

$$\min_{\mathbf{s}} \sum_{i=1}^{N} \left[\phi_i^*(s_i) + \psi_i(x_i)\right]$$

s.t.

$$x_{i+1} = f_i(x_i, s_i, t)$$

LOWER LEVEL PROBLEMS:

$$\min_{u_i} \phi_i(s_i) = \int_0^{s_i} L_i(z_i(t), u_i(t))dt$$

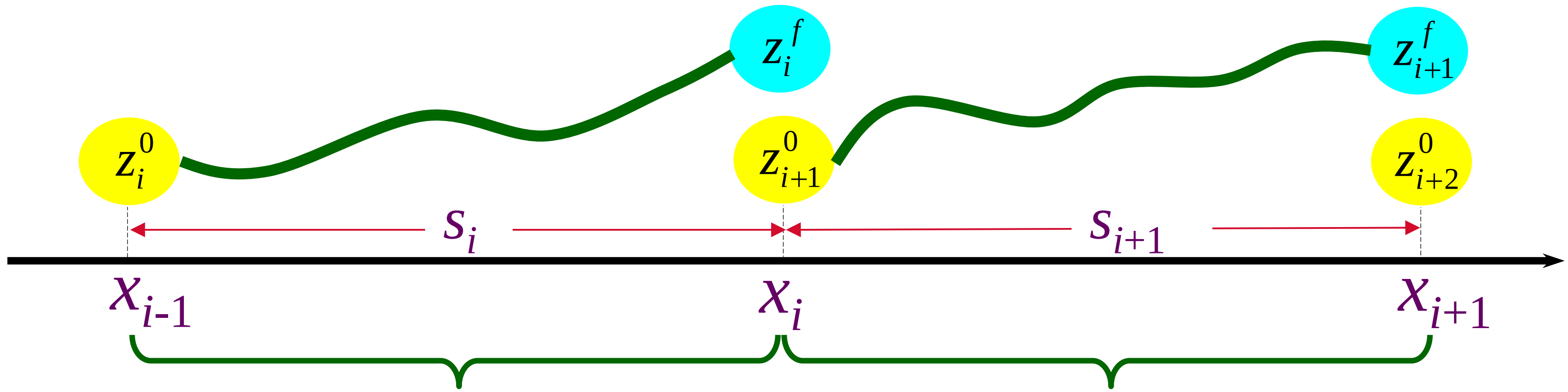
s.t.

$$\dot{z}_i = g_i(z_i, u_i, t)$$

FIXED s_i

HIERARCHICAL DECOMPOSITION

CONTINUED



$$\min_{u_i(z_i^0, z_i^f, s_i)} \phi_i(s_i) \qquad \min_{u_{+1i}(z_{i+1}^0, z_{i+1}^f, s_{i+1})} \phi_{i+1}(s_{i+1})$$

$$u_i^*(z_i^0, z_i^f, s_i)$$

$$\theta_i(z_i^0, z_i^f, s_i) \equiv \min_{u_i} \phi_i(z_i, u_i, s_i)$$

...

THE *REALLY* CHALLENGING PROBLEM!

$$\min_{\mathbf{z}^0, \mathbf{z}^f, \mathbf{s}} \sum_{i=1}^{N} \left[\theta_i(z_i^0, z_i^f, s_i) + \psi_i(x_i) \right]$$

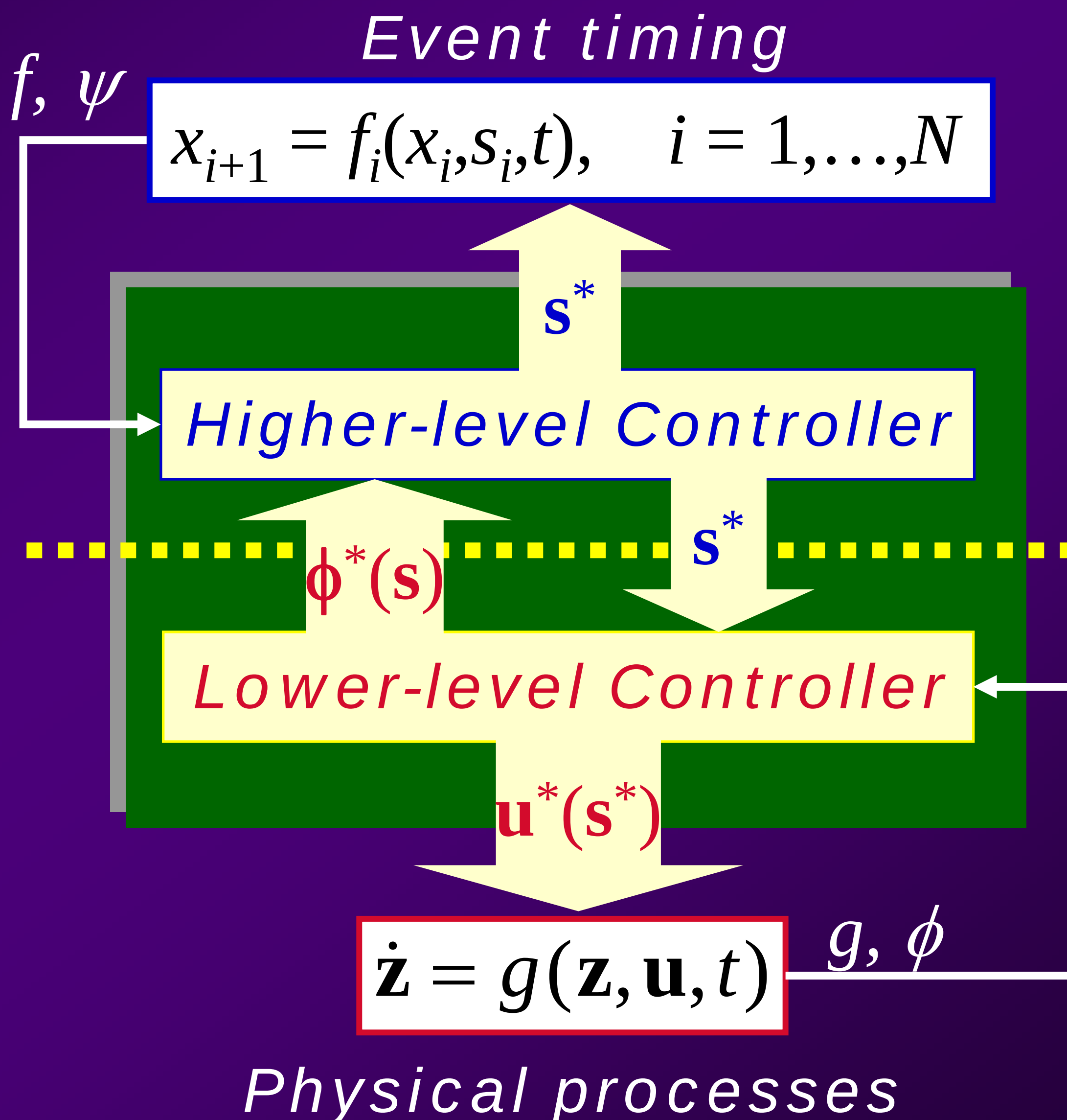
s.t.

$$x_{i+1} = f_i(x_i, u_i, t)$$

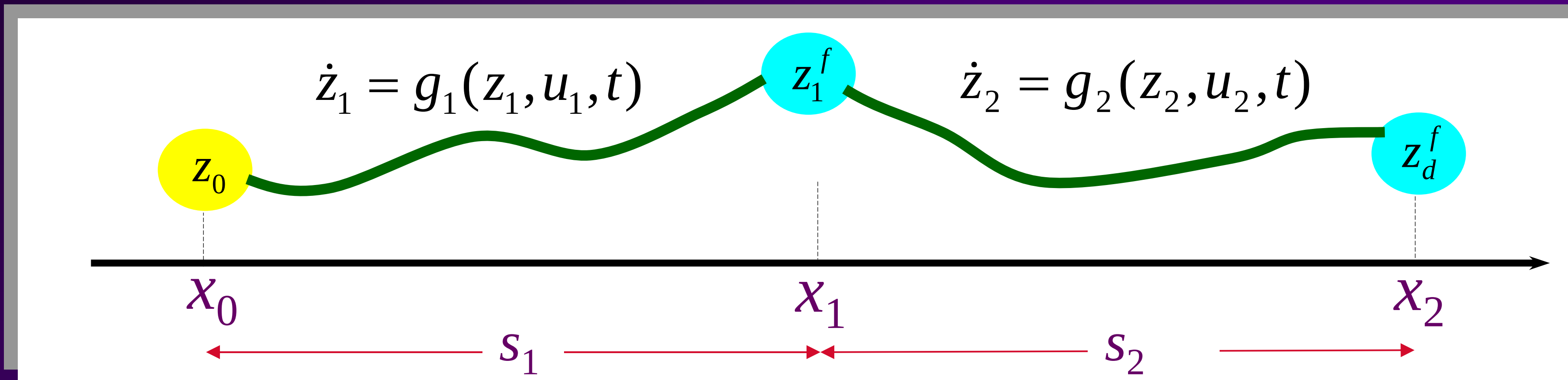
HYBRID CONTROLLER STRUCTURE

Hybrid controller steps:

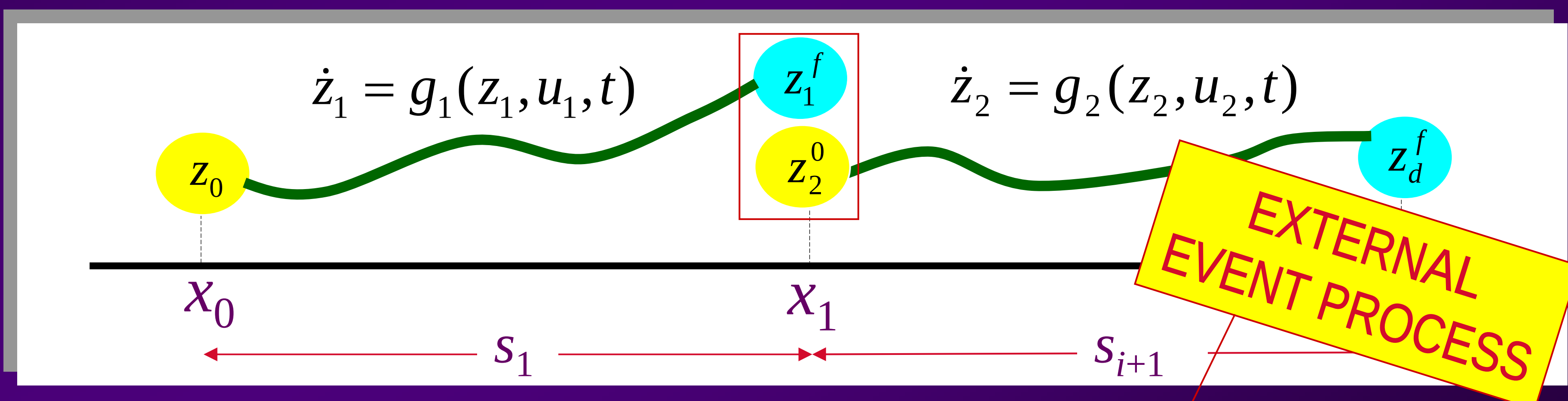
- System identification
- Lower-level solution
- Higher-level solution
- Lower-level solution
- Operation...



TWO TYPES OF PROBLEMS…

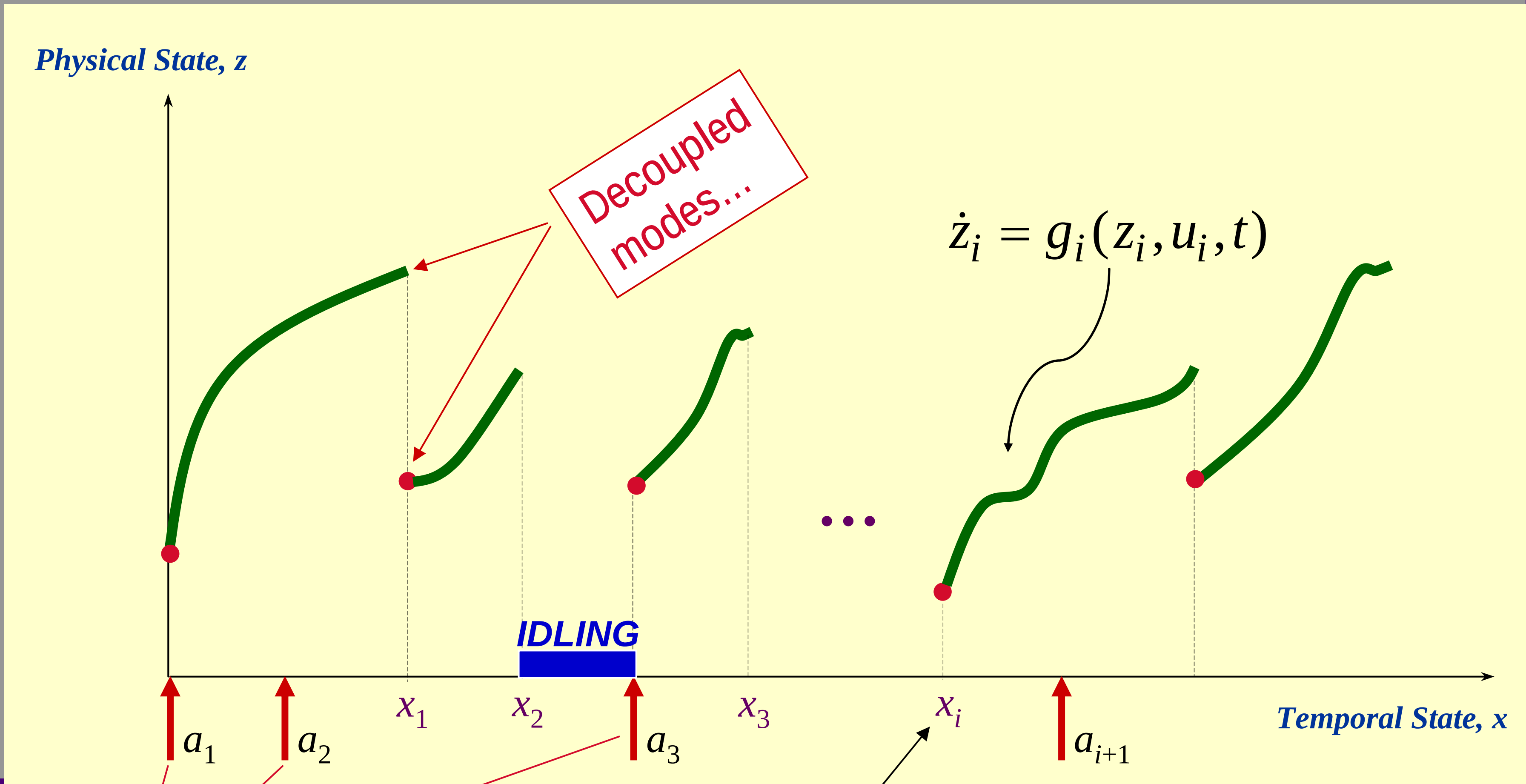


1. A single event process controls switching dynamics: $x_{i+1} = x_i + s_i(z_i, u_i)$



2. Multiple event processes control switching dynamics: $x_{i+1} = \max(x_i, a_{i+1}) + s_i(z_i, u_i)$

EXAMPLE

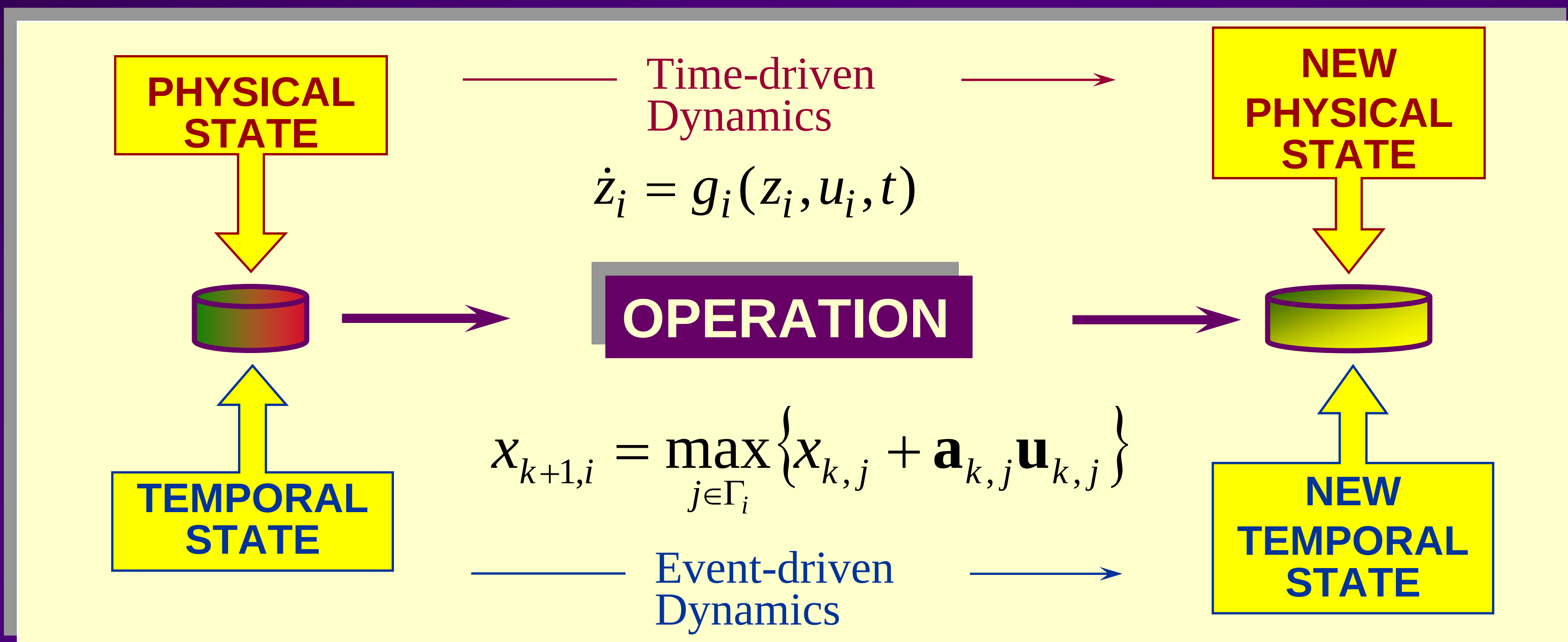


$$x_i = \max\{x_{i-1}, a_i\} + s_i(z_i, u_i)$$

External event process: *i*th mode cannot start before a_{i+1}

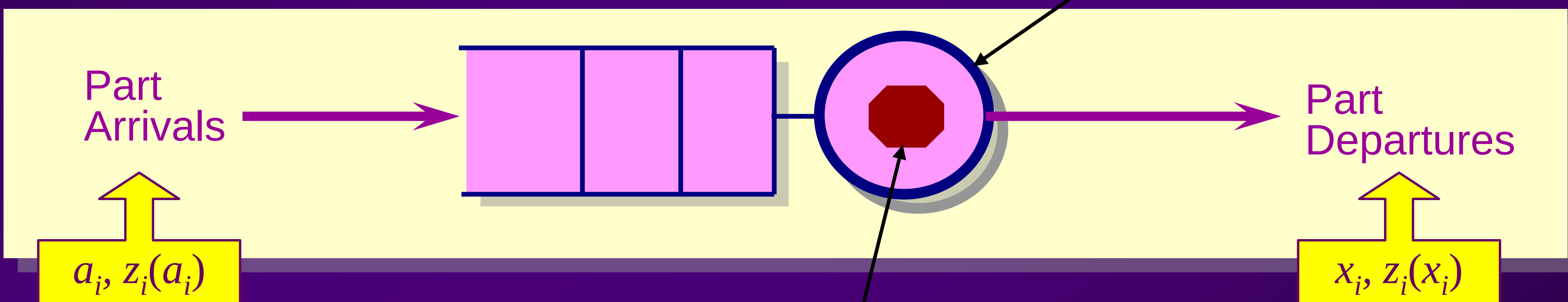
Throughout a manuf. process, each part is characterized by

- A **PHYSICAL** state (e.g., size, temperature, strain)
- A **TEMPORAL** state (e.g., total time in system, total time to due-date)



EVENT-DRIVEN COMPONENT

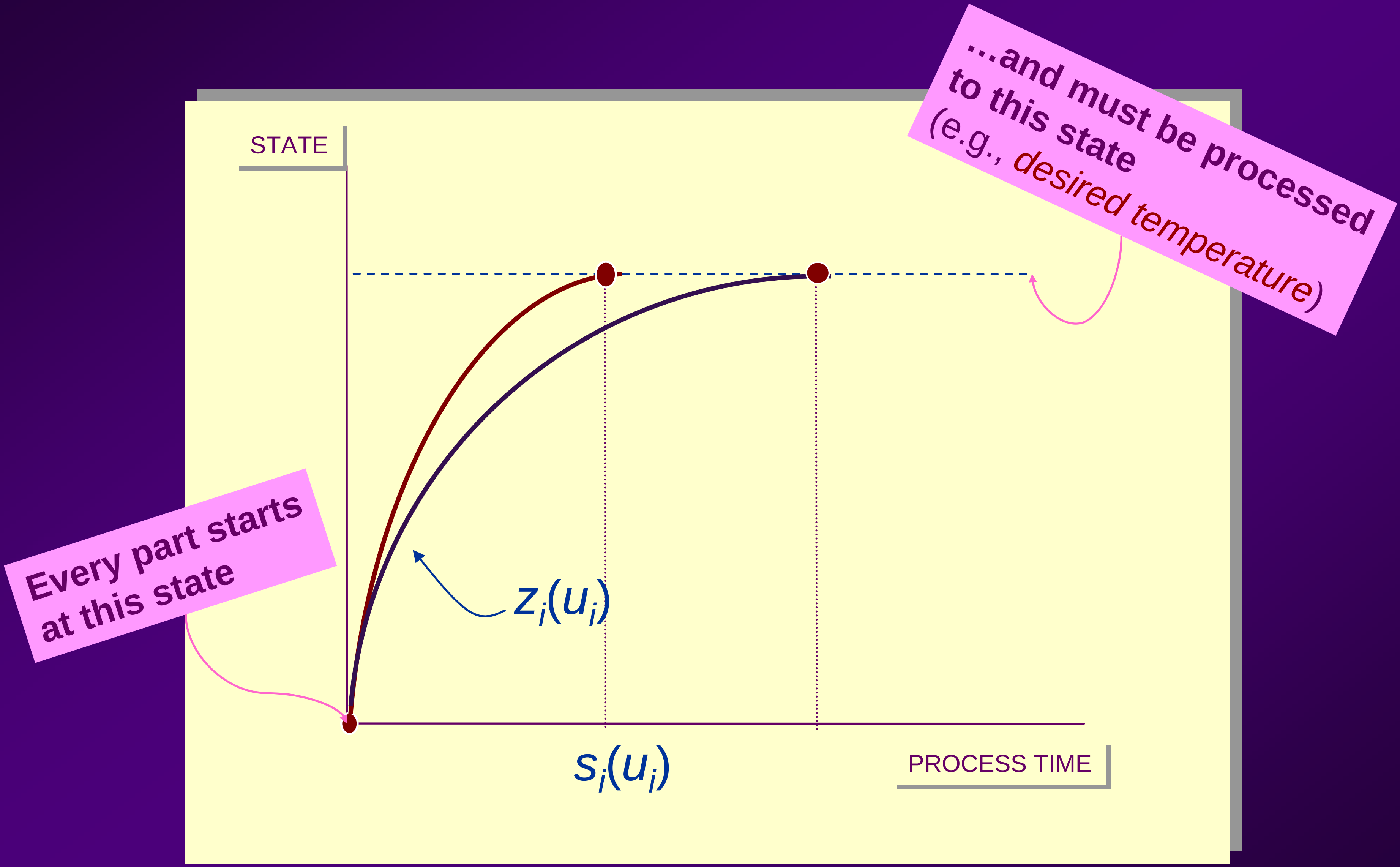
$$x_i = \max\{x_{i-1}, a_i\} + s_i(u_i)$$



TIME-DRIVEN COMPONENT

$$u_i \longrightarrow \dot{z}_i(t) = g(z_i, u_i, t)$$

EXAMPLE



STATE
…and must be processed to this state
(e.g., desired temperature)
$z_i(u_i)$
Every part starts at this state
$s_i(u_i)$
PROCESS TIME

LOWER LEVEL PROBLEM

Parameterized by switching times

LQ PROBLEM:

$$\min_{u_i} \phi_i(z_i, u_i, s_i) = \frac{1}{2} h(z_{fi} - z_{di})^2 + \int_0^{s_i} \frac{1}{2} r u_i^2(t) dt$$

$$s.t. \quad \dot{z}_i = a z_i + b u_i, \quad z_i(0) = \varsigma_i$$

Penalize final state deviation

STANDARD LQ SOLUTION METHOD:

$$\phi_i^*(s_i) = \frac{1}{2} h(z_{fi}^* - z_{di})^2 + \int_0^{s_i} \frac{1}{2} r u_i^*(t) dt$$

HIGHER LEVEL PROBLEM

$$\min_{\mathbf{s}} \sum_{i=1}^{N} \left[\phi_i^*(s_i) + \psi_i(x_i) \right] \quad s.t. \quad x_i = \max\{x_{i-1}, a_i\} + s_i(u_i)$$

Cost of optimal process control over interval [0, s_i]

Cost related to event timing

Given arrival sequence (INPUT)

Processing time (CONTOLLABLE)

$$\text{EXAMPLE :} \quad \psi_i(x_i) = (x_i - \tau_i)^2$$

HOW DO WE SOLVE THE HIGHER LEVEL PROBLEM?

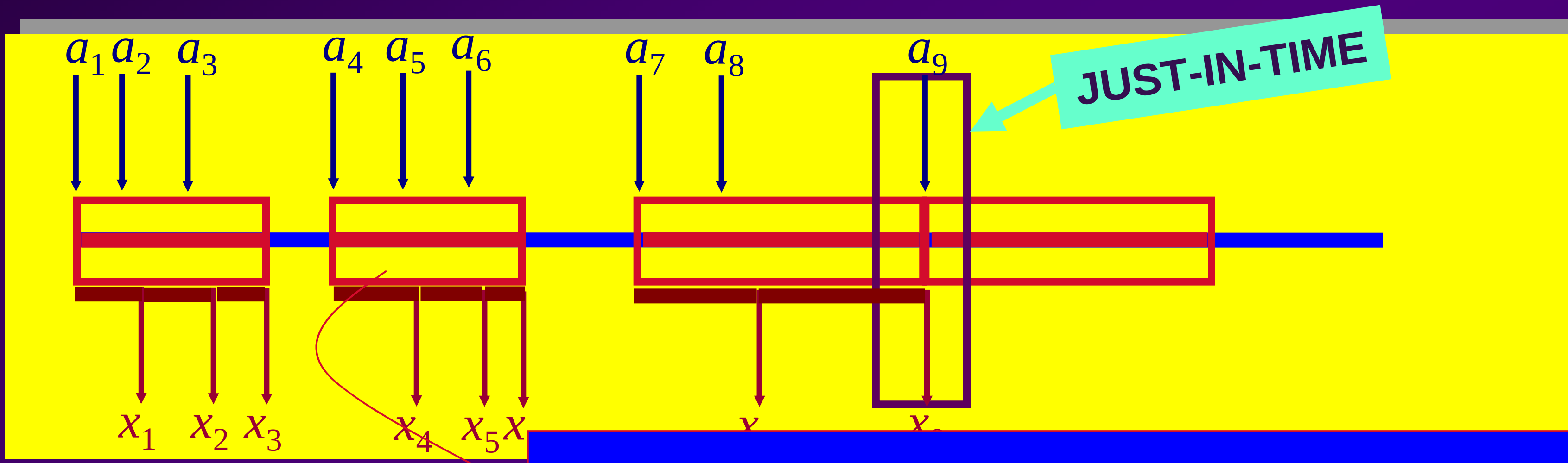
$$\min_{\mathbf{s}} \sum_{i=1}^{N} \left[\phi_i^*(s_i) + \psi_i(x_i) \right] \quad s.t. \quad \boxed{x_i = \max\{x_{i-1}, a_i\} + s_i(u_i)}$$

Even if these are convex,
problem is still NOT convex in s!

Causes nondifferentiabilities!

Even though problem is
NONDIFFERENTIABLE and NONCONVEX,
optimal solution shown to be *unique*.

[*Cassandras, Pepyne, Wardi, IEEE TAC 2001*]



Each "block" corresponds to a *Constrained Convex Optimization* problem

$\Rightarrow$ search over 2^{N-1} possible *Constrained Convex Optimization* problems

BUT

algorithms that only need *N Constrained Convex Optimization* problems have been developed $\Rightarrow$ SCALEABILITY

[*Cho, Cassandras, Intl. J. Rob. and Nonlin.Control, 2001*]

INTERACTIVE JAVA APPLETS

- *http://vita.bu.edu/cgc/hybrid*

  - Single-stage model

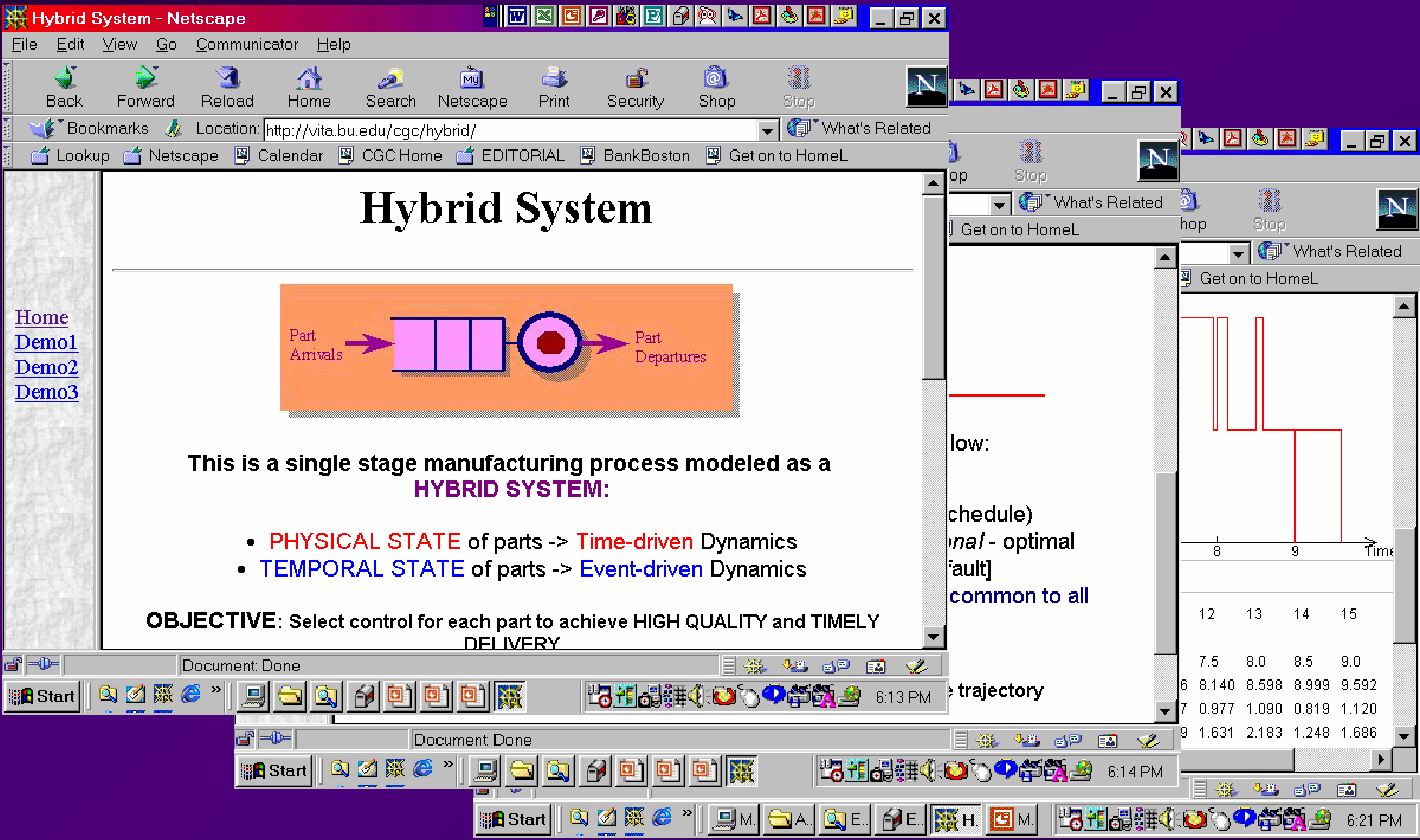
  - Backward-recursive TPBVP solver with critical job identification

- *http://vita.bu.edu/cgc/newhybrid*

  - 3-stage model

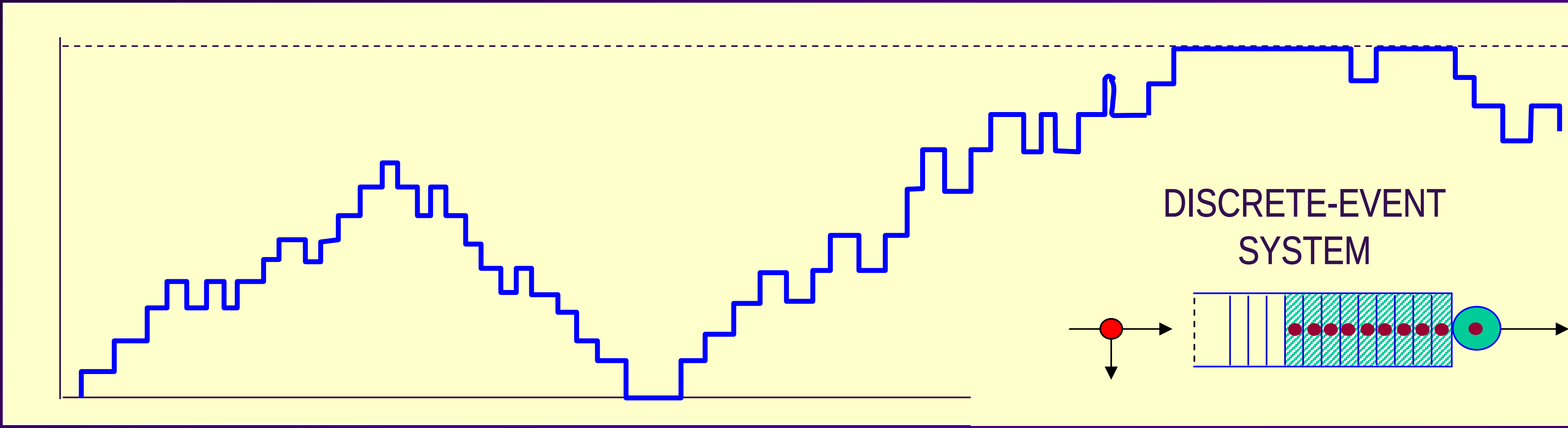
  - Bezier approximation with standard TPBVP solver

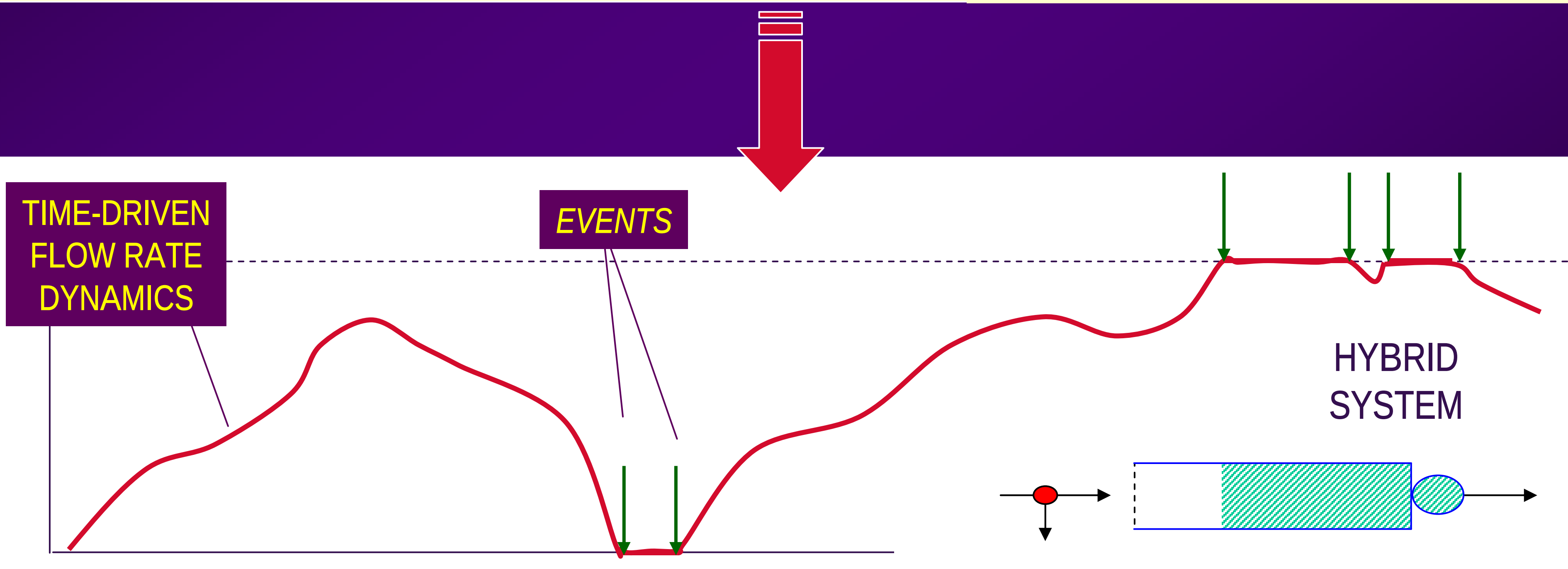
http://vita.bu.edu/cgc/hybrid



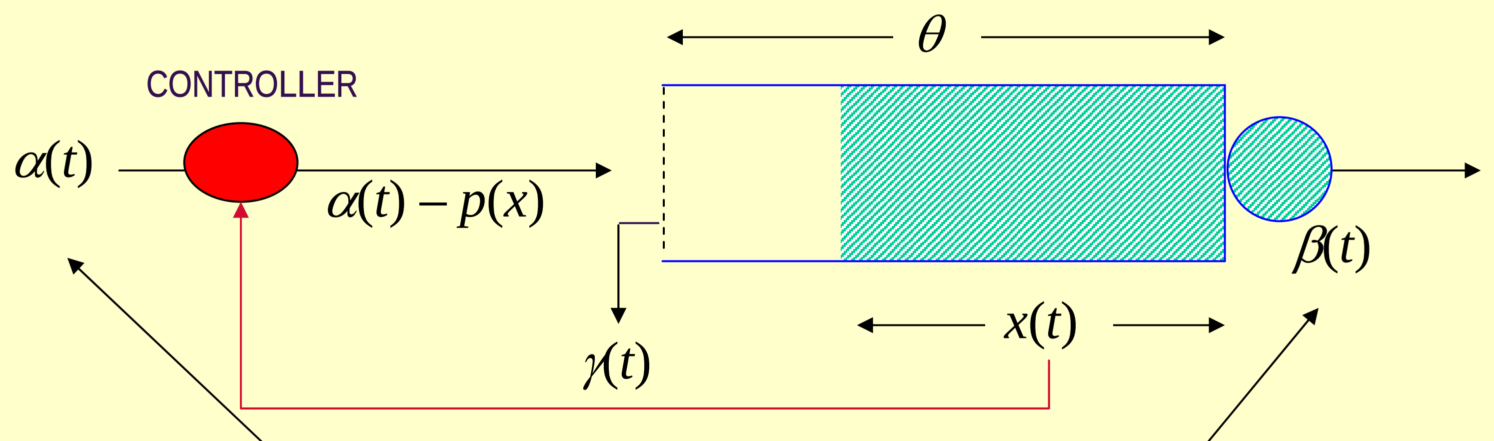
ABSTRACTION

COMPLEX DES ABSTRACTION





STOCHASTIC FLOW MODELS (SFM)



$\alpha(t)$, $\beta(t)$: *arbitrary stochastic processes (piecewise continuously differentiable)*

$$\frac{dx}{dt} = \begin{cases} 0 & x(t)=0,\ \lambda(t)-p(0)\le 0 \\ 0 & x(t)=\theta,\ \lambda(t)-p(\theta)\ge 0 \\ \lambda(t)-p(x(t)) & \text{otherwise} \end{cases}$$

$\lambda(t) = \alpha(t) - \beta(t)$

feedback

WHY SFM?

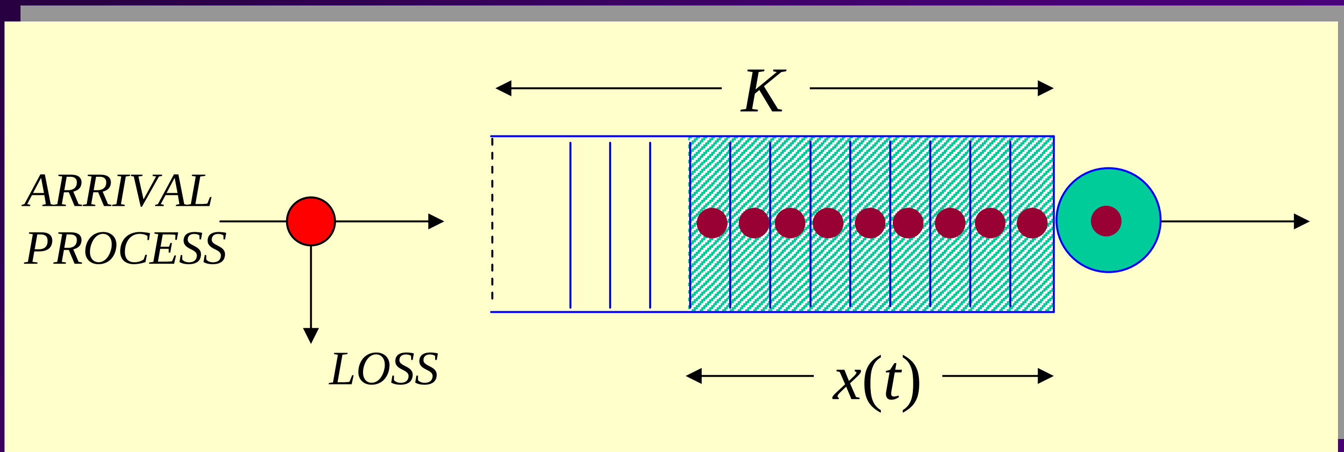
- "Lower resolution" model of "real" system intended to capture *just enough* info. on system dynamics

- Aggregates many events into simple continuous dynamics, preserves only events that cause drastic change

  $\Rightarrow$ computationally efficient
  (e.g., *orders of magnitude faster simulation*)

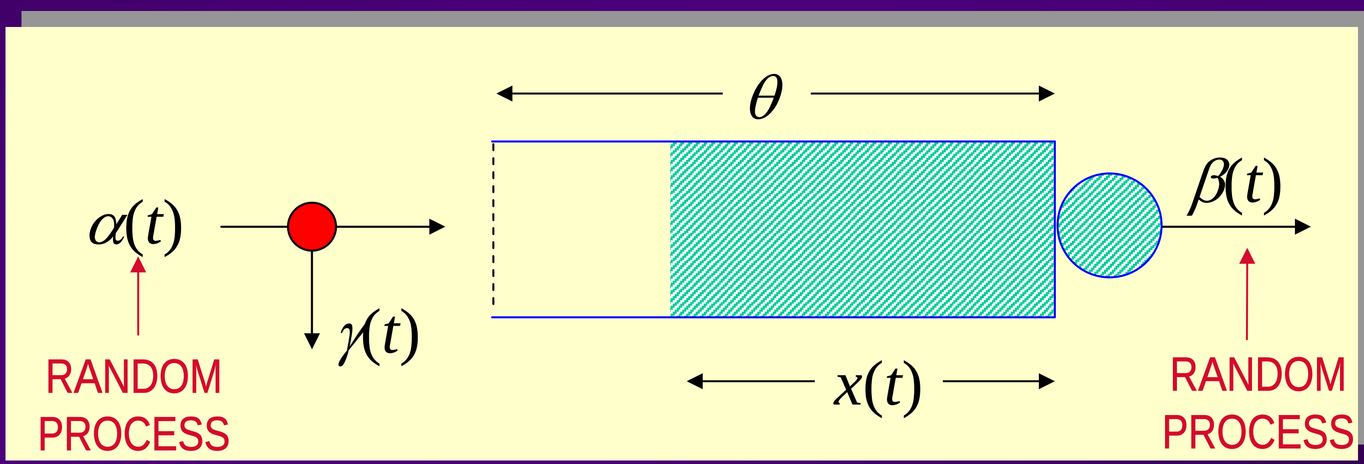
- Intended for developing CONTROL schemes rather than for PERFORMANCE ANALYSIS

MOTIVATING EXAMPLE: *THRESHOLD-BASED BUFFER CONTROL*



"REAL" SYSTEM

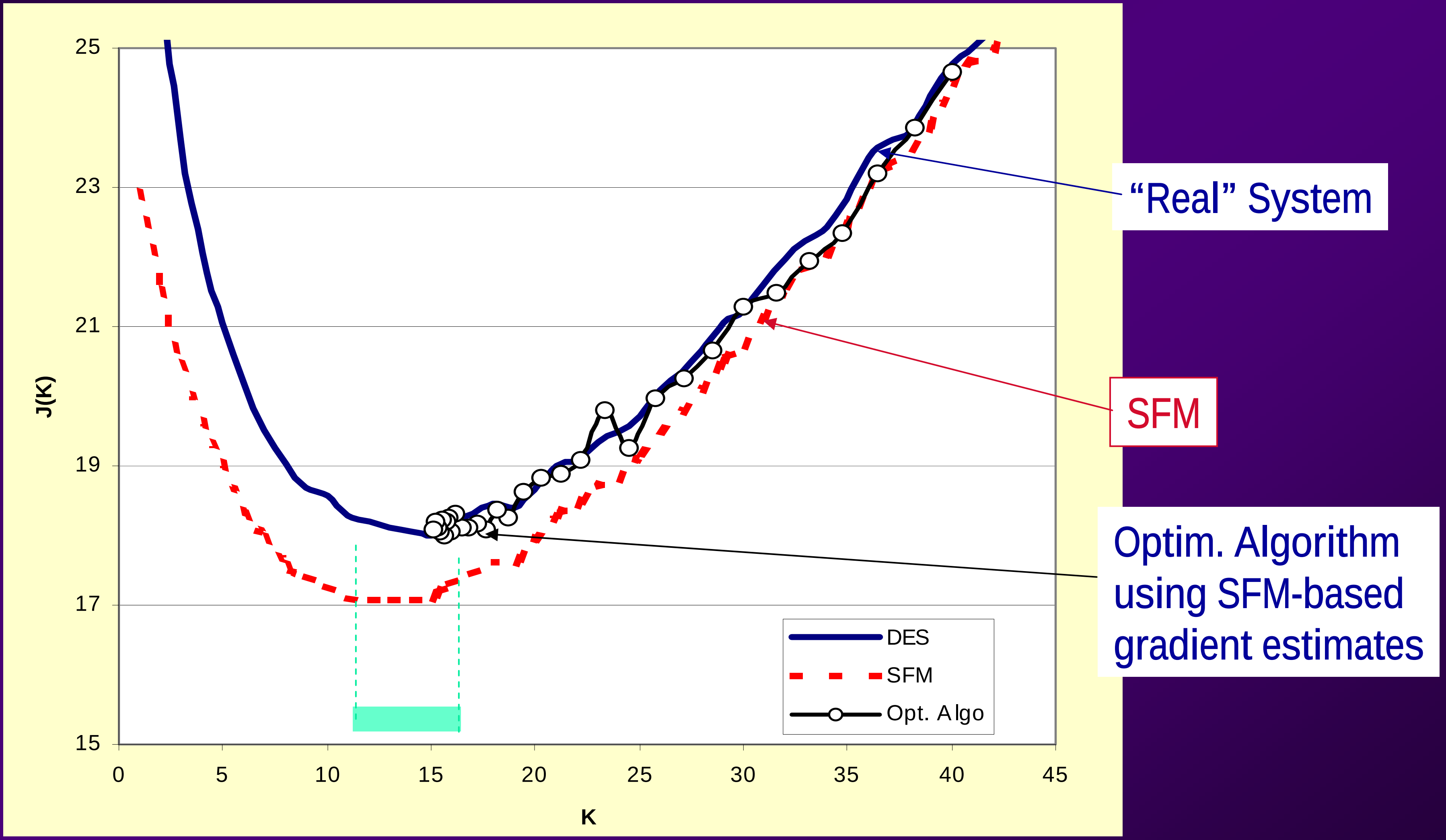
$$J_T(K) \;=\; \bar{Q}_T(K) + R \cdot \bar{L}_T(K)$$



Stochastic Flow Model (SFM)

$$J_T^{SFM}(\theta) \;=\; \bar{Q}_T^{SFM}(\theta) + R \cdot \bar{L}_T^{SFM}(\theta)$$

MOTIVATING EXAMPLE *CONTINUED*



Cassandras et al, 2002

OPTIMIZATION OF SFMS

- *Stochastic* Optimal Control problems too hard!..

- Parametric optimization:
  use gradient estimates with on-line opt. algorithms

$$\theta_{n+1} = \theta_n - \eta_n H_n(\theta_n, \omega_n^{SFM}), \quad n = 0, 1, \ldots$$

- Need efficient ways to estimate performance sensitivities

INFINITESIMAL PERTURBATION ANALYSIS (IPA)

"Brute Force" Sensitivity Estimation:

$\theta \longrightarrow$ DES $\longrightarrow \hat{J}(\theta)$

$\theta+\Delta\theta \longrightarrow$ DES $\longrightarrow \hat{J}(\theta+\Delta\theta)$

$$\Rightarrow \left[\frac{dJ}{d\theta}\right]_{est} = \frac{\hat{J}(\theta+\Delta\theta)-\hat{J}(\theta)}{\Delta\theta}$$

Finite Perturbation Analysis (FPA):

θ, $\Delta\theta \longrightarrow$ DES $\longrightarrow \hat{J}(\theta)$

DES $\longrightarrow$ FPA $\longrightarrow \hat{J}(\theta+\Delta\theta)$

$$\Rightarrow \left[\frac{dJ}{d\theta}\right]_{est} = \frac{\hat{J}(\theta+\Delta\theta)-\hat{J}(\theta)}{\Delta\theta}$$

Infinitesimal or *Smoothed* Perturbation Analysis (IPA, SPA):

$\theta \longrightarrow$ DES $\longrightarrow \hat{J}(\theta)$

DES $\longrightarrow$ IPA/SPA $\longrightarrow \left[\frac{dJ}{d\theta}\right]_{est}$

INFINITESIMAL PERTURBATION ANALYSIS (IPA)

OBJECTIVES:

- Obtain sample performance derivatives that depend ONLY on observed sample path data:

Performance

$$L'_T(\theta) \equiv \frac{dL_T(\theta)}{d\theta}$$

Parameter

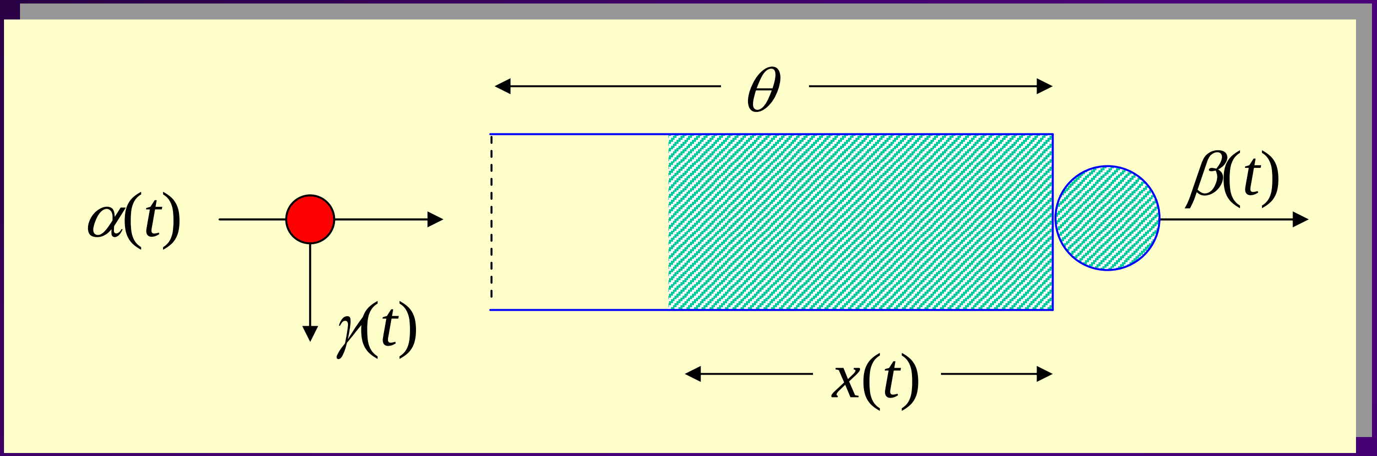
- Prove unbiasedness:

$$\frac{dE[L_T(\theta)]}{d\theta} = E\left[\frac{dL_T(\theta)}{d\theta}\right]$$

- Then, use gradient estimates to drive on-line opt. algorithms:

$$\theta_{n+1} = \theta_n + \eta_n L'_T(\theta_n), \quad n = 0,1,\ldots$$

THRESHOLD-BASED BUFFER CONTROL



$$J_T(\theta) = Q_T(\theta) + RL_T(\theta)$$

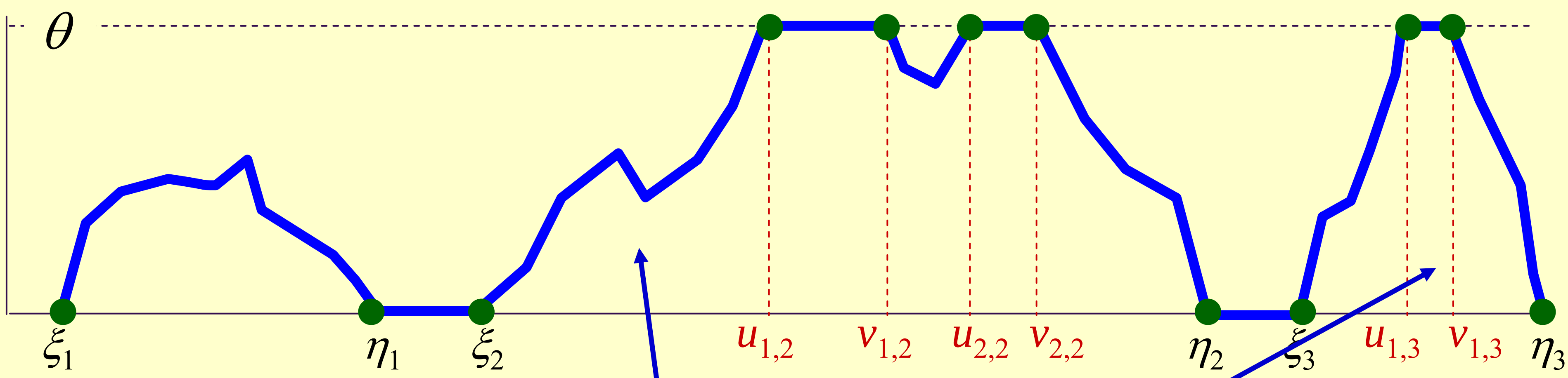
WORK

LOSS

$$Q_T(\theta) = \int_0^T x(\theta;t)dt$$

$$L_T(\theta) = \int_0^T \gamma(\theta;t)dt$$

THRESHOLD-BASED BUFFER CONTROL *CONTINUED*



BUFFERING PERIOD: $\mathcal{B}_k = (\xi_k, \eta_k(\theta)), \quad k = 1, \ldots, K$

OVERFLOW PERIOD: $\mathcal{F}_{i,k} = [u_{i,k}(\theta), v_{i,k}], \; i = 1, \ldots, M$

SET OF BPs WITH AT LEAST ONE OVERFLOW:

$\Phi(\theta) \coloneqq \{k \in \{1, \ldots, K\} \; : \; x(t) = \theta, \; \alpha(t) - \beta(t) > 0$ for some $t \in (\xi_k, \eta_k(\theta))\}$

$B(\theta) = |\Phi(\theta)|$

IPA – MAIN RESULTS

Cassandras et al, 2002

THEOREM 1 (*Loss* IPA derivative):

$$L_T'(\theta) = -B(\theta)$$

- Simple *count* of Buffering Periods with at least one overflow
- Nonparametric (independent of θ and any model assumption)

THEOREM 2 (*Work* IPA derivative):

$$Q_T'(\theta) = \sum_{k\in\Phi(\theta)} [\underbrace{\eta_k(\theta) - u_{k,1}(\theta)}]$$

- Simple *timer* for each Buffering Period with at least one overflow
- Nonparametric (independent of θ and any model assumption)

Let $\boldsymbol{N(T)}$ be the number of events in $\boldsymbol{[0, T]}$.

THEOREM:

1. If $E[N(T)] < \infty$**, then** $L_T^{'}(\theta)$ **is an unbiased estimator of** $\dfrac{dE[L_T(\theta)]}{d\theta}$

2. $Q_T^{'}(\theta)$ **is an unbiased estimator of** $\dfrac{dE[Q_T(\theta)]}{d\theta}$

THE FUTURE...

- COMPLEXITY

  - GETTING AROUND IT THROUGH STRUCTURAL PROPERTIES

  - FINDING THE "RIGHT" MODELING RESOLUTION

- COMPUTATIONAL CHALLENGES

  - REAL-TIME IMPLEMENTATIONS, EMBEDDED SYSTEMS

  - SOFTWARE TOOLS (for *Verification*, *Optimization*, etc.)

- APPLICATION DOMAINS

  COOPERATIVE CONTROL, AUTOMOTIVE,

  COMPUTER NETWORKS, LOGISTICS, BIOMEDICAL

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… All 13 Former

+ Current PhD Students

…Co[illegible]ag[illegible]s

Y.C. [illegible]

Wei- [illegible] ***Go*** [illegible]

Yorai Wardi

Young Ch [illegible]

Xiren Cao

Ted Dj [illegible] ***ris***

MUCHAS GRACIAS

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