FROM DISCRETE EVENT TO HYBRID SYSTEMS

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OUTLINE

- HYBRID SYSTEMS IN COMPLEX SYSTEM...
  ... *DECOMPOSITION* AND *ABSTRACTION*

- WHAT’S A HYBRID SYSTEM...

- *DECOMPOSITION*: HYBRID SYSTEM $\rightarrow$ DES
  - SOLVING OPTIMAL CONTROL PROBLEMS

- *ABSTRACTION*: DES $\rightarrow$ HYBRID SYSTEM
  - ANALYSIS OF STOCHASTIC FLOW MODELS (SFM)

- THE FUTURE...
TIME-DRIVEN SYSTEMS

EVENT-DRIVEN SYSTEMS

HYBRID SYSTEMS

CONTINUOUS

DISCRETE

1980

1990

2000
HYBRID SYSTEM

TIME-DRIVEN SYSTEM

EVENT-DRIVEN SYSTEM

LESS COMPLEX

MORE COMPLEX

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What exactly does that mean?
TIME-DRIVEN SYSTEM

HYBRID SYSTEM

EVENT-DRIVEN SYSTEM

LESS COMPLEX

MORE COMPLEX

ABSTRACTION (AGGREGATION)

ZOOM OUT
WHAT IS THE RIGHT ABSTRACTION LEVEL?

TOO FAR...
model not
detailed enough

TOO CLOSE...
too much
undesirable
detail

JUST RIGHT...
good model

CREDIT: W.B. Gong

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WHAT'S A HYBRID SYSTEM?
WHAT’S A HYBRID SYSTEM?

\[ \dot{z}_1 = g_1(z_1, u_1, t) \]

\[ x_1 = f_1(x_0, z_1, u_1, t) \]

\[ \dot{z}_2 = g_2(z_2, u_2, t) \]

\[ x_2 = f_2(x_1, z_2, u_2, t) \]
WHAT’S A HYBRID SYSTEM?

TIME DRIVEN:
\[ \dot{z}_i = g_i(z_i, u_i, t) \]

EVENT-DRIVEN:
\[ x_{i+1} = f_i(x_i, z_i, u_i, t) \]

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Physical State, \( z \)

\[
\dot{z}_i = g_i(z_i, u_i, t)
\]

\[
x_{i+1} = f_i(x_i, u_i, t)
\]

SWITCHING TIMES HAVE THEIR OWN DYNAMICS!

WHAT’S A HYBRID SYSTEM?

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CONTINUED
1. Autonomous Switching, e.g., Hysteresis

\[ \dot{x} = \begin{cases} x + u & x < \Delta \\ -x + u & x \geq \Delta \\ x - u & x \leq -\Delta \end{cases} \]
2. External Switching, e.g., Zeno’s bouncing ball

\[
\begin{align*}
\dot{x} &= v_x, \quad \dot{v}_x = 0 \\
\dot{y} &= v_y, \quad \dot{v}_y = -mg
\end{align*}
\]
3. Controlled Switching, e.g., *Interconnected tanks*
Key questions facing manufacturing system integrators:

• How to integrate ‘process control’ with ‘operations control’?

• How to improve product quality within reasonable time?

**PROCESS CONTROL**

- Physicists
- Material Scientists
- Chemical Engineers
- ...

**OPERATIONS CONTROL**

- Industrial Engineers, OR
- Schedulers
- Inventory Control
- Factory Control
- ...

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HYBRID SYSTEM EXAMPLES - MANUFACTURING
Throughout a manuf. process, each part is characterized by

- A **PHYSICAL** state (e.g., size, temperature, strain)
- A **TEMPORAL** state (e.g., total time in system, total time to due-date)
COOPERATIVE CONTROL

EVENT: threat sensed

TIME-DRIVEN DYNAMICS

BASE

TARGET

EVENT: info. communicated by team member

THREATS
GENERAL MODELING FRAMEWORKS

• Hybrid Automata
  - [Branicky et al., 1998]

• Mixed Logical Dynamical Systems
  - [Bemporad and Morari, 1999]

• etc.
  - [Proc. of IEEE Special Issue, 2000]
**Time-Driven Dynamics** ($STATE = z$):

\[
\dot{z}(t) = g(z,u,t) \quad \text{or:} \quad z_{k+1} = g_k(z_k,u_k)
\]

**Event-Driven Dynamics** ($STATE = \text{Event Times } x_{k,i}$):

\[
x_{k+1,i} = \max_{j \in \Gamma_i} \{ x_{k,j} + a_{k,j} u_{k,j} \}
\]

Event counter $k = 1,2,...$

Event index $i \in E = \{1,...,n\}$
DECOMPOSITION
HIERARCHICAL DECOMPOSITION

PLANNING

DISCRETE-EVENT PROCESSES

PHYSICAL PROCESSES

???
HYBRID CONTROL SYSTEM

What exactly does that mean?
WHAT’S A HYBRID SYSTEM?

Physical State, $z$

Temporal State, $x$

$\dot{z}_i = g_i(z_i, u_i, t)$

$x_{i+1} = f_i(x_i, u_i, t)$

Switching Times

hybrid

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OPTIMAL CONTROL PROBLEMS

- Get to a desired final physical state $z_N$ in minimum time $x_N$, subject to $N-1$ switching events.

- Minimize deviations from $N$ desired physical states: $(z_i - q_i)^2$
  and deviations from target desired times: $(x_i - \tau_i)^2$

In general:

$$\min_u \sum_{i=1}^{N} \int_{x_{i-1}}^{x_i} L_i(z_i(t),u_i(t))dt$$

subject to

$$\dot{z}_i = g_i(z_i,u_i,t)$$
$$x_{i+1} = f_i(x_i,u_i,t)$$

Temporal state

Physical state
OPTIMAL CONTROL PROBLEMS

Problems we consider:

\[
\min_u \sum_{i=1}^{N} [\phi_i(x_i, x_{i-1}) + \psi_i(x_i)]
\]

Cost under \(u_i(t)\) over \([x_{i-1}, x_i]\)

Cost of switching time \(x_i\)

where:

\[
\phi_i(x_i, x_{i-1}) = \int_{x_{i-1}}^{x_i} L_i(z_i(t), u_i(t))dt
\]

Let: \(s_i = x_i - x_{i-1}\)

Assuming stationarity:

\[
\phi_i(x_i, x_{i-1}) = \phi_i(s_i)
\]
HIERARCHICAL DECOMPOSITION

\[
\min_{u} \sum_{i=1}^{N} \left[ \phi_i(s_i) + \psi_i(x_i) \right]
\]

s.t.
\[
\dot{z}_i = g_i(z_i, u_i, t)
\]
\[
x_{i+1} = f_i(x_i, u_i, t)
\]

HIGHER LEVEL PROBLEM:

\[
\min_{s} \sum_{i=1}^{N} \left[ \phi_i^*(s_i) + \psi_i(x_i) \right]
\]

s.t.
\[
x_{i+1} = f_i(x_i, s_i, t)
\]

LOWER LEVEL PROBLEMS:

\[
\min_{u_i} \phi_i(s_i) = \int_{0}^{s_i} L_i(z_i(t), u_i(t)) dt
\]

s.t.
\[
\dot{z}_i = g_i(z_i, u_i, t)
\]
HIERARCHICAL DECOMPOSITION CONTINUED

\[ \min_{u_i(z_i^0, z_i^f, s_i)} \phi_i(s_i) \]

\[ \min_{u_{i+1}(z_{i+1}^0, z_{i+1}^f, s_{i+1})} \phi_{i+1}(s_{i+1}) \]

\[ u_i^*(z_i^0, z_i^f, s_i) \]

\[ \theta_i(z_i^0, z_i^f, s_i) = \min_{u_i} \phi_i(z_i, u_i, s_i) \]

\[ \min_{z_i^0, z_i^f, s_i} \sum_{i=1}^{N} \left[ \theta_i(z_i^0, z_i^f, s_i) + \psi_i(x_i) \right] \]

s.t.

\[ x_{i+1} = f_i(x_i, u_i, t) \]

THE REALLY CHALLENGING PROBLEM!
HYBRID CONTROLLER STRUCTURE

Hybrid controller steps:

- System identification
- Lower-level solution
- Higher-level solution
- Operation...

Higher-level Controller

Lower-level Controller

Event timing

\[ x_{i+1} = f_i(x_i, s_i, t), \quad i = 1, \ldots, N \]

Physical processes

\[ \dot{z} = g(z, u, t) \]
TWO TYPES OF PROBLEMS...

1. A single event process controls switching dynamics: \[ x_{i+1} = x_i + s_i(z_i,u_i) \]

2. Multiple event processes control switching dynamics: \[ x_{i+1} = \max(x_i, a_{i+1}) + s_i(z_i,u_i) \]
External event process: $i$th mode cannot start before $a_{i+1}$

\[ x_i = \max \{x_{i-1}, a_i\} + s_i(z_i, u_i) \]

Decoupled modes...

\[ \dot{z}_i = g_i(z_i, u_i, t) \]
Throughout a manuf. process, each part is characterized by

- A **PHYSICAL** state (e.g., size, temperature, strain)
- A **TEMPORAL** state (e.g., total time in system, total time to due-date)

\[ \dot{z}_i = g_i(z_i, u_i, t) \]

\[ x_{k+1,i} = \max_{j \in \Gamma_i} \left\{ x_{k,j} + a_{k,j} u_{k,j} \right\} \]

**Time-driven Dynamics**

**Event-driven Dynamics**
\[ x_i = \max \{x_{i-1}, a_i\} + s_i(u_i) \]

\[ \dot{z}_i(t) = g(z_i, u_i, t) \]
Every part starts at this state

...and must be processed to this state (e.g., desired temperature)

\[ z_i(u_i) \]

\[ s_i(u_i) \]
LOWER LEVEL PROBLEM

LQ PROBLEM:

$$\min_{u_i} \phi_i(z_i, u_i, s_i) = \frac{1}{2} h(z_{fi} - z_{di})^2 + \int_0^{s_i} \frac{1}{2} ru_i^2(t)dt$$

s.t. \quad \dot{z}_i = az_i + bu_i, \quad z_i(0) = \zeta_i$$

Parameterized by switching times

Penalize final state deviation

STANDARD LQ SOLUTION METHOD:

$$\phi_i^*(s_i) = \frac{1}{2} h(z_{fi}^* - z_{di})^2 + \int_0^{s_i} \frac{1}{2} r u_i^*(t)dt$$

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HIGHER LEVEL PROBLEM

\[ \min_s \sum_{i=1}^{N} \left[ \phi_i^*(s_i) + \psi_i(x_i) \right] \quad \text{s.t.} \quad x_i = \max \{ x_{i-1}, a_i \} + s_i(u_i) \]

**Cost of optimal process control over interval \([0, s]\)**

**Given arrival sequence (INPUT)**

**Processing time (CONTROLLABLE)**

**EXAMPLE:** \[ \psi_i(x_i) = (x_i - \tau_i)^2 \]
How do we solve the higher level problem?

\[ \min_s \sum_{i=1}^{N} \left[ \phi_i^*(s_i) + \psi_i(x_i) \right] \quad \text{s.t.} \quad x_i = \max \left\{ x_{i-1}, a_i \right\} + s_i(u_i) \]

Even if these are convex, problem is still NOT convex in \( s \! \! \text{!} \)

Even though problem is \textbf{NONDIFFERENTIABLE} and \textbf{NONCONVEX}, optimal solution shown to be \textit{unique}.

\[ \text{[Cassandras, Pepyne, Wardi, IEEE TAC 2001]} \]
Each “block” corresponds to a \textit{Constrained Convex Optimization} problem

\[ \Rightarrow \text{search over } 2^{N-1} \text{ possible } \textit{Constrained Convex Optimization} \text{ problems} \]

\textbf{BUT}

\[ \text{algorithms that only need } N \text{ } \textit{Constrained Convex Optimization} \text{ problems have been developed } \Rightarrow \text{SCALEABILITY} \]

• **http://vita.bu.edu/cgc/hybrid**
  - Single-stage model
  - Backward-recursive TPBVP solver with critical job identification

• **http://vita.bu.edu/cgc/newhybrid**
  - 3-stage model
  - Bezier approximation with standard TPBVP solver
Hybrid System

This is a single stage manufacturing process modeled as a HYBRID SYSTEM:

- PHYSICAL STATE of parts -> Time-driven Dynamics
- TEMPORAL STATE of parts -> Event-driven Dynamics

OBJECTIVE: Select control for each part to achieve HIGH QUALITY and TIMELY DELIVERY

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ABSTRACTION
COMPLEX DES ABSTRACTION

HYBRID SYSTEM

DISCRETE-EVENT SYSTEM

TIME-DRIVEN FLOW RATE DYNAMICS

EVENTS

HYBRID SYSTEM

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\[ \alpha(t) - p(x) \]

\[ \lambda(t) = \alpha(t) - \beta(t) \]

\[ \frac{dx}{dt} = \begin{cases} 
0 & x(t) = 0, \lambda(t) - p(0) \leq 0 \\
0 & x(t) = \theta, \lambda(t) - p(\theta) \geq 0 \\
\lambda(t) - p(x(t)) & \text{otherwise} 
\end{cases} \]
“Lower resolution” model of “real” system intended to capture just enough info. on system dynamics

Aggregates many events into simple continuous dynamics, preserves only events that cause drastic change

⇒ computationally efficient
  (e.g., orders of magnitude faster simulation)

Intended for developing CONTROL schemes rather than for PERFORMANCE ANALYSIS
MOTIVATING EXAMPLE: THRESHOLD-BASED BUFFER CONTROL

ARRIVAL PROCESS $\rightarrow$ LOSS $\rightarrow$ $x(t)$ $\rightarrow$ “REAL” SYSTEM

$J_T(K) = \tilde{Q}_T(K) + R \cdot \tilde{L}_T(K)$

$\alpha(t)$ $\rightarrow$ $\gamma(t)$ $\rightarrow$ $x(t)$ $\rightarrow$ RANDOM PROCESS

$\theta$ $\rightarrow$ $\beta(t)$ $\rightarrow$ RANDOM PROCESS

Stochastic Flow Model (SFM)

$J_T^{SFM}(\theta) = \tilde{Q}_T^{SFM}(\theta) + R \cdot \tilde{L}_T^{SFM}(\theta)$

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MOTIVATING EXAMPLE

"Real" System

SFM

Optim. Algorithm using SFM-based gradient estimates

Cassandras et al, 2002

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Stochastic Optimal Control problems too hard!

Parametric optimization:
use gradient estimates with on-line opt. algorithms

\[ \theta_{n+1} = \theta_n - \eta_n H_n(\theta_n, \omega_n^{SFM}), \quad n = 0, 1, \ldots \]

Need efficient ways to estimate performance sensitivities
"Brute Force" Sensitivity Estimation:

\[ \frac{dJ}{d\theta}_{est} = \frac{\hat{J}(\theta + \Delta \theta) - \hat{J}(\theta)}{\Delta \theta} \]

Finite Perturbation Analysis (FPA):

\[ \frac{dJ}{d\theta}_{est} = \frac{\hat{J}(\theta + \Delta \theta) - \hat{J}(\theta)}{\Delta \theta} \]

Infinitesimal or Smoothed Perturbation Analysis (IPA, SPA):

\[ \left[ \frac{d J}{d \theta} \right]_{est} \]
OBJECTIVES:

• Obtain sample performance derivatives that depend ONLY on observed sample path data:

\[ L'_T(\theta) \equiv \frac{dL_T(\theta)}{d\theta} \]

• Prove unbiasedness:

\[ \frac{dE[L_T(\theta) \theta]}{d\theta} = E\left[ \frac{dL_T(\theta)}{d\theta} \right] \]

• Then, use gradient estimates to drive on-line opt. algorithms:

\[ \theta_{n+1} = \theta_n + \eta_n L'_T(\theta_n), \quad n = 0, 1, \ldots \]
THRESHOLD-BASED BUFFER CONTROL

\[ J_T(\theta) = Q_T(\theta) + RL_T(\theta) \]

\[ Q_T(\theta) = \int_0^T x(\theta; t) dt \]

\[ L_T(\theta) = \int_0^T \gamma(\theta; t) dt \]
BUFFERING PERIOD: \[ \mathcal{B}_k = (\xi_k, \eta_k(\theta)) \quad k = 1, \ldots, K \]

OVERFLOW PERIOD: \[ \mathcal{F}_{i,k} = [u_{i,k}(\theta), v_{i,k}] \quad i = 1, \ldots, M \]

SET OF BPs WITH AT LEAST ONE OVERFLOW:
\[ \Phi(\theta) \coloneqq \{k \in \{1, \ldots, K\} : x(t) = \theta, \alpha(t) - \beta(t) > 0 \text{ for some } t \in (\xi_k, \eta_k(\theta))\} \]
\[ \mathcal{B}(\theta) = |\Phi(\theta)| \]
THEOREM 1 (Loss IPA derivative):

\[ L_T'(\theta) = -B(\theta) \]

- Simple count of Buffering Periods with at least one overflow
- Nonparametric (independent of \( \theta \) and any model assumption)

THEOREM 2 (Work IPA derivative):

\[ Q_T'(\theta) = \sum_{k \in \Phi(\theta)} [\eta_k(\theta) - u_{k,1}(\theta)] \]

- Simple timer for each Buffering Period with at least one overflow
- Nonparametric (independent of \( \theta \) and any model assumption)
Let $N(T)$ be the number of events in $[0, T]$.

**THEOREM:**

1. If $E[N(T)] < \infty$, then $L_T'(\theta)$ is an unbiased estimator of $\frac{dE[L_T(\theta)]}{d\theta}$

2. $Q_T'(\theta)$ is an unbiased estimator of $\frac{dE[Q_T(\theta)]}{d\theta}$
THE FUTURE...

➢ COMPLEXITY
  - GETTING AROUND IT THROUGH STRUCTURAL PROPERTIES
  - FINDING THE “RIGHT” MODELING RESOLUTION

➢ COMPUTATIONAL CHALLENGES
  - REAL-TIME IMPLEMENTATIONS, EMBEDDED SYSTEMS
  - SOFTWARE TOOLS (for Verification, Optimization, etc.)

➢ APPLICATION DOMAINS
  COOPERATIVE CONTROL, AUTOMOTIVE,
  COMPUTER NETWORKS, LOGISTICS, BIOMEDICAL
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