Energy-Based Lifetime Maximization and Security of Wireless-Sensor Networks With General Nonideal Battery Models

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***Abstract*—We study the problem of maximizing the lifetime of a sensor network by means of routing and initial energy allocation over its nodes. We consider a general state space battery model and show that similar results to our previous work with simpler battery dynamics are still valid. In particular, we show that under this general dynamic battery model, there exists an optimal policy consisting of time-invariant routing probabilities in a fixed topology network and these can be obtained by solving a set of nonlinear programming (NLP) problems. Moreover, we show that the problem can be reformulated as a single NLP problem. In addition, we consider a joint routing and initial energy allocation problem over the network nodes with the same network lifetime maximization objective. We prove that the solution to this problem is given by a policy that depletes all node energies at the same time and that the corresponding energy allocation and routing probabilities are obtained by solving an NLP problem. Finally, we examine a network's performance under security threats, typified by faked-cost attacks, in terms of its lifetime and its normalized throughput. We illustrate how the optimal routing probabilities, as well as the network lifetime, are robust under such forms of routing attacks even though its normalized throughput can be significantly reduced.**

***Index Terms*—Energy-aware systems, optimal control, optimization, sensor networks .**

I. Introduction

A WIRELESS-SENSOR network (WSN) is formed by small autonomous nodes communicating over wireless links. Nodes have sensing, processing, and communicating capabilities. They are mainly battery powered and tightly constrained in terms of energy, processing, and storage capacities, therefore requiring careful resource management [2]. Applications of such networks include exploration, surveillance, and environmental monitoring. Power consumption is a key issue in WSNs, since it directly impacts their lifetime in the likely absence of human intervention for most applications of interest. Since the majority of power consumption is due to the radio component [3], nodes rely on short-range communication and form a multihop network to deliver information to a base station.

Routing schemes in WSNs aim to deliver data from the data sources (nodes with sensing capabilities) to a data sink (typically, a base station) in an energy-efficient and reliable way. A survey of several routing algorithms may be found in [4]. Most proposed algorithms are based on shortest path routing, for example, [5] and [6], or multipath approaches, for example, [7], and may indirectly reduce energy usage, but they do not explicitly use energy consumption models to address the optimality of a routing policy with respect to energy-aware metrics. Such "energy awareness" has motivated a number of minimum-energy routing algorithms which typically seek paths minimizing the energy per packet consumed, for example, [8]. However, seeking a minimum energy path can rapidly deplete energy from some nodes and ultimately reduce the full network's lifetime by destroying its connectivity. Shah and Rabaey [9] proposed an energy-aware routing (EAR) policy which does not attempt to use a single optimal path, but rather a number of suboptimal paths that are probabilistically selected. In [10], a similar problem is studied with the inclusion of uncertainties in several WSN parameters. In a network utility maximization framework, [11] proposes a fully asynchronous distributed algorithm based on dual decomposition for a general utility function, while [12] proposes centralized and decentralized algorithms to solve a multiperiod scheduling problem considering energy constraints and periodic sensing requirements.

The importance of prolonging the lifetime of a WSN has motivated studies of routing with *network lifetime* as an explicit performance metric. This is usually defined as the time until the first node depletes its battery [1], [13]. In [13], this optimal routing problem was solved based on two assumptions: 1) a battery is "ideal" in the sense that it depletes linearly with respect to the quantity of information forwarded, independent of physical dynamics of the battery itself and 2) only fixed routing probabilities over time were sought. In a recent paper [1], we addressed this problem with the goal of determining routing probabilities in order to maximize the lifetime of a WSN subject to a *dynamic* energy consumption model for each node, thus relaxing both of these assumptions. In particular, we used a kinetic battery model (KBM) [14]–[16] for the batteries powering the WSN nodes and proved that in a fixed network topology, there exists an optimal policy consisting of

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time-invariant routing probabilities determined through a set of relatively simple nonlinear programming (NLP) problems. We also considered a problem where, in addition to routing, we allocate total initial energy over the network nodes with the same network lifetime maximization objective. We showed that the solution to this problem is given by a policy that depletes all node energies *at the same time* and that the corresponding energy allocation and routing probabilities are obtained by solving an NLP problem. The conclusion from [1], therefore, is that even when the dynamic behavior of batteries is taken into account, the solution of the network lifetime maximization problem is robust to the battery behavior and leads to optimal routing policies which are static, similar to those obtained in [13] under the simplifying assumptions of ideal batteries and static routing. Furthermore, the solution of the optimal routing problem in [1] leads to individual node lifetimes being the same or almost the same as those of others, hence, the definition of network "lifetime" as the time until the first node depletes its battery is indeed a good characterization of the overall network's lifetime.

In view of these results, the question we address in this paper is whether considering different, more elaborate, nonideal battery models preserves the time-invariant nature of an optimal routing policy as shown in [1]. In other words, is the relatively simple nature of the KBM previously used to capture battery behavior responsible for this property or is this inherent in the problem regardless of how detailed a battery model one uses? There are three specific contributions in this paper. First, we generalize the results obtained in [1] for both the optimal routing and the joint routing and initial energy allocation problems for lifetime maximization by adopting the most general nonideal battery model available in the literature and show that the time-invariant nature of a maximal network lifetime routing policy is preserved. This leads to the conclusion that optimal policies for WSNs are indeed robust with respect to the battery model used, although, naturally, the corresponding network lifetime value may be very different (therefore, accurately predicting the lifetime benefits from the increased accuracy of such general nonideal battery models.) The second contribution is to reduce the computational complexity of the method used in [1] for deriving an optimal routing policy. In particular, in [1], this was accomplished by solving a set of NLP problems, whereas here we provide a much more efficient single NLP formulation.

The third contribution of this paper is to investigate WSN performance under common forms of security threats. This is motivated by the fact that energy-aware routing policies are often probabilistic in nature, thus making it harder for attackers to identify an "ideal node" to take over. At the same time, such a probabilistic routing policy can be easily implemented as a deterministic policy as well by simply transforming these probabilities to packet flows over links. We explore the network performance under one of the most severe routing attacks in WSN, namely, the sink-hole attack [17]. Although we limit ourselves to a simple empirical study, it becomes clear that the optimal policy we have derived is significantly more robust to common forms of cyberattacks than other proposed energy-aware routing policies.

In Section II, we formulate the maximum lifetime optimization problem using nonideal energy sources based on a detailed energy consumption model due to Rakhmatov *et al.* in [18]. In Section III, we show that for a fixed network topology, there exists an optimal routing policy which is time invariant and we identify a set of NLP problems which can be solved to obtain an explicit fixed optimal routing vector and the corresponding WSN lifetime. In view of the existence of fixed optimal routing probabilities, we also introduce a single NLP problem which results in optimal routing and lifetime at the same time. We also show that this optimal policy is robust with respect to the battery model used. In Section IV, we consider a joint optimal routing and initial energy allocation problem and show that it is optimal to set a routing vector and initial node energies so that all nodes have the same lifetime. In Section V, we analyze the network performance when the network is under a "sink hole" type of routing attack in terms of its normalized throughput as a performance metric. In particular, we compare the WSN performance by adopting our optimal routing policy and the energy-aware routing policy introduced in [19]. Numerical examples are included to illustrate our analytical results.

II. Optimal Control Problem Formulation

A. Network Model

We begin by reviewing the WSN model used in [1], with a single source node and one base station and fixed topology. Consider a network with $N+1$ nodes where 0 and N denote the source and destination (base station) nodes, respectively. Except for the base station whose energy supply is not constrained, a limited amount of energy is available to all other nodes. Let $r_i(t)$ be the residual energy of node i, $i = 0, \dots, N-1$, at time t. The dynamics of $r_i(t)$ depend on the battery model used at node i, which will be presented in the next subsection. The distance between nodes i and j at time t is denoted by $d_{i,j}(t)$; since we assume a fixed topology, we will treat $d_{i,j}(t)$ as time-invariant in the sequel. The nodes in the network may be ordered according to their distance to the destination node N so that $d_{1,N} \geq d_{2,N} \geq \cdots \geq d_{i,N} \geq \cdots \geq d_{N-1,N}$ and assume that $d_{0,N} > d_{i,N}$ for all $i = 1, \dots, N-1$.

Let O_i denote the set of nodes to which node i can send packets. We assume full coverage of the network and define $O_i = \{j : j > i,\ d_{i,j} < d_{i,N}\}$, where $j > i$ implies that $d_{i,N} > d_{j,N}$, that is, a node only sends packets to those nodes that are closer to the destination, and $d_{i,j} < d_{i,N}$ means that a node cannot send packets to another node which is further away from it relative to the destination node N. We will use the notation $i \prec j$, if $j \in O_i$. Let $w_{i,j}(t)$ be the routing probability of a packet from node i to node j at time t. The vector $w(t) = [w_{0,1}(t), \dots, w_{0,N-1}(t), \dots, w_{N-2,N-1}(t)]'$ defines the control in our problem. We do not include $w_{0,N}(t), \dots, w_{i,N}(t), \dots, w_{N-1,N}(t)$ in the definition of $w(t)$, since it is clear that $w_{i,N}(t)$ is an implicit control variable given by $w_{i,N}(t) = 1 - \sum_{i \prec j,\ j<N} w_{i,j}(t),\ i = 0, \dots, N-2$.

For simplicity, the data sending rate of source node 0 is normalized to 1 and let $G_i(w)$ denote the data packet inflow rate to node i. Given the definitions from before, we can

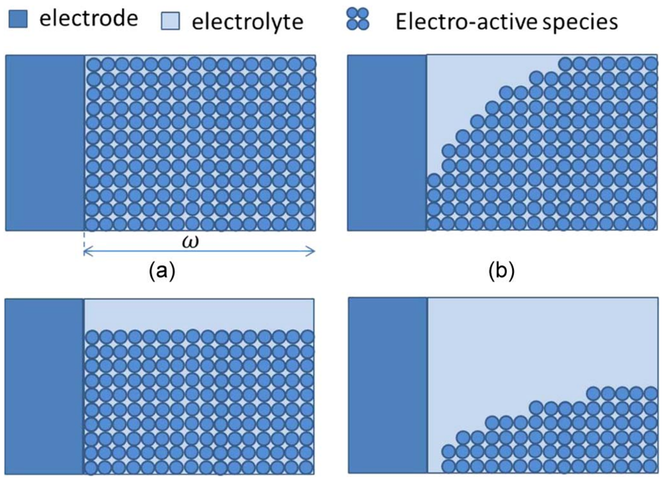

Fig. 1. Battery operation: (a) Charged battery. (b) Before recovery. (c) After recovery. (d) Discharged battery.

express $G_i(w)$ through the following flow conservation recursive equation, where $G_0(w) = 1$

$$G_i(w) = \sum_{k \prec i} w_{k,i}(t) G_k(w), \quad i = 1, \ldots, N. \tag{1}$$

B. Dynamic Battery Model

Under the assumption that an electrochemical battery cell is "ideal," a constant voltage throughout the discharge process and a constant capacity for all discharge profiles are both maintained over time. However, in real batteries, the *rate capacity effect* [20] leads to the loss of capacity with increasing load current and the *recovery effect* [21] makes the battery appear to regain portions of its capacity after some resting time. Due to these phenomena, the voltage, as well as energy amount delivered by the battery, heavily rest on the discharge profile. Therefore, when dealing with energy optimization, it is necessary to take this into account, along with nonlinear variations in a battery's capacity.

There are several proposed models to describe a nonideal battery overviewed in [22]. Accordingly, models are broadly classified as electrochemical, circuit based, stochastic, and analytical. Among all, analytical models, such as the kinetic battery model (KBM) [14], [15] or diffusion-based models [23]–[25], provide a tradeoff between accuracy and computational complexity. A detailed analysis of two analytical battery models—the KBM and diffusion models derived by Rakhmatov *et al.* [18]—is given in [26] where it is shown that the KBM is a first-order approximation of the popular Rakhmatov-Vrudhula-Wallach (RVW) diffusion model [27].

The results obtained in [1] adopting the KBM pave the way for an investigation of the same problem using a more accurate model. In what follows, we briefly review a linear state-space model [24] derived from the diffusion-based model [18]. In Fig. 1, the battery operation based on the diffusion model is illustrated. We assume the distance between electrodes (anode and cathode) is 2ω. As shown in Fig. 1, during a rest time, the electrolyte concentration is constant over the length of ω [Fig. 1(a)]. Under a load, $i(t)$, due to the electrochemical reaction, the concentration of the electrolyte is reduced near the electrode and creates a gradient [Fig. 1(b)], which causes the diffusion of species toward the electrode. Then, during an idle period, this diffusion makes the electrolyte concentration gradually become uniform over the length ω showing the battery recovery effect [Fig. 1(c)]. Finally, when the electrolyte concentration drops to a predetermined cutoff level, C_{cutoff}, the battery is said to be depleted while it has some unused capacity. This phenomenon describes the rate capacity effect [Fig. 1(d)].

A 1-D diffusion equation describing the concentration behavior inside a battery [18] is given by

$$\begin{aligned} J(x,t) &= -D\frac{\partial C(x,t)}{\partial x} \\ \frac{\partial C(x,t)}{\partial t} &= D\frac{\partial^2 C(x,t)}{\partial x^2} \end{aligned} \tag{2}$$

where $C(x,t)$ represents the electrolyte concentration at time t at a distance $x \in [0,\omega]$ from the electrode. $J(x,t)$ stands for the electrolyte flux at time t at distance x and D denotes a constant diffusion coefficient. Let the initial concentration be a constant C^*. As described in [18], applying the following two boundary conditions:

$$D\frac{\partial C(x,t)}{\partial x}\bigg|_{x=0} = \frac{i(t)}{\nu AF}, \quad D\frac{\partial C(x,t)}{\partial x}\bigg|_{x=\omega} = 0$$

where A is the area of the electrode, F is Faraday's constant, and ν is a scaling factor, the final solution for the concentration of the electrolyte at the electrode $(x=0)$ is (using $*$ to denote convolution)

$$C(0,t) = C^* - \frac{i(t)}{\nu\omega FA} * \left(1 + 2\sum_{m=1}^{\infty} e^{-\frac{\pi^2 m^2}{\omega^2}Dt}\right). \tag{3}$$

Defining $\rho(t) = 1 - (C(0,t)/C^*)$ at $t=0$, we have $C(0,0) = C^*$ and $\rho(0) = 0$. Note that during discharge, $C(0,t)$ decreases, hence, $\rho(t)$ increases. When the battery is depleted (electrolyte concentration reaches C_{cutoff}), $\rho(t)$ reaches the corresponding threshold $\rho_{\text{cutoff}} = (1 - C_{\text{cutoff}}/C^*)$. In order to derive a state-space realization as in [24], we define $y(t) = \rho(t)/\rho_{\text{cutoff}}$ which results in $y(0) = 0$ and $y(T) = 1$ at the failure time $t = T$. Replacing the infinite sum in (3) by a finite one with M terms, we obtain

$$\begin{aligned} y(t) &= \frac{i(t)}{\alpha} * 1 + \frac{i(t)}{\alpha} * 2\sum_{m=1}^{M} e^{-\delta_m t} \\ &= [1\ 1, \ldots, 1]\begin{bmatrix} \frac{i(t)}{\alpha} * 1 \\ \frac{2i(t)}{\alpha} * e^{-\delta_1 t} \\ \cdots \\ \frac{2i(t)}{\alpha} * e^{-\delta_M t} \end{bmatrix} \end{aligned} \tag{4}$$

where $\delta_m = \pi^2 m^2 D/\omega^2$ and $\alpha = C^* \nu\omega F A \rho_{\text{cutoff}}$. Next, we define the state vector $\mathbf{x}(t) = [x_0(t), \ldots, x_M(t)]^T$ such that

$$\begin{aligned} \dot{x}_0(t) &= \frac{1}{\alpha} i(t) \\ \dot{x}_m(t) &= \frac{2}{\alpha} i(t) - \delta_m x_m(t) \quad && m \in \{1, 2, \ldots, M\} \\ x_m(0) &= 0 && m \in \{0, 1, \ldots, M\} \end{aligned} \tag{5}$$

which can be written as

$$\begin{aligned} x_0(t) &= \frac{i(t)}{\alpha} * 1 \\ x_m(t) &= \frac{2i(t)}{\alpha} * e^{-\delta_m t} \qquad m \in \{1, 2, \ldots, M\}. \end{aligned} \tag{6}$$

Substituting (6) into (4), we have

$$y(t) = [1\ 1, \ldots, 1] \begin{bmatrix} x_0(t) \\ x_1(t) \\ \cdots \\ x_M(t) \end{bmatrix} = [1\ 1, \ldots, 1]\mathbf{x}(t). \tag{7}$$

For each node $i = 0, \ldots, N-1$, $y_i(t)$ is the battery status indicator at time t. Setting $y_i(0) = 0$, it follows that $y_i(T) = 1$ which indicates that the battery is out of charge at the failure time $t = T$.

In our WSN environment, the battery workload $i(t)$ is due to three factors: 1) the energy needed to sense a bit; E_{sense}; 2) the energy needed to receive a bit E_{rx}; and 3) the energy needed to transmit a bit E_{tx}. If the distance between two nodes is d, we have

$$E_{tx} = p(d), \quad E_{rx} = C_r, \quad E_{\text{sense}} = C_e \tag{8}$$

where C_r, C_e are given constants depending on the communication and sensing characteristics of nodes, and $p(d) \geq 0$ is a function monotonically increasing in d; the most common such function is $p(d) = C_f + C_s d^\beta$ where C_f, C_s are given constants and β is a constant dependent on the medium involved. We shall use this energy model but ignore the sensing energy, that is, set $C_e = 0$. Clearly, this is a relatively simple energy model that does not take into consideration the channel quality or the Shannon capacity of each wireless channel. The ensuing optimal control analysis is not critically dependent on the exact form of the energy consumption model attributed to communication, although the ultimate optimal value of $w(t)$ obviously is. Before proceeding, as in [1], we define the following constants:

$$k_{i,j} = p(d_{i,j}) - p(d_{i,N}), \quad i < j < N \tag{9}$$

$$k_{0,N} = p(d_{0,N}) \tag{10}$$

$$k_{i,N} = C_r + p(d_{i,N}), \quad i = 1, \ldots, N-1 \tag{11}$$

where $d_{i,j}$ is the distance between nodes i and j. Note that we may allow these constants to be time dependent if the network topology is not fixed, that is, $d_{i,j}(t)$ is time varying. Let us now combine the adopted battery model above with (8). Then, similar to [1], we can show that the workload of node 0 $u_0(t)$ is given by

$$u_0(t) = G_0(w) \left[\sum_{0 \prec j, j < N} w_{0,j}(t) k_{0,j} + k_{0,N} \right] \tag{12}$$

where $G_0(w) = 1$. Also, for any node $i = 1, \ldots, N-1$, where we must include the energy for both receiving and transmitting data packets, we can show that

$$u_i(t) = G_i(w) \left[\sum_{i \prec j, j < N} w_{i,j}(t) k_{i,j} + k_{i,N} \right]. \tag{13}$$

Defining $g_i(w) = \sum_{i<j,j<N} w_{i,j}(t) k_{i,j} + k_{i,N}$ the dynamic model (5) and (7) for nodes $i = 0, \ldots, N-1$ becomes

$$\begin{aligned} \dot{\mathbf{x}}_i(t) &= A_i \mathbf{x}_i(t) + b_i G_i(w(t))\, g_i(w(t)) \\ y_i(t) &= c\mathbf{x}_i(t) \end{aligned} \tag{14}$$

$$\begin{aligned} A_i &= \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & -\delta_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\delta_M \end{bmatrix} \\ &= \text{diag}[0, -\delta_1, \ldots, -\delta_M]_{(M+1)\times(M+1)} \end{aligned} \tag{15}$$

$$b_i = \left[\frac{1}{\alpha}, \frac{2}{\alpha}, \ldots, \frac{2}{\alpha} \right]^T \qquad c = [1\ 1, \ldots, 1]_{1\times(M+1)}. \tag{16}$$

This is a more general, high-dimensional model compared to the KBM considered in [1] where there are only two state equations

$$\dot{r}_i(t) = -G_i(w(t))\, g_i(w(t)) + k(b_i(t) - r_i(t)) \tag{17}$$

$$\dot{b}_i(t) = -k(b_i(t) - r_i(t)) \tag{18}$$

in which k is a crucial parameter modeling the "recovery effect" in the battery dynamics, similar to the role that the D parameter plays in (2).

Note that we consider identical battery characteristics for all nodes in the network, that is, $A_i = A_j$, $b_i = b_j$ for all $i, j = 0, \ldots, N-1$ (we will discuss the reason for this assumption later in Remark 1). The vectors $\mathbf{x}_i(t) = [x_{i0}, \ldots, x_{iM}]^T$ for $i = 0, \ldots, N-1$ define the state variables for our problem. Finally, observe that by controlling the routing probabilities $w_{i,j}(t)$ in (12) and (13), we directly control node i's battery discharge process.

C. Optimal Control Problem Formulation

Our objective is to maximize the WSN lifetime by controlling the routing probabilities $w_{i,j}(t)$. The maximum lifetime optimal control problem is formulated as follows:

$$\min_{w(t)} -\int_0^T dt \tag{19}$$

s.t. for $i = 0, \ldots, N-1$

$$\dot{\mathbf{x}}_i(t) = A\mathbf{x}_i(t) + bG_i\left(w(t)\right) g_i\left(w(t)\right) \tag{20}$$

$$y_i(t) = c\mathbf{x}_i(t)$$

$$A = \mathrm{diag}[0, -\delta_1, \ldots, -\delta_M]_{(M+1)\times(M+1)}$$

$$b = \left[\frac{1}{\alpha}, \frac{2}{\alpha}, \ldots, \frac{2}{\alpha}\right]^T \quad c = [1\ 1, \ldots, 1]_{1\times(M+1)}$$

$$G_i\left(w(t)\right) = \sum_{k\prec i} w_{k,i}(t) G_k\left(w(t)\right) \tag{21}$$

$$G_0\left(w(t)\right) = 1$$

$$g_i\left(w(t)\right) = \sum_{i\prec j, j<N} w_{i,j}(t)k_{ij} + k_{i,N} \tag{22}$$

$$\sum_{i\prec j, j<N} w_{i,j}(t) \le 1, \quad 0 \le w_{i,j}(t) \le 1 \tag{23}$$

$$\min_{i=0,\ldots,N-1} y_i(T) = 1 \tag{24}$$

where $\mathbf{x}_i(t) = [x_{i0}, \ldots, x_{iM}]^T$ are the state variables representing node i's battery dynamics for $i = 0, \ldots, N-1$ and $y_i(t) = \sum_{j=0}^M x_{ij}(t)$ is the battery status indicator at node i. Control constraints are specified through (23), where the first inequality follows from the fact that $\sum_{i\prec j<N} w_{i,j}(t) + w_{i,N}(t) = 1$. Finally, (24) provides boundary conditions for $\mathbf{x}_i(t)$, $i = 0, \ldots, N-1$, at $t = T$ requiring that the terminal time is the earliest instant when $y_i(t) = \sum_{j=0}^M x_{ij}(t) = 1$ for any node i (recall that $y_i(T) = 1$ indicates battery depletion). In other words, at $t = T$, we require that the maximal value over all $\{y_0(T), \ldots, y_{N-1}(T)\}$ is 1 or, equivalently, $T = \inf_{t\ge 0}\{t : y_i(t) = 1$ for at least some $i = 0, \ldots, N-1\}$.

This is a classic minimum (maximum) time optimal control problem except for two complicating factors: 1) The boundary condition (24) which involves the nondifferentiable min function, and 2) the control constraints (23). In what follows, we will use $w^*(t)$ to denote the optimal routing vector, which provides a (not necessarily unique) solution to this problem.

III. Optimal Control Problem Solution

Our analysis is similar to that in [1], but it is complicated by the high-dimensional dynamics in (20). We begin with the Hamiltonian for this optimal control problem

$$\begin{aligned} H(w,t,\lambda) &= -1 + \sum_{i<N}[\lambda_{i0}\dot{x}_{i0} + \lambda_{i1}\dot{x}_{i1} + \cdots + \lambda_{iM}\dot{x}_{iM}] \\ &= -1 + \sum_{i<N}\left[\lambda_{i0}\frac{1}{\alpha}G_i(w(t))\, g_i(w(t)) + \cdots \right. \\ &\quad \left. + \lambda_{iM}\left(\frac{2}{\alpha}G_i(w(t))g_i(w(t)) - \delta_M x_{iM}\right)\right] \end{aligned} \tag{25}$$

where $\lambda_{i0}(t), \ldots, \lambda_{iM}(t)$ are the costates corresponding to $x_{i0}(t), \ldots, x_{iM}(t)$ at node i, which must satisfy

$$\begin{cases} \dot{\lambda}_{i0}(t) = -\frac{\partial H}{\partial x_{i0}} = 0 \\ \dot{\lambda}_{im}(t) = -\frac{\partial H}{\partial x_{im}} = -\delta_m \lambda_{im}(t) \quad m = 1, \ldots, M. \end{cases} \tag{26}$$

Due to the nature of the state boundary conditions in (24), it is hard to derive explicit expressions for the costates $\lambda_{ij}(t)$. Thus, we proceed by considering each possible case of a node dying first, which we will refer to as "scenario S_i" under which $1 = y_i(T) \ge y_j(T)$, $j \ne i$ for some fixed node i.

A. Analysis of scenario S_i

Under S_i, we have the terminal time constraints $y_i(T) = 1$ and $y_j(T) \le 1$ for all $j \ne i$. Consequently, all $y_j(t)$; hence, $\mathbf{x}_j(t)$, $j \ne i$, are unconstrained at $t = T$. The next theorem establishes the property that under a fixed network topology, there exists a static optimal routing policy, that is, there exists a vector $w^*(t)$ which is time invariant.

Theorem 1: If $1 = y_i(T) \ge y_j(T)$, $j \ne i$, for some i and the network topology is fixed, that is, $d_{ij}(t) = d_{i,j} =$ constant for all $i, j = 0, \ldots, N-1$, then there exists a time-invariant solution of (19)–(24): $w^*(t) = w^*(T)$.

Proof: See the Appendix.

Note that there may exist multiple optimal control policies, including some that may be time varying. Theorem 1 asserts that there is at least one which is time-invariant, that is, $w^*(t) = w^*(T) = w^*$, and it remains to obtain the values of $w^*_{i,j}$, $i = 0, \ldots, N-2$, and $j = 1, \ldots, N-1$ by explicitly solving the optimization problem (41). This requires knowledge of all $y_i(t)$, $t \in [0, T]$ in order to determine the values of all $y_i(T)$ and, hence, identifying the node i such that $1 = y_i(T) \ge y_j(T)$ and use the values of $y_j(T)$, $j \ne i$. This can be accomplished by solving the differential equations (14)–(16), whose initial conditions are given as $x_{im}(0) = 0$, $i = 0, \ldots, N-1$, and $m = 0, \ldots, M$, with $w(t) = w$ being the unknown optimal routing vector. It is straightforward to obtain $x_{ij}(t)$ as follows:

$$x_{i0}(t) = \frac{1}{\alpha}G_i(w)g_i(w)t$$

$$x_{ij}(t) = \frac{2}{\alpha\delta_j}G_i(w)g_i(w)(1 - e^{-\delta_j t}),\ j = 1, \ldots, M.$$

Recall that $y_i(t) = \sum_{j=0}^M x_{ij}(t)$, the "critical time" T_i^* such that $y_i(T_i^*) = 1$ and $0 < y_i(t) < 1$ for all $t \in [0, T_i^*)$ is the solution of the nonlinear equation in T

$$\frac{1}{\alpha}G_i(w)g_i(w)T + \sum_{j=1}^M \frac{2}{\alpha\delta_j}G_i(w)g_i(w)(1 - e^{-\delta_j T}) = 1 \tag{27}$$

which we write as $T_i^*(w)$. Thus, we may rewrite the S_i optimization problem as follows:

$$\begin{aligned} \mathbf{P}_i :\ &\min_w G_i(w)g_i(w) \\ &\text{s.t.}\quad (21)-(23), \quad T_i^*(w) \le T_j^*(w), \quad j \ne i \end{aligned}$$

where $T_i^*(w)$ is the solution of (27) for all $i = 0, \ldots, N-1$. Note that $\mathbf{P}_i$ may not always have a feasible solution.

Based on our analysis thus far, if we focus on a fixed scenario S_i, the solution to the optimal control problem is simply the solution of the NLP problem $\mathbf{P}_i$. However, since we do not know which node will die first, determining the value of i such that $T_i^*(w) \le T_j^*(w)$ for all $j \neq i$ requires solving *all* $\mathbf{P}_i$ problems and finding the best policy among them. This is accomplished through the following algorithm, referred to as **A1**.

Algorithm A1

1) Solve problem $\mathbf{P}_i$ for $i = 0, \ldots, N-1$ to obtain $T_i^*(w)$.
2) Set $T_i^*(w) = -1$ if a problem is infeasible.
3) The optimal lifetime is given by $\max_i\{T_i^*(w)\}$ and the corresponding optimal policy w^* is the one obtained for the associated problem $\mathbf{P}_i$.

B. Robustness Property of the Optimal Routing Policy

In this section, we show that the optimal routing vector w^* obtained through Algorithm **A1** is robust with respect to the diffusion coefficient constant D. This is similar to the robustness property established in [1, Lemma 3 and Theor. 2], where it is shown that the solution of problem $\mathbf{P}_i$ is robust with respect to the parameter k of the KBM in (17) and (18). Here, the intuition behind this property lies in the nature of the NLPs $\mathbf{P}_i$: observe that the solution depends on the values of $G_i(w)g_i(w)$ and the associated constraints (21)–(23), while the only effect of the parameter D enters through the inequalities $T_i^*(w) \le T_j^*(w)$, $j \neq i$. Therefore, if a solution is obtained under $D = 0$ and these inequalities are still satisfied when $D > 0$, then the actual routing policy remains unchanged, while the value of the resulting optimal network lifetime is generally different. Let $w^i(D)$ denote the solution of problem $\mathbf{P}_i$ when the RVW model is invoked with parameter D, including the case $D = 0$. The corresponding node lifetimes are denoted by $T_i^*(w^i, D)$. The robustness property we identify rests on the following Theorem:

Theorem 2: The optimal routing policy under $D = 0$, is unaffected when $D > 0$, i.e.,

$$w^*(0) = w^*(D) \quad \text{for any } D > 0. \tag{28}$$

Proof: See the Appendix.

Remark 1: It should be noted that the robustness property of the optimal solution may not be valid if nodes have different battery parameters, that is, A_i, b_i in (15) and (16) are not all the same. However, the time-invariant nature of the optimal routing vector in Theorem 1 remains unaffected.

C. Optimal Routing by Solving a Single NLP

Based on Theorem 1, when the topology of the network is fixed, there is at least one optimal routing policy which is time-invariant. Now, by defining a new variable T as the network lifetime (the first node whose battery is depleted), we merge

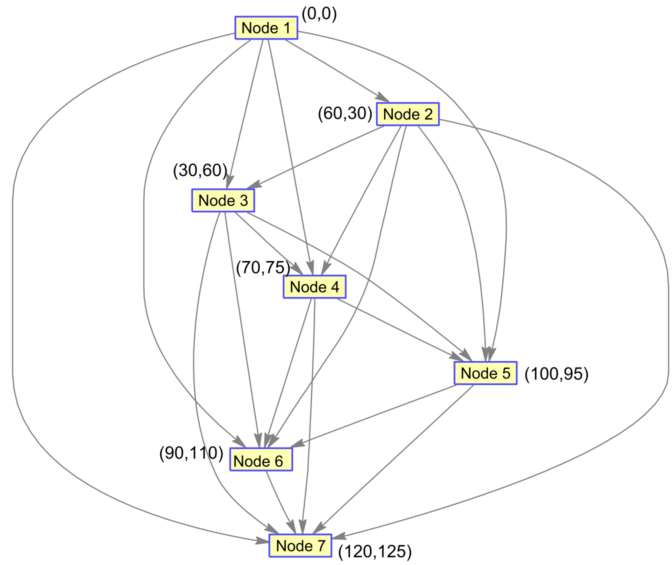

Fig. 2. Network topology.

Algorithm A1 into a single NLP problem which determines an optimal routing vector and the network lifetime at the same time as follows:

$$\begin{aligned} &\max_{w} \; T \\ &\text{s.t.} \quad (21)-(23), \quad T \le T_i^*(w). \end{aligned} \tag{29}$$

Note that $T_i^*(w)$ is the parametric solution of the node i lifetime based on the energy dynamics considered for the battery. We consider the following three cases:

1) For nodes with ideal battery dynamics, the energy consumption is directly proportional to the battery load, that is, $\partial r_i(t)/\partial t = -i(t)$, $T_i^*(w) = R_i/G_i(w)g_i(w)$, where R_i is the initial energy of node i.
2) If the KBM we used in [1] describes the battery dynamics, we have

$$\begin{aligned} \dot{r}_i(t) &= -i_i(t) + k\left(b_i(t) - r_i(t)\right) \\ \dot{b}_i(t) &= -k\left(b_i(t) - r_i(t)\right) \end{aligned}$$

and the battery lifetime $T_i^*(w)$ is the solution of the following equation:

$$\begin{aligned} R_i &- \frac{G_i(w)g_i(w)}{2}T \\ &- \frac{1}{2}\left[B_i - R_i - \frac{G_i(w)g_i(w)}{2k}\right]\left(e^{-2kT} - 1\right) = 0. \end{aligned}$$

3) If we consider the diffusion model (14)–(16) to describe the battery dynamics, $T_i^*(w)$ is the solution of (27).

D. Simulation Examples

In order to illustrate the results of our analysis, we consider the 7-node network shown in Fig. 2 where node coordinates are given next to each node. Nodes 1 and 7 are the source and

TABLE I
OPTIMAL ROUTING PROBABILITIES AND NETWORK LIFETIME FOR A 7-NODE NETWORK WITH DIFFERENT DIFFUSION COEFFICIENTS

Routing Probability	(I) δ_m=0	(II) $\delta_m = (0.273)^2 m^2$
$w_{1,2}$	0.495974	0.506774
$w_{1,3}$	0.504026	0.493226
$w_{1,4}$	0	0
$w_{1,5}$	0	0
$w_{1,6}$	0	0
$w_{2,3}$	0.091704	0.132088
$w_{2,4}$	0.279738	0.15061
$w_{2,5}$	0.218011	0.334065
$w_{2,6}$	0.157214	0.173207
$w_{3,4}$	0.376457	0.183185
$w_{3,5}$	0.197358	0.407184
$w_{3,6}$	0.132955	0.218149
$w_{4,5}$	0.326819	0.34064
$w_{4,6}$	0.28053	0.300537
$w_{5,6}$	0.295149	0.418332
Lifetime	26916.66	80723.17

TABLE II
NODE LIFETIMES UNDER $w^*(0)$ WHEN $\delta_m = (0.273)^2 m^2$

i	1	2	3
$T_i(w^*(0))$	80723.17	110347.25	100335.6
i	4	5	6
$T_i(w^*(0))$	303883.87	608865.92	550302.16

base, respectively, while the rest are relay nodes. We solve the problem for a 2-state model $(M = 1)$ and set $C_s = 0.0001$, $C_f = C_r = 0.05$, and $\beta = 2$ in the energy model. We assume $\alpha_i = 40375$, $i = 1, \ldots, 6$ [28]. Table I shows the optimal routing probabilities and network lifetime for different values for δ_m ($D = 0$ and $D > 0$) obtained through **Algorithm A1**. Note that D is a constant multiplier in δ_m. To validate the robustness property discussed in Theorem 2, we apply the routing vector obtained when $\delta_m = 0$ (column 2 of Table I), $w^*(0)$ to the $T_i(w)$ equations with $\delta_m = (0.273)^2 m^2$ to see if it results in the same network lifetime of 80723.17 (we do not provide specific units, but, based on standard known data, distance units in feet and time units in minutes are reasonable for the RVW model). Table II shows node lifetimes under $w^*(0)$ when $\delta_m = (0.273)^2 m^2$. It is observed that by adopting $w^*(0)$, node 1 dies first and the network lifetime is equal to that obtained by solving the NLP problem (29) when $\delta_m = (0.273)^2 m^2$. This illustrates the robustness property as expected.

Remark 2: We should point out that solving (27) to obtain a parametric solution for node i lifetime $T_i(w)$ is a hard task when we consider the battery model with more than two state variables $(M > 1)$. However, the robustness property of the optimal solution with respect to the diffusion coefficient D, obtained in Theorem 2, allows us to find the optimal routing vector for the simpler case when $D = 0$ and the same routing vector is optimal for other cases with $D > 0$. Assuming $D = 0$ (consequently $\delta_m = 0$) in (5) and (7), we obtain a closed-form expression for the lifetime of node i as $T_i(w) = \alpha/(1 + 2M)G_i(w)g_i(w)$. We can then find the optimal routing vector for any value of M by solving a single NLP problem (29).

Reduction in Computational Complexity: In order to investigate the reduction in the computational effort needed to find the optimal routing probabilities using the proposed "single NLP" formulation, we solve (29) for all three battery dynamic models discussed in Section III-C and compare the CPU times with those needed when implementing **Algorithm A1**. For the network in Fig. 2, we adopt the diffusion-based model ($\delta_m = (0.273)^2 m^2$), the KBM (with $k = 0.02$), and the ideal battery model. The corresponding CPU times are as shown in Table III where one can see that the new formulation offers a reduction in computation time of an order of magnitude or more, with the understanding that this reduction depends on the size of the network and its topology. Note that in order to obtain an optimal routing vector using **Algorithm A1**, one should solve $(N - 1)$ NLP problems.

TABLE III
CPU TIME UNDER DIFFERENT BATTERY DYNAMICS USING SINGLE NLP FORMULATION COMPARED TO **ALGORITHM A1**

Battery Dynamics	Single NLP	**Algorithm A1**
Diffusion model	430.62 sec	28098 sec
KBM	61.27 sec	450.12 sec
Ideal Battery	59.56 sec	500.16 sec

Remark 3: The extension to a network with multiple source nodes is straightforward. Let us assume a network with k source nodes, each with a data generation rate of u_k. Let us also assume that the source nodes do not act as relay nodes and that each node routes data to nodes which are closer to the base station. The rest of the analysis is the same as the problem with a single-source network. The optimal control problem remains as in (19)–(24) except that the inflow rate to each node becomes

$$G_i(w(t)) = u_i \qquad \forall i \in N_s$$
$$G_i(w(t)) = \sum_{j \prec i} w_{j,i}(t) G_j(w(t)) \qquad \forall i \neq N_s$$

where N_s is the set of all source nodes. Beginning with the Hamiltonian and defining Scenario S_i as we did in Section III-A, one can show that there exists a time-invariant optimal routing policy for networks with multisource nodes and fixed topology.

IV. JOINT OPTIMAL ROUTING AND INITIAL ENERGY ALLOCATION

In this section, we go a step beyond routing as a mechanism through which we can control the WSN resources by also controlling the allocation of initial energy over its nodes in order to maximize the lifetime. An application where this problem arises is in a network with rechargeable nodes. In this case, solving the joint optimal routing and initial energy allocation problem provides optimal recharging amounts by maximizing the network lifetime which may not correspond to full charges for all nodes. Unlike our analysis for this problem in [1], here the battery model works based on changes in electrolyte concentration; therefore, finding an optimal initial energy allocation for the nodes is equivalent to finding an optimal initial electrolyte concentration for each one. Consequently, we need to relate the battery residual energy to the equivalent electrolyte concentration. We assume a linear relationship as follows: since we consider identical batteries for all nodes, we define R_{nom} to be the rated energy of the battery. Then, for each node, we have $R_i = \text{SoC}_i \cdot R_{\text{nom}}$, where SoC_i denotes the "state of charge" of node i. One of the methods used to find the SoC of a battery is by measuring the specific gravity (SG) of its electrolyte. For example, for a lead-acid battery, as the SoC decreases through discharge, sulfuric acid is consumed and its concentration in water decreases. Consequently, the SG of the solution is reduced in direct proportion to the SoC [29]. We assume a linear relationship between SoC and SG such that $\mathrm{SoC}_i = a \cdot SG_i + b$, where a and b can be calculated based on available SoC versus SG lookup tables. Note that the electrolyte concentration is proportional to the SG of the solution, that is, $SG_i = k \cdot C_i$, where C_i stands for the electrolyte concentration at node i, and k is a constant coefficient which can be calculated based on the molecular wight of the electrolyte and mass percent of the solution. Finally, initial energy is a linear function of the initial electrolyte concentration

$$R_i = m \cdot C_i + n \tag{30}$$

where $m = R_{\mathrm{nom}} ak$ and $n = R_{\mathrm{nom}} b$. Let us define the total initial energy available as $\bar{R}$ and let $R = [R_0, \ldots, R_{N-1}]$. Using (30), we define corresponding terms for electrolyte concentrations as $\bar{C}$ and $C = [C_0, \ldots, C_{N-1}]$. From Theorem 1, we know that the optimal routing policy is fixed unless the topology of the network changes. Then, we can formulate the following problem:

$$\begin{aligned}
&\max_{\substack{C_i, i=0,\ldots,N-1\\ w_{i,j}, j=1,\ldots,N-1}} \quad T\\
&\text{s.t. } T \le T_i^*(w, C_i), \quad i = 0, \ldots, N-1\\
&\sum_{i \prec j,\ j<N} w_{i,j} \le 1,\ 0 \le w_{i,j} \le 1,\ i,j = 0,\ldots,N,\ i \prec j\\
&\quad -\frac{n}{m} < C_i < \frac{R_{\mathrm{nom}} - n}{m}, \quad \sum_{i=0}^{N-1} C_i = \bar{C}\\
&\bar{C} = \frac{\bar{R} - Nn}{m}.
\end{aligned} \tag{31}$$

This is an NLP problem where the control variables are the routing probabilities $w_{i,j}$ and the initial concentrations C_i for nodes $i = 0, \ldots, N-1$. Looking at (30), the constraints on C_i above are to ensure that the equivalent R_i stays between 0 and R_{nom} and that $\sum_{i=0}^{N-1} R_i = \bar{R}$. In this case, $T_i^*(w, C_i)$ is the solution of (27) for all $i = 0, \ldots, N-1$, which is now dependent on w and C_i. Recalling that $\alpha_i = \nu\omega F A \rho_{\mathrm{cutoff}} C_i$, we observe that unlike the problem discussed in the previous section, α is not identical for all nodes in the network. Differentiating (27) with respect to α_i, we obtain

$$G_i(w) g_i(w) \frac{\partial T}{\partial \alpha_i} + \sum_{j=1}^{M} \left(2 G_i(w) g_i(w) \frac{\partial T}{\partial \alpha_i} e^{-\delta_j T} \right) = 1$$

which yields

$$\frac{\partial T}{\partial \alpha_i} = \frac{1}{G_i(w) g_i(w) + 2\sum_{j=1}^{M} G_i(w) g_i(w) e^{-\delta_j T}} > 0.$$

Observe that $\partial T/\partial C_i = (\partial T/\partial \alpha_i) \cdot (\partial \alpha_i/\partial C_i) = \nu\omega F A \rho_{\mathrm{cutoff}} (\partial T/\partial \alpha_i)$, which results in $\partial T/\partial C_i > 0$.

If the solution of problem (31) is (w^*, C^*), then $T_i^*(w^*, C_i^*)$ is the solution of (27) under this routing vector and initial electrolyte concentration at node i. The following theorem establishes the fact that this optimal solution is such that all nodes deplete their energy at the same time.

Theorem 3: The solution of problem (31) satisfies

$$T^* = T_0^*\left(w^*, C_0^*\right) = T_1^*\left(w^*, C_1^*\right) = \cdots = T_{N-1}^*\left(w^*, C_{N-1}^*\right). \tag{32}$$

Proof: See the Appendix.

Remark 4: In order to guarantee (32), it is necessary that $T_i^*(w^*, C_i^*) < \infty$. Looking at (27) and recalling that $g_i(w) > 0$, this is equivalent to assuming that $G_i(w) > 0$, that is, that no node is left unutilized.

Based on Theorem 3, we can simplify the NLP problem (31). In particular, we solve it in two steps. In Step 1, assuming a fixed routing policy w, we determine the corresponding optimal initial energy distribution policy by solving the set of equations

$$\begin{aligned}
&T_0^*(w, C_0) = T_1^*(w, C_1) = \cdots = T_{N-1}^*(w, C_{N-1})\\
&\text{s.t.} \quad \sum_{i=0}^{N-1} C_i = \bar{C}.
\end{aligned} \tag{33}$$

Its solution is defined to be $C^*(w)$ with an associated lifetime $T^*(w)$. Then, in Step 2, we search over the feasible set of w given by (23) to determine the optimal $T^*(w)$ by using a standard nonlinear optimization solution procedure. As also observed in [1], we should point out, however, that solving problem (33) to obtain parametric solutions for $T^*(w)$ and $C^*(w)$ is not a simple task and common solvers fail to accomplish it. Instead, we can proceed by selecting one of the parametric equations for $T_i^*(w, C_i)$ as an objective function and add (33) as constraints to a new NLP problem below, whose solution we can obtain with standard optimization solvers

$$\begin{aligned}
&\max_{C_i, w_{i,j}, j=1,\ldots,N-1} \quad T_i^*(w, C_i)\\
&\text{s.t. } T_i^*(w, C_i) - T_j^*(w, C_j) = 0 \quad i,j = 0,\ldots,N-1, i \ne j\\
&\sum_{i \prec j,\ j<N} w_{i,j} \le 1,\ 0 \le w_{i,j} \le 1,\ i,\ j = 0,\ldots,N,\ i \prec j\\
&\quad -\frac{n}{m} < C_i < \frac{R_{\mathrm{nom}} - n}{m}, \quad \sum_{i=0}^{N-1} C_i = \bar{C}.
\end{aligned} \tag{34}$$

A. Simulation Examples

We provide a numerical example for the joint optimal routing and initial energy allocation problem using the network in Fig. 3 with node coordinates shown next to each node. We set $m = 43.75$, $n = -200$ in (30), $R_{\mathrm{nom}} = 25$, $\bar{R} = 100$ ($\bar{C} = 29.71$), $\alpha = 40375$, $\delta_m = 0.273^2 m^2$ and other numerical values as before. Table IV shows the optimal routing probabilities and initial energies of all nodes. Note that the WSN lifetime for

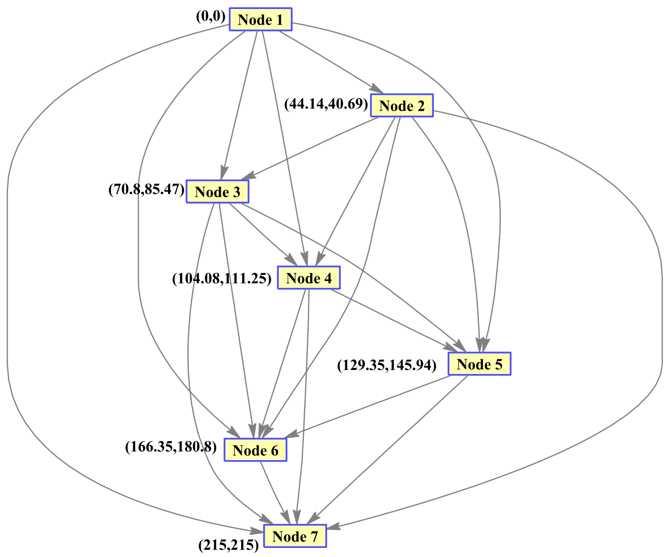

Fig. 3. Network topology.

TABLE IV
OPTIMAL ROUTING PROBS., 7-NODE NETWORK, NONIDEAL BATTERIES

$w_{i,j}$	1	2	3	4	5	6
1	N/A	1	0	0	0	0
2	N/A	N/A	0.9784	0.0154	0.0041	0.0015
3	N/A	N/A	N/A	0.7978	0.1914	0.0097
4	N/A	N/A	N/A	N/A	0.7082	0.2879
5	N/A	N/A	N/A	N/A	N/A	0.8325
R_i	25	16.6	14	14	14.4	16

this case is 98353 which is equal to the network lifetime when we consider all batteries initially fully charged ($R_i = R_{\text{nom}}$, $i = 1, \ldots, N-1$) and we just control the routing vector as discussed in Section III. However, here we observe that only the source node needs a fully charged battery. Finally, the fact that the network lifetime coincides with all individual node lifetimes (as expected by Theorem 3) provides a strong justification for the definition of network lifetime as the time when the first node depletes its energy.

V. NETWORK PERFORMANCE UNDER SECURITY THREATS

In this section we compare the WSN's performance under our optimal routing policy and the probabilistic routing policy introduced in [9] when a cyber-attack takes place. We limit ourselves to an example aimed at simply illustrating the advantages of the optimal routing policy we have derived for a specific form of attack. In [9] an Energy Aware Routing (EAR) policy is proposed in which a number of suboptimal paths are probabilistically selected with the intent of extending the network lifetime by spreading the traffic and forcing nodes in the network to deplete their energy at the same time. In EAR, each node builds a cost information table and propagates local cost information to other nodes. Costs are determined by the residual energy of each node and by the distances between them. Each node also maintains a routing probability table determined by local cost information. In this method, the routing probabilities

TABLE V
OPTIMAL ROUTING PROBS., 7-NODE NETWORK, IDEAL BATTERIES

$w_{i,j}$	1	2	3	4	5	6
1	N/A	0.3705	0.6295	0	0	0
2	N/A	N/A	0.1792	0.2045	0.1843	0.1960
3	N/A	N/A	N/A	0.2665	0.2046	0.3338
4	N/A	N/A	N/A	N/A	0.2133	0.3539
5	N/A	N/A	N/A	N/A	N/A	0.3627

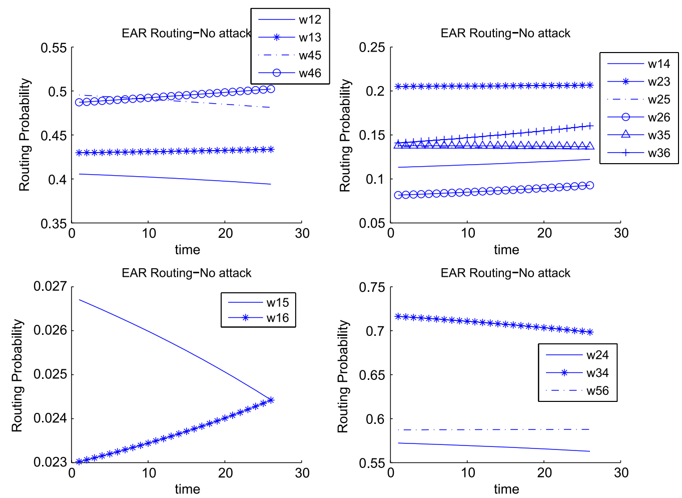

Fig. 4. Routing probability updates under EAR policy.

are set periodically. At the beginning of each period, the routing probabilities are computed recursively as follows:

$$w_{i,j} = \frac{C_{ij}^{-1}}{\sum_{k \in O\{i\}} C_{ik}^{-1}} \tag{35}$$

$$C_{ij} = d_{ij}^{k_1} r_j^{k_2} + \mathcal{C}_j \quad \text{for all } j \in O\{i\} \tag{36}$$

$$\mathcal{C}_i = \sum_{k \in O\{i\}} w_{i,j} C_{ij} \tag{37}$$

where $w_{i,j}$ is the routing probability on the edge (i, j), C_{ij} is the cost of sending a data packet from node i to the destination via node j and $\mathcal{C}_i$ is the average cost of sending a packet from node i to the base station (Note that $\mathcal{C}_N = 0$ where N is the base station). Moreover, r_j is the residual energy of node j and k_1 and k_2 are weighting factors which can be chosen to find the minimum energy path or the path with the most energy or a combination of the above [9]. Since the EAR method works based on the residual battery energy assuming ideal battery dynamics, we likewise use the same settings and determine the optimal routing vector and the network lifetime assuming ideal battery dynamics, i.e., Case 1) of problem (29).

Consider the network topology shown in Fig. 2. Table V shows the optimal routing probabilities obtained by solving (29) under normal (no threat) conditions. Under this routing policy, the network lifetime is 33.33. Fig. 4 shows the routing probability updates obtained using the EAR policy by computing routing probabilities, $w_{i,j}$s, through (35)–(37) periodically when $k_1 = 5$ and $k_2 = 1$. Under the EAR routing policy, the network lifetime is 25.94. As expected, our optimal routing policy results in the longer lifetime compared to the EAR

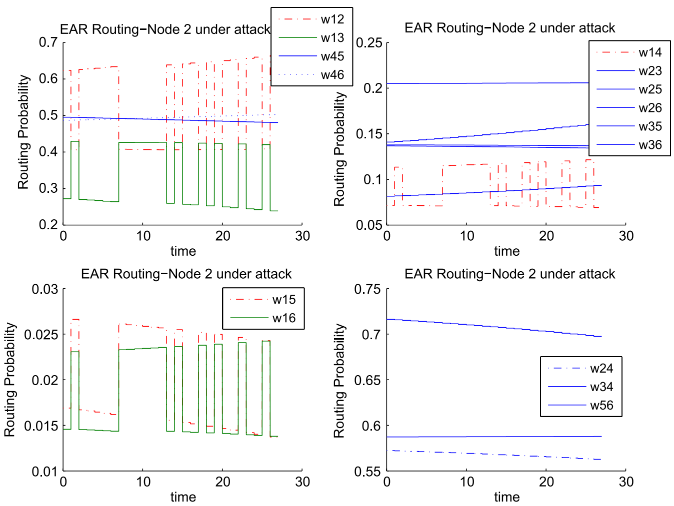

Fig. 5. Routing probability updates under EAR policy when node 2 is under attack.

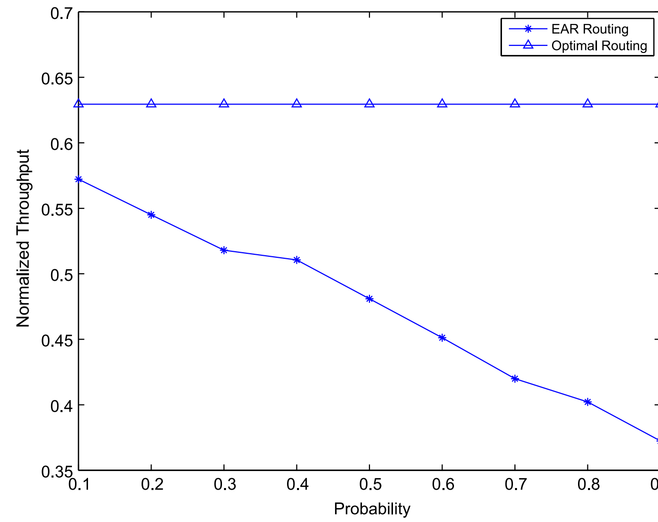

Fig. 6. Normalized throughput versus probability of broadcasting faked-cost.

solution. Next, we investigate the network performance under a "sink-hole attack," one of the most severe routing attacks in sensor networks [17], for the two routing policies. Under a sink-hole attack, a compromised node broadcasts a fake low cost to the neighboring nodes, thus enticing all such nodes to route packets to it. We will assume an attacker uses the following strategy:

1) The attacker compromises one node in the network randomly.
2) At each time kT, where T is the updating period for the routing probabilities, the compromised node will: 1) broadcast a fake near-zero cost ($\mathcal{C}_i$) to all nodes with probability p to attract more flow; 2) act as a normal node with probability $(1 - p)$.
3) The compromised node corrupts all the packets it has received and forwards them to other nodes to deplete their energy.

In particular, we compare the network performance under the attack in terms of the normalized throughput (the ratio of the number of uncorrupted packets to the total number of packets) for the EAR and our optimal policy. Recall that in the EAR policy, each node i needs to know its neighbors' residual energies, r_j, and average costs, $\mathcal{C}_j$, $\forall j \in O_i$, to update its routing table. Thus, it is vulnerable to faked-cost-based attacks. We will further illustrate this through the same network in Fig. 2. Assume that node 2 is under sink-hole attack and that in each updating period it broadcasts faked-cost information to its neighbors with probability $p = 0.5$. Fig. 5 shows how routing probability updates are affected in this scenario. Based on the network topology, node 1 is the only node that sends data to node 2. One can observe how routing probabilities from source node, node 1, to the other nodes, $[w_{12}\ w_{13}\ w_{14}\ w_{15}\ w_{16}]$, are affected at the periods in which node 2 broadcasts faked-cost data. On the other hand, our optimal policy uses the network topology to calculate routing probabilities and is robust with respect to this kind of attacks. However, the normalized throughput will be affected in both routing policies. Fig. 6 shows the normalized throughput as a function of the probability of broadcasting faked-cost, p, when node 2 is under sink-hole attack. It can be observed that for this specific example, under our optimal policy the normalized throughput drops to 63%, but it is not sensitive to p. However, under the EAR policy it drops significantly as p increases. This happens because our routing policy is calculated based on the network topology and consequently robust with respect to p. Hence, the inflow rate to the compromised node as well as the normalized throughput, are not affected by the propagated faked-cost. On the other hand, in the EAR routing strategy, the data inflow rate to the compromised node increases with p which drops the normalized throughput correspondingly.

Remark 5: Depending on the network topology, it is possible that the optimal routing policy dictates all data packets to be routed through a specific node, i, which gives $G_i = 1$, (e.g., assume $w_{1,2} = 1$ in the previous examples). Under a sink-hole attack, if this node is the compromised one, the normalized throughput drops to zero. Clearly, this node should be a top priority in terms of protection against routing attacks. One way to address this problem is to purposely deviate from the optimal solution by routing a fraction q of data packets via node i and the remaining $1 - q$ through other nodes. This randomization-by-design degrades the network lifetime from its optimal value under normal operation (no attack), but protects the network against becoming completely useless when under attack by increasing its normalized throughput to $1 - q$.

VI. Conclusions and Future Work

We have shown that an optimal routing policy for maximizing a fixed topology sensor network's lifetime is time invariant even when the batteries used as energy sources for the nodes are modeled so as to take into account "nonideal" phenomena such as the rate capacity effect and the recovery effect with a detailed dynamic battery model of which the KBM used in our prior work [1] is a special case. The associated fixed routing probabilities may be obtained by solving a set of relatively simple Non-Linear Programming (NLP) problems. In addition, this optimal policy is independent of battery parameters. The fact that the optimal routing probabilities are fixed is a strong indication leading to reduce **Algorithm A1** to a *single* NLP. The robustness property suggests to find the optimal routing by adopting the ideal battery dynamics. This reduces the problem to a single LP. However, in order to have a precise prediction for the network lifetime one should calculate $T_i^*(w)$ by applying optimal routing as discussed in Section III-C using appropriate battery dynamic. We have also considered a joint routing and initial energy allocation problem over the network nodes with the same network lifetime maximization objective and shown that the solution to this problem is given by a policy that depletes all node energies at the same time; the associated energy allocation and routing probabilities are obtained by solving an NLP problem.

Extensions to networks with multiple sources and base stations are expected to be straightforward. However, extensions to a changing network topology are more challenging. Finally, our solutions so far are centralized, so that an obvious direction to pursue is to seek distributed versions of the same problems.

Regarding issues of network security, we have limited ourselves to simple empirical evidence that our optimal routing policy is characterized by robustness properties relative to other energy-aware policies when it comes to certain common types of cyber-attacks. Clearly, a much more extensive investigation of how the probabilistic nature of routing policies can be exploited to react to security threats in terms of maintaining acceptable performance levels when operating under various attack conditions. This includes the randomization-by-design possibility mentioned in Remark 4.

Appendix

Proof of Theorem 1: To derive explicit expressions for $\lambda_{i0}(t), \ldots, \lambda_{iM}(t)$ it is necessary to use boundary conditions $\lambda_{i0}(T), \ldots, \lambda_{iM}(T)$. Since $0 \leq y_i(t) \leq 1$ for all i and $t \in [0, T]$, the optimal control problem under S_i is state-unconstrained except for $y_i(T) = \sum_{j=0}^{M} x_{ij}(T) = 1$. Thus, the terminal state constraint function $\Phi(\mathbf{x}_i(T), \ldots, \mathbf{x}_{N-1}(T))$ is reduced to $\sum_{j=0}^{M} x_{ij}(T)$ and the costate boundary conditions are given by

$$\begin{cases} \lambda_{im}(T) = \nu \frac{\partial \Phi(\mathbf{x}_i(T), \ldots, \mathbf{x}_{N-1}(T))}{\partial x_{im}} = \nu & m = 0, \ldots, M \\ \lambda_{jm}(T) = 0 \quad j \neq i \quad m = 0, \ldots, M \end{cases}$$

where ν is an unspecified scalar constant. This allows us to solve the costate equations in (26) to obtain for $t \in [0, T]$

$$\begin{cases} \lambda_{i0}(t) = \nu \\ \lambda_{im}(t) = \nu e^{-\delta_m (t-T)}, \qquad m = 1, \ldots, M \\ \lambda_{jm}(t) = 0 \qquad j \neq i \qquad m = 0, \ldots, M. \end{cases} \tag{38}$$

Using (38) in (25), we can simplify the Hamiltonian as follows:

$$H(w, t, \lambda) = -1 + \lambda_{i0} \frac{1}{\alpha} G_i(w(t)) g_i(w(t)) + \sum_{j=1}^{M} \lambda_{ij} \left(\frac{2}{\alpha} G_i(w(t)) g_i(w(t)) - \delta_j x_{ij} \right). \tag{39}$$

Observe that the control variables $w_{i,j}(t)$ appear only in $G_i(w(t))$ and $g_i(w(t))$ in the problem formulation (19)–(24). Thus, we can set $U_i(t) = G_i(w(t)) g_i(w(t)),\ i = 0, \ldots, N-1$ to be the effective control variables with $U_l \leq U_i(t) \leq U_u$, where $U_l \geq 0$ and U_u are, respectively, the lower bound and upper bound of $U_i(t)$ for all $t \in [0, T]$. Note that both are constant since their determination depends exclusively on (21), (22) subject to (23), independent of the states. In particular, they depend on the fixed network topology and the values of the energy parameters $k_{i,j}, k_{i,N}$ in (22). Applying the Pontryagin minimum principle to (39)

$$U_i^*(t) = \arg \min_{U_l \leq U_i(t) \leq U_u} H(U_i, t, \lambda^*)$$

implies that the optimal control is of bang-bang type

$$U_i^*(t) = \begin{cases} U_u & \text{if } \nu < 0 \\ U_l & \text{if } \nu > 0. \end{cases} \tag{40}$$

Moreover, the optimal solution must satisfy the transversality condition $(\lambda^*(d\Phi/dt) + L)_{t=T} = 0$ where $L = -1$ and we have seen that $\Phi(\mathbf{x}_i(T), \ldots, \mathbf{x}_{N-1}(T)) = \sum_{j=0}^{M} x_{ij}(T)$. Therefore

$$-1 + \nu \sum_{j=0}^{M} \dot{x}_{ij}(T) = 0$$

and it follows that $\nu = 1/\dot{y}_i(T)$. Since $y_i(T) = 1,\ y_i(0) = 0$ and $0 < y_i(t) < 1$ for all $t \in [0, T)$, we have $\dot{y}_i(T) > 0$, therefore, $\nu > 0$. By (40), $U_i^*(t) = U_l$ for all $t \in [0, T]$. We conclude that the optimal control problem under S_i is reduced to the following optimization problem:

$$\begin{aligned} &\min_{w(t)} \quad G_i(w(t)) g_i(w(t)) \\ &\text{s.t.} \quad (21)-(23) \text{ and } 1 = y_i(T) \geq y_j(T), \quad j \neq i. \end{aligned} \tag{41}$$

When $t = T$, the solution of this problem is $w^*(T)$ and depends only on $y_j(T),\ j \neq i$, and, as already argued, the fixed network topology and the values of the fixed energy parameters $k_{i,j}, k_{i,N}$ in (22). The same applies to any other $t \in [0, T)$, therefore, there exists a time-invariant optimal control policy $w^*(t) = w^*(T)$, which minimizes the Hamiltonian and proves the theorem. ∎

Proof of Theorem 2: Let $y_i^D(t)$ denote the battery status indicator of node i under $D \geq 0$. Recall that $\delta_m = \pi^2 m^2 D/\omega^2$;

therefore, $\delta_m = 0$ when $D = 0$ and the state equations in (5) for node i become

$$\dot{x}_{i0}(t) = \frac{1}{\alpha} i(t)$$
$$\dot{x}_{im}(t) = \frac{2}{\alpha} i(t) \quad m \in \{1, 2, \ldots, M\}.$$

Hence,

$$\dot{y}_i^0(t) = \sum_{j=0}^{M} \dot{x}_{ij}(t) = G_i\left(w^i(0)\right) g_i\left(w^i(0)\right) \frac{(1+2M)}{\alpha}.$$

Therefore, for any $j \neq i$, we have

$$\frac{\dot{y}_i^0(t)}{\dot{y}_j^0(t)} = \frac{G_i\left(w^i(0)\right) g_i\left(w^i(0)\right)}{G_j\left(w^i(0)\right) g_j\left(w^i(0)\right)}. \tag{42}$$

When $D > 0$, by fixing the routing vector $w(t)$ to $w^i(0)$ and solving the differential (14)–(16) with initial condition $\mathbf{x}_i(0) = 0$, we obtain

$$y_i^D(t) = \frac{1}{\alpha} G_i\left(w^i(0)\right) g_i\left(w^i(0)\right) t + \sum_{j=1}^{M} \frac{2}{\alpha \delta_j} G_i\left(w^i(0)\right) g_i\left(w^i(0)\right)(1 - e^{-\delta_j t}). \tag{43}$$

Recall that $\delta_m = \pi^2 m^2 D / \omega^2$, D is a constant multiplier in δ_j. Consequently, we have

$$\dot{y}_i^D(t) = \frac{1}{\alpha} G_i\left(w^i(0)\right) g_i\left(w^i(0)\right) \left(1 + 2\sum_{j=1}^{M} e^{-\delta_j t}\right).$$

Therefore

$$\frac{\dot{y}_i^D(t)}{\dot{y}_j^D(t)} = \frac{G_i\left(w^i(0)\right) g_i\left(w^i(0)\right)}{G_j\left(w^i(0)\right) g_j\left(w^i(0)\right)} \quad D > 0$$

which is identical to (42). Thus, under $D > 0$, the inequalities $T_i^*(w^i, D) \leq T_j^*(w^i, D)$ remain just as valid as $T_i^*(w^i, 0) \leq T_j^*(w^i, 0)$ under $D = 0$ and it follows that the solution $w^i(D)$ is unaffected relative to $w^i(0)$. Note that **Algorithm A1** gives $w^*(D)$ as the solution of the NLP $\mathbf{P}_i$ such that $\max_i\{T_i^*(w)\} = T_i^*(w^i(D))$ for some i for any $D \geq 0$. Hence, $w^*(0) = w^i(0) = w^i(D) = w^*(D)$. ■

Proof of Theorem 3: The proof is similar to the same problem considered in [1] using the KBM battery model (see proof of [1, Theorem 3]). The critical fact needed in the proof is $\partial T / \partial C_i > 0$ (replacing $\partial T / \partial R_i > 0$ in [1]). ■

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