Optimal Routing and Energy Allocation for Lifetime Maximization of Wireless Sensor Networks With Nonideal Batteries

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***Abstract*—An optimal control approach is used to solve the problem of routing in sensor networks where the goal is to maximize the network's lifetime. In our analysis, the energy sources (batteries) at nodes are not assumed to be "ideal" but rather behaving according to a dynamic energy consumption model, which captures the nonlinear behavior of actual batteries. We show that in a fixed topology case there exists an optimal policy consisting of time-invariant routing probabilities, which may be obtained by solving a set of relatively simple nonlinear programming (NLP) problems. We also show that this optimal policy is, under very mild conditions, robust with respect to the battery model used. Further, we consider a joint routing and initial energy allocation problem over the network nodes with the same network lifetime maximization objective. We prove that the solution to this problem is given by a policy that depletes all node energies at the same time and that the corresponding energy allocation and routing probabilities are obtained by solving an NLP problem. Numerical examples are included to illustrate the optimality of the time-invariant policy and its robustness with respect to the battery model used.**

***Index Terms*—Optimal control, power-limited system, routing, sensor network.**

I. Introduction

A WIRELESS SENSOR NETWORK (WSN) is a spatially distributed wireless network consisting of low-cost autonomous nodes, which are mainly battery powered and have sensing and wireless communication capabilities [20]. Applications of such networks include exploration, surveillance, and environmental monitoring. Power consumption is a key issue in WSNs, since it directly impacts their lifetime in the likely absence of human intervention for most applications of interest. Since the majority of power consumption is due to the radio component [30], nodes rely on short-range communication and form a multihop network to deliver information to a base station. Routing schemes in WSNs aim to deliver data from the data sources (nodes with sensing capabilities) to a data sink (typically, a base station) in an energy-efficient and reliable way. A survey of the state-of-the-art routing algorithms is provided in [1].

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In this paper, we focus on the problem of routing in a WSN with the objective of optimizing performance metrics that reflect the limited energy resources of the network while also preventing common security vulnerabilities. Most proposed routing protocols in WSNs are based on shortest path algorithms, e.g., [25], [23]. Such algorithms usually require each node to maintain a global cost (or state) information table, which is a significant burden for resource-constrained WSNs. In order to deal with node failures, Ganesan *et al.* [14] proposed a multipath routing algorithm, so that a failure on the main path can be recovered without initiating a network-wide flooding process for path rediscovery. Since flooding consumes considerable energy, this routing method can extend the network's lifetime when there are failures. On the other hand, finding multiple paths and sending packets through them also consumes energy, thus adversely impacting the lifetime of the network if there are no failures.

The routing policies mentioned earlier may indirectly reduce energy usage in WSNs, but they do not explicitly use energy consumption models to address optimality of a routing policy with respect to energy-aware metrics. Such "energy awareness" has motivated a number of minimum-energy routing algorithms, which typically seek paths minimizing the energy per packet consumed (or maximizing the residual node energy) to reach a destination, e.g., [31]. However, seeking a minimum energy (or maximum residual energy) path can rapidly deplete energy from some nodes and ultimately reduce the full network's lifetime by destroying its connectivity. Thus, an alternative performance metric is the *network lifetime*. The definition of the term "lifetime" for WSNs varies. Some researchers, e.g., [7], define the network lifetime as the time until the first node depletes its battery; however, this may just as well be defined as the time until the data source cannot reach the data sink [5]. In what follows, we will adopt the former definition, i.e., the time until the first node depletes its battery. As our results will show, it is often the case that an optimal policy controlling the WSN's resources leads to individual node lifetimes being the same or almost the same as those of others, hence this definition is a good characterization of the overall network's lifetime.

Along the lines of energy-aware routing, Shah and Rabaey [29] proposed an Energy Aware Routing (EAR) policy, which does not attempt to use a single optimal path, but rather a number of suboptimal paths that are probabilistically selected with the intent of extending the network lifetime by "spreading" the traffic and forcing nodes in the network to deplete their energies at the same time. In [24], a similar problem is studied with the inclusion of uncertainties in several WSN parameters. From a network security viewpoint, deterministic routing policies (i.e., policies where source nodes send data through one or more fixed paths) are highly vulnerable to attacks that can compromise a node and easily falsify cost information, leading to Denial of Service (DoS) attacks [35]. In order to reduce the effect of such attacks, probabilistic routing is an interesting alternative, since this makes it difficult for attackers to identify an "ideal" node to take over. In this sense, the EAR policy is attractive because of its probabilistic routing structure, even though it does not attempt to provide optimal routing probabilities for network lifetime maximization. It is worth mentioning, however, that a routing policy based on probabilities can easily be implemented as a deterministic policy as well by transforming these probabilities to packet flows over links and using simple mechanisms to ensure that flows are maintained over time.

The network lifetime maximization problem studied in [7] is based on two assumptions. First, it assumes that the energy in a battery depletes linearly with respect to the quantity of information forwarded, and does not depend on the physical dynamics of the battery itself. Second, it seeks fixed routing probabilities over time, even though the dynamic behavior of the WSN may in fact imply that a time-dependent (possibly based on state feedback closed-loop) routing policy may be optimal. More generally, routing problems in WSNs are based on ideal battery models where a battery maintains a constant voltage throughout the discharge process and a constant capacity for all discharge profiles, neither of which is generally true. In fact, the energy amount delivered by a battery heavily depends on the discharge profile and it is generally not possible to extract all the capacity stored in it [22]. This dynamic behavior also leads to the conjecture that an optimal routing policy should consider the battery state over time and should, therefore, be time-dependent rather than fixed. Thus, an optimal control problem formulation for the network lifetime maximization problem seems to be a natural setting.

In this paper, we adopt an optimal control setting with the goal of determining routing probabilities so as to maximize the lifetime of a WSN subject to a *dynamic* energy consumption model for each node. In particular, we will use a *Kinetic Battery Model* (KBM) [18], [28], which has successfully been applied in other power management applications. We will then show that in a fixed network topology case there exists an optimal policy consisting of *time-invariant* routing probabilities. We subsequently show that the optimal control problem may be converted into a set of relatively simple nonlinear programming (NLP) problems. Moreover, we show that, under a very mild condition, this optimal routing policy is in fact robust with respect to the battery model used, i.e., the routing probabilities are not affected by the battery model used, although naturally the estimated WSN lifetime itself is significantly longer under a nonideal battery model, primarily due to the recovery effect mentioned earlier. We also consider an alternative problem where, in addition to routing, we allocate a total initial energy over the network nodes with the same network lifetime maximization objective; the idea here is that a proper allocation of energy can further increase the network lifetime. We show that the solution to this problem is given by a policy that depletes all node energies *at the same time* and that the corresponding energy allocation and routing probabilities are obtained by solving again an NLP problem. We note that when the battery behavior is reduced to a simple idealized model, our setting recovers that of [36] and [21] where it was shown that the set of NLP subproblems can in fact be transformed into the linear programming (LP) formulation in [7]. It was also shown in [21] that the initial energy allocation problem can be reformulated into a shortest path problem on a graph where the arc weights equal the link energy costs.

In Section II, we formulate the maximum lifetime optimization problem using nonideal energy sources at nodes that have their own dynamics. We adopt a standard energy consumption model along with the aforementioned KBM. In Section III, we show that for a fixed network topology there exists an optimal routing policy which is time invariant and identify a set of NLP problems, which can be solved to obtain an explicit fixed optimal routing vector and the corresponding WSN lifetime. We also derive sufficient conditions under which this optimal policy is robust with respect to the battery model used. In Section IV, we consider a joint optimal routing and initial energy allocation problem. We show that in this case (under some conditions), it is optimal to set a routing vector and initial node energies, so that all nodes have the same lifetime. An explicit solution can again be obtained by solving an NLP problem. Numerical examples are included to illustrate our analytical results.

II. Optimal Control Problem Formulation

In order to simplify our analysis, we will consider a WSN with a single source node and one base station and will assume a fixed topology. It will become clear that our methodology can be extended to multiple sources and one base station, as well as time-varying topologies, although the main fixed optimal routing result will obviously no longer hold in general.

A. Network Model

Consider a network with $N+1$ nodes, where 0 and N denote the source and destination (base station) nodes, respectively. Except for the base station whose energy supply is not constrained, a limited amount of energy is available to all other nodes. Let $r_i(t)$ be the residual energy of node i, $i = 0, \ldots, N-1$, at time t. The dynamics of $r_i(t)$ depend on the battery model used at node i; we will discuss in Section II-B the KBM we will adopt. The distance between nodes i and j at time t is denoted by $d_{i,j}(t)$; since we assume a fixed topology, we will treat $d_{i,j}(t)$ as time-invariant in the sequel. The nodes in the network may be ordered according to their distance to the destination node N, so that $d_{1,N} \geq d_{2,N} \geq \cdots \geq d_{i,N} \geq \cdots \geq d_{N-1,N}$ and assume that $d_{0,N} > d_{i,N}$ for all $i = 1, \ldots, N-1$.

Let O_i denote the set of nodes to which node i can send packets. We assume full coverage of the network and define $O_i = \{j : j > i, d_{i,j} < d_{i,N}\}$, where $j > i$ implies that $d_{i,N} > d_{j,N}$, i.e., a node only sends packets to those nodes that are closer to the destination, and $d_{i,j} < d_{i,N}$ means that a node cannot send packets to another node which is further away from it relative to the destination node N. We will use the notation $i \prec j$, if $j \in O_i$. Let $w_{i,j}(t)$ be the routing probability of a packet from node i to node j at time t. The vector $\mathbf{w}(t) = [w_{0,1}(t), \ldots, w_{0,N-1}(t), \ldots, w_{N-2,N-1}(t)]'$ defines the control in our problem. We do not include $w_{0,N}(t), \ldots, w_{i,N}(t), \ldots, w_{N-1,N}(t)$ in the definition of $\mathbf{w}(t)$, since it is clear that $w_{i,N}(t)$ is an implicit control variable given by $w_{i,N}(t) = 1 - \sum_{i \prec j, j<N} w_{i,j}(t), i = 0, \ldots, N-2$.

For simplicity, the data sending rate of source node 0 is normalized to 1 and let $G_i(\boldsymbol{w})$ denote the data packet inflow rate to node i. Given the definitions above, we can express $G_i(\boldsymbol{w})$ through the following flow conservation recursive equation:

$$G_i(\mathbf{w}) = \sum_{k \prec i} w_{k,i}(t) G_k(\mathbf{w}), \quad i = 1, \ldots, N \tag{1}$$

where $G_0(\mathbf{w}) = 1$.

B. Energy Consumption Model

Under the assumption that an electrochemical battery cell is "ideal," a constant voltage throughout the discharge process and a constant capacity for all discharge profiles are both maintained over time. However, in real batteries, the *rate capacity effect* [12] leads to the loss of capacity with increasing load current, and the *recovery effect* [19] makes the battery appear to regain portions of its capacity after some resting time. Due to these phenomena, the voltage as well as energy amount delivered by the battery heavily rest on the discharge profile. Therefore, when dealing with energy optimization, it is necessary to consider this along with nonlinear variations in a battery's capacity. As a result, there are several proposed models to describe a nonideal battery; a detailed overview is given in [16]. Accordingly, models are broadly classified as: electrochemical [13], [12], [26], circuit-based [15], [8], stochastic [10], [9], [11], [28], and analytical [27], [32], [17]. Electrochemical models possess the highest accuracy, but their complexity makes them impractical for most real-time applications. Electrical-circuit models are much simpler and, therefore, computationally less expensive but their accuracy leads to errors, which may be reduced at the expense of added complexity [8]. Stochastic models use a discrete time Markov chain with $N+1$ states to represent the number of charge units available in the battery. Since N is large (in the order of 10^7), these models are also limited by high computational requirements. Last but not the least, analytical models, including diffusion-based models [32], [37], [2] and the KBM [18], [28], use only a few equations to capture the battery's main features. While diffusion-based models are hard to combine with a performance model [16], a KBM combines speed with sufficient accuracy, as reported, for instance, in embedded system applications [18]. Empirical evidence for the accuracy of the KBM is also provided in [28]. The KBM was successfully used to study problems of optimal single and multibattery power control in [33], [34] with results consistent with the use of a more elaborate linear state space model [37] derived from the popular RVW diffusion-based model [27]. In what follows, we briefly review the KBM.

The KBM models a battery as two wells of charge, as shown in Fig. 1. The available-charge well (R-well) directly supplies electrons to the load, whereas the bound-charge well (B-well) only supplies electrons to the R-well. The energy levels in the two wells are denoted by $r(t)$ and $b(t)$, respectively. The rate of energy flow from the B-well to the R-well is $k(b(t) - r(t))$, where k depends on the battery characteristics. The output $u(t)$ is the workload of the battery at time t. The battery is said to be depleted when $r(t) = 0$. If a battery has rechargeability capabilities, we modify the KBM by adding a controllable input flow $h(t)$. For the sake of generality, we distribute the inflow $h(t)$ to both wells by adding a constant coefficient β ($0 \le \beta \le 1$), as seen in Fig. 1. The resulting model is

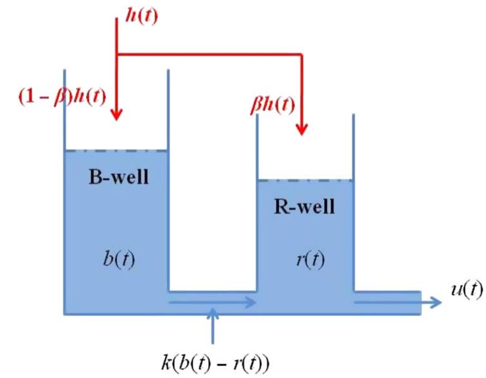

Fig. 1. Kinetic battery model including recharging.

$$\dot{r}(t) = -c_1 u(t) + c_2 \beta h(t) + k(b(t) - r(t)) \tag{2}$$
$$\dot{b}(t) = c_2(1-\beta)h(t) - k(b(t) - r(t)) \tag{3}$$

where c_1, c_2 are battery-specific influencing factors for the discharge outflow $u(t)$ and the recharge inflow $h(t)$, respectively; since, in general, a battery discharges faster than it can recharge, we assume $c_1 > c_2 \ge 0$ where the special case $c_2 = 0$ simply means the battery is not rechargeable. The state variables $r(t), b(t)$ are physically constrained, so that $b(0) \ge r(0)$ and $b(t) \le \bar{B}$, where $\bar{B}$ is the battery capacity.

In our WSN environment, the battery workload $u(t)$ is due to three factors (e.g., see [4], [3]): the energy needed to sense a bit E_{sense}, the energy needed to receive a bit E_{rx}, and the energy needed to transmit a bit E_{tx}. If the distance between two nodes is d, we have

$$E_{\text{tx}} = p(d), \quad E_{\text{rx}} = C_r, \quad E_{\text{sense}} = C_e \tag{4}$$

where C_r, C_e are given constants dependent on the commu nication and sensing characteristics of nodes, and $p(d) \ge 0$ is a function monotonically increasing in d; the most common such function is $p(d) = C_f + C_s d^\beta$ where C_f, C_s are given constants and β is a constant dependent on the medium involved. We shall use this energy model but ignore the sensing energy, i.e., set $C_e = 0$. Clearly, this is a relatively simple energy model that does not consider the channel quality or the Shannon capacity of each wireless channel. The ensuing optimal control analysis is not critically dependent on the exact form of the energy consumption model attributed to communication, although the ultimate optimal value of $\mathbf{w}(t)$ obviously is. Before proceeding, it is convenient to define the following constants:

$$k_{i,j} = p(d_{i,j}) - p(d_{i,N}), \quad i < j < N \tag{5}$$

$$k_{0,N} = p(d_{0,N}) \tag{6}$$

$$k_{i,N} = C_r + p(d_{i,N}), \quad i = 1, \ldots, N-1 \tag{7}$$

where $d_{i,j}$ is the distance between nodes i and j. Note that we may allow these constant to be time-dependent if the network topology is not fixed, i.e., $d_{i,j}(t)$ is time-varying. Let us now combine the KBM model above with (4). Although the ability to recharge a battery offers an interesting possibility for routing control, we shall not consider it in this paper, i.e., set $c_2 = 0$ in (2) and (3). Moreover, for simplicity, we set $c_1 = 1$. Then, starting with node 0, the workload $u_0(t)$ at that node is given by

$$\begin{aligned}
u_0(t) &= \sum_{0 \prec j} w_{0,j}(t) p(d_{0,j}) \\
&= \sum_{0 \prec j, j < N} w_{0,j}(t) p(d_{0,j}) + w_{0,N}(t) p(d_{0,N}) \\
&= \sum_{0 \prec j, j < N} w_{0,j}(t) p(d_{0,j}) + \left[1 - \sum_{0 \prec j, j < N} w_{0,j}(t)\right] p(d_{0,N}) \\
&= \sum_{0 \prec j, j < N} w_{0,j}(t) [p(d_{0,j}) - p(d_{0,N})] + p(d_{0,N}) \\
&= \sum_{0 \prec j, j < N} w_{0,j}(t) k_{0,j} + k_{0,N} \\
&= G_0(\mathbf{w}) \left[\sum_{0 \prec j, j < N} w_{0,j}(t) k_{0,j} + k_{0,N}\right]
\end{aligned}$$

where we have used the fact that $G_0(\mathbf{w}) = 1$. Similarly, for any node $i = 1, \ldots, N-1$, where we must include the energy for both receiving and transmitting data packets, we get

$$u_i(t) = G_i(\mathbf{w}) \left[\sum_{i \prec j, j < N} w_{i,j}(t) k_{i,j} + k_{i,N}\right].$$

Defining $g_i(\mathbf{w}) = \sum_{i < j, j < N} w_{i,j}(t) k_{i,j} + k_{i,N}$, the KBM equations (2) and (3) for nodes $i = 0, \ldots, N-1$ become

$$\dot{r}_i(t) = -G_i(\mathbf{w}(t)) g_i(\mathbf{w}(t)) + k(b_i(t) - r_i(t)) \tag{8}$$

$$\dot{b}_i(t) = -k(b_i(t) - r_i(t)). \tag{9}$$

The vectors $\boldsymbol{r}(t) = [r_0(t), \ldots, r_{N-1}(t)]$ and $\boldsymbol{b}(t) = [b_0(t), \ldots, b_{N-1}(t)]$ define the state variables for our problem. Observe that controlling the routing probabilities $w_{i,j}(t)$ means indirectly controlling node i's battery discharge process.

C. Optimal Control Problem Formulation

Our objective is to maximize the WSN lifetime by controlling the routing probabilities $w_{i,j}(t)$ (equivalently, the flows through all network links). The WSN lifetime is defined as $T = \min_{0 \leq i < N} T_i$, where T_i is given by $T_i = \inf\{t : r_i(t) = 0, t \geq 0\}$. Thus, our objective is to maximize T. Using the energy consumption model we have developed above, the optimal control problem is formulated as follows:

$$\min_{\mathbf{w}(t)} - \int_0^T dt \tag{10}$$

s.t

$$\dot{r}_i(t) = -G_i(\mathbf{w}(t)) g_i(\mathbf{w}(t)) + k(b_i(t) - r_i(t)), \quad r_i(0) = R_i \tag{11}$$

$$\dot{b}_i(t) = -k(b_i(t) - r_i(t)), \quad b_i(0) = B_i \tag{12}$$

$$G_i(\mathbf{w}(t)) = \sum_{k \prec i} w_{k,i}(t) G_k(\mathbf{w}), \quad i = 1, \ldots, N-1 \tag{13}$$

$$G_0(\mathbf{w}(t)) = 1$$

$$g_i(\mathbf{w}(t)) = \sum_{i \prec j, j < N} w_{i,j}(t) k_{i,j} + k_{i,N} \tag{14}$$

$$\sum_{i \prec j, j < N} w_{i,j}(t) \leq 1, \quad 0 \leq w_{i,j}(t) \leq 1, \quad i = 0, \ldots, N-1 \tag{15}$$

$$\min_{i=0,\ldots,N-1} r_i(T) = 0 \tag{16}$$

where $r_i(t)$, $b_i(t)$ are the state variables representing node i's instantaneous battery energy level, $i = 0, \ldots, N-1$. Control constraints are specified through (15), where the first inequality follows from the fact that $\sum_{i \prec j < N} w_{i,j}(t) + w_{i,N}(t) = 1$. Finally, (16) provides boundary conditions for $r_i(t)$, $i = 0, \ldots, N-1$, at $t = T$ requiring that the terminal time is the earliest instant when $r_i(t) = 0$ for any node i. In other words, at $t = T$, we require that the minimal value over all $\{r_0(T), \ldots, r_{N-1}(T)\}$ is 0 or, equivalently, $T = \inf_{t \geq 0}\{t : r_i(t) = 0 \text{ for at least some } i = 0, \ldots, N-1\}$.

This is a classic minimum (maximum) time optimal control problem except for two complicating factors: 1) the boundary condition (16), which involves the nondifferentiable $\min$ function, and 2) the control constraints (15). In what follows, we will use $\mathbf{w}^*(t)$ to denote the optimal routing vector, which provides a (not necessarily unique) solution to this problem.

Remark 1: Note that there is an additional state constraint imposed by the capacity of every node battery, i.e., $b_i(t) \leq B_i$. However, it is easy to show (see [33]) that as long as $R_i < B_i$, it is always true that $r_i(t) < b_i(t) < \bar{B}_i$ for all $t > 0$, so that this constraint is never active in our problem (intuitively, since all batteries are being discharged and never recharged, it is not possible for a capacity to be reached except at $t = 0$). Moreover, if $B_i = R_i$, then $r_i(t) < b_i(t) < \bar{B}_i$ as long as $G_i(\mathbf{w}(t)) g_i(\mathbf{w}(t)) > 0$ for all $t > 0$. Since $k_{i,N} > 0$ in (7) for all $i = 0, \ldots, N-1$, this is always true unless a node i is not used in the network, i.e., $w_{k,i}(t) = 0$ for all $k \prec i$. In addition, observe that when the battery is "at rest," i.e., there is no load in (11), it is easy to show that $\lim_{t \to \infty}(b_i(t) - r_i(t)) = 0$. Therefore, we normally set initial conditions, so that $B_i = R_i$.

III. Optimal Control Problem Solution

We begin with the Hamiltonian for this optimal control problem:

$$H(\mathbf{w},t,\lambda) = -1 + \sum_{i<N} [\lambda_{i1}(t)(-G_i(\mathbf{w}(t))g_i(\mathbf{w}(t)) + k(b_i(t)-r_i(t))) - \lambda_{i2}(t)(k(b_i(t)-r_i(t)))] \tag{17}$$

where $\lambda_{i1}(t)$, $\lambda_{i2}(t)$ are the costates corresponding to $r_i(t)$ and $b_i(t)$ at node i, which must satisfy

$$\begin{cases} \dot{\lambda}_{i1}(t) = -\frac{\partial H}{\partial r_i} = k[\lambda_{i1}(t) - \lambda_{i2}(t)] \\ \dot{\lambda}_{i2}(t) = -\frac{\partial H}{\partial b_i} = -k[\lambda_{i1}(t) - \lambda_{i2}(t)] \end{cases}. \tag{18}$$

To derive explicit expressions for $\lambda_{i1}(t)$, $\lambda_{i2}(t)$, it is necessary to use boundary conditions $\lambda_{i1}(T)$, $\lambda_{i2}(T)$. This is complicated by the nature of the state boundary conditions in (16). Thus, we proceed by considering each possible case of a node dying first, which we will refer to as "scenario S_i" under which $0 = r_i(T) \le r_j(T)$, $j \neq i$ for some fixed node i.

A. Analysis of Scenario S_i

Under S_i, we have the terminal time constraints $r_i(T) = 0$ and $r_j(T) \ge 0$ for all $j \neq i$. Consequently, all $r_j(t)$, $j \neq i$ are unconstrained at $t = T$. The next theorem establishes the property that under a fixed network topology, there exists a static optimal routing policy, i.e., there exists a vector $\mathbf{w}^*(t)$, which is time invariant.

Theorem 1: If $0 = r_i(T) \le r_j(T)$, $j \neq i$, for some i and the network topology is fixed, i.e., $d_{ij}(t) = d_{i,j} = \text{constant}$ for all $i, j = 0, \ldots, N-1$, then there exists a time-invariant solution of (10)–(16):

$$\mathbf{w}^*(t) = \mathbf{w}^*(T).$$

Proof: See Appendix.

Note that there may exist multiple optimal control policies, including some that may be time varying. Theorem 1 asserts that there is at least one which is time-invariant, i.e., $\mathbf{w}^*(t) = \mathbf{w}^*(T) = \mathbf{w}^*$, and it remains to obtain the values of $w^*_{i,j}$, $i = 0, \ldots, N-2$ and $j = 1, \ldots, N-1$, by explicitly solving the optimization problem (37). This requires knowledge of all $r_i(t)$, $t \in [0, T]$ in order to determine the values of all $r_i(T)$ and hence identify the node i, such that $0 = r_i(T) \le r_j(T)$ and use the values of $r_j(T)$, $j \neq i$. This can be accomplished by solving the differential equations (11) and (12), whose initial conditions are given, with $\mathbf{w}(t) = \mathbf{w}$, the unknown optimal routing vector. It is straightforward to obtain $r_i(t)$ and hence show that the "critical time" T_i^* such that $r_i(T_i^*) = 0$ and $r_i(t) > 0$ for all $t \in [0, T_i^*)$ is the solution of the nonlinear equation in T:

$$R_i - \frac{G_i(\mathbf{w})g_i(\mathbf{w})}{2}T - \frac{1}{2}\left[B_i - R_i - \frac{G_i(\mathbf{w})g_i(\mathbf{w})}{2k}\right](e^{-2kT} - 1) = 0 \tag{19}$$

which we write as $T_i^*(\mathbf{w})$. Thus, we may rewrite the S_i optimization problem as follows:

$$\begin{aligned} &\mathbf{P}_i : \min_{\mathbf{w}} G_i(\mathbf{w})g_i(\mathbf{w}) \\ &\text{s.t.} \quad (13)\text{–}(15), \quad T_i^*(\mathbf{w}) \le T_j^*(\mathbf{w}), \quad j \neq i \end{aligned}$$

where $T_i^*(\mathbf{w})$ is the solution of (19) for all $i = 0, \ldots, N-1$. We will refer to this as problem $\mathbf{P}_i$ and note that it may not always have a feasible solution. The following lemma establishes upper and lower bounds for $T_i^*(\mathbf{w})$ based on which necessary conditions for $\mathbf{P}_i$ to have a feasible solution may be derived. Before proceeding, we return to the definitions of the energy consumption constants in (5)–(7) and recall that $k_{i,N} > 0$ for all $i = 0, \ldots, N-1$. Moreover, since $d_{i,j} < d_{i,N}$ if $i \prec j$ and $j < N$, we have

$$k_{i,j} = p(d_{i,j}) - p(d_{i,N}) < 0, \quad \text{if } i \prec j \text{ and } j < N. \tag{20}$$

Let us also define

$$\gamma(i) = \arg \min_{i \prec j, j<N} k_{i,j}. \tag{21}$$

From the definition of $k_{i,j}$, this is the nearest node in the output node set of i.

Lemma 1: For all $i \neq 0$,

$$\frac{R_i}{k_{i,N}} \le T_i^*(\mathbf{w}) \le \infty \tag{22}$$

and for $i = 0$

$$\frac{R_0}{k_{0,N}} \le T_0^*(\mathbf{w}) \le \frac{R_0}{k_{0,\gamma(0)} + k_{0,N} - k\bar{B}_0}. \tag{23}$$

Proof: See Appendix.

Note that it is possible for $k_{0,\gamma(0)} + k_{0,N} - k\bar{B}_0$ to be negative. In practice, however, values of the battery parameter k are small and likely to make the contribution of $k\bar{B}_0$ much smaller than $k_{0,\gamma(0)} + k_{0,N}$. Lemma 1 allows us to determine necessary conditions for $\mathbf{P}_i$ to have a feasible solution. In particular, if $i \neq 0$ and

$$\frac{R_i}{k_{iN}} > \frac{R_0}{k_{0\gamma(0)} + k_{0N} - k\bar{B}_0} \tag{24}$$

then $T_i^*(\mathbf{w}) > T_0^*(\mathbf{w})$ and $\mathbf{P}_i$ has no feasible solution. Thus, the necessary condition for $\mathbf{P}_i$ ($i > 0$) to have a feasible solution is

$$\frac{R_i}{k_{iN}} \le \frac{R_0}{k_{0\gamma(0)} + k_{0N} - k\bar{B}_0}. \tag{25}$$

B. Algorithm for Solving the Optimal Control Problem

Based on our analysis thus far, if we focus on a fixed scenario S_i, the solution to the optimal control problem is simply the solution of the NLP problem $\mathbf{P}_i$. However, since we do not know which node will die first, determining the value of i such that $T_i^*(\mathbf{w}) \le T_j^*(\mathbf{w})$ for all $j \neq i$ requires solving all $\mathbf{P}_i$ problems and find the best policy among them. Since not all $\mathbf{P}_i$ problems have feasible solutions, we can use (25) to check for feasibility before solving the associated NLP problem. The complete algorithm, referred to as A1, to solve this optimal control problem is as follows.

Algorithm A1:

1) Solve problem $\mathbf{P}_0$ to obtain $T_0^*(\boldsymbol{w})$.
2) For $0 < i < N$, if $\frac{R_i}{k_{iN}} > \frac{R_0}{k_{0\gamma(0)}+k_{0N}-k\bar{B}_0}$, set $T_i^*(\boldsymbol{w}) = -1$ (no feasible solution exists); otherwise solve problem $\mathbf{P}_i$ and obtain $T_i^*(\boldsymbol{w})$ if it exists.
3) The optimal lifetime is given by $\max_i \{T_i^*(\boldsymbol{w})\}$ and the corresponding optimal policy $\boldsymbol{w}^*$ is the one obtained for the associated problem $\mathbf{P}_i$.

If the network topology is such that every node i can communicate with every downstream node j, then the algorithm can be substantially simplified due to the following result.

Lemma 2: For a single-source fixed topology network such that $O_i = \{j : j = i+1, \ldots, N\}$ for all $i = 0, \ldots, N-1$, then the source node lifetime is no longer than any other node lifetime under the optimal routing policy $\boldsymbol{w}^*$, i.e.,

$$T_0^*(\boldsymbol{w}^*) \leq T_i^*(\boldsymbol{w}^*), \quad \text{for all } i = 1, \ldots, N-1.$$

Proof: See Appendix.

This lemma allows us to reduce the original optimal control problem to a single problem $\mathbf{P}_0$ as follows:

$$\mathbf{P}_0 : \min_{\boldsymbol{w}} g_0(\boldsymbol{w})$$
$$\text{s.t.} \quad (13)\text{–}(15) \text{ and } T_0^*(\boldsymbol{w}) \leq T_i^*(\boldsymbol{w}), \quad i > 0 \tag{26}$$

where we have used the fact that $G_0(\boldsymbol{w}) = 1$. Clearly, this provides a much simpler approach to the solution.

Remark 2: Our analysis can recover the ideal battery case by setting $k = 0$ in (11) and (12). We can then obtain $T_i^*(\boldsymbol{w})$ for a fixed routing vector $\boldsymbol{w}$ from $\dot{r}_i(t) = -G_i(\boldsymbol{w})g_i(\boldsymbol{w})$, $r_i(0) = R_i$ as $T_i^*(\boldsymbol{w}) = R_i[G_i(\boldsymbol{w})g_i(\boldsymbol{w})]^{-1}$, which greatly simplifies the process of obtaining a solution through Algorithm A1. In this case, as shown in [21], the set of NLP problems $\mathbf{P}_i$ can be transformed into the LP formulation in [7].

C. A Robustness Property of the Optimal Routing Policy

In this section, we show that the optimal routing vector $\boldsymbol{w}^*$ obtained through Algorithm A1 under the ideal battery assumption, i.e., $k = 0$ in (11) and (12), is often unchanged when the nonideal battery model ($k > 0$) is used. The intuition behind such a robustness property lies in the nature of the NLPs $\mathbf{P}_i$ in Section III-A: observe that the solution depends on the values of $G_i(\boldsymbol{w})g_i(\boldsymbol{w})$ and the associated constraints (13)–(15), while the only effect of the parameter k enters through the inequalities $T_i^*(\boldsymbol{w}) \leq T_j^*(\boldsymbol{w})$, $j \neq i$. Therefore, if a solution is obtained under $k = 0$ (a much easier problem which, as we have seen, can be reduced to an LP) and these inequalities are still satisfied when $k > 0$, then there is no need to re-solve the $\mathbf{P}_i$ NLPs. Naturally, when this property holds, the value of the resulting optimal network lifetime is generally different, but the actual routing policy remains unchanged.

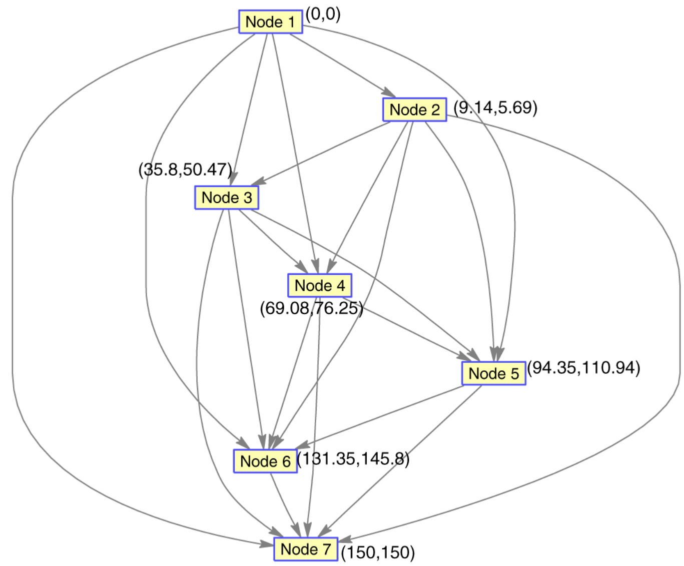

Fig. 2. Network topology-1.

Let $\boldsymbol{w}^i(k)$ denote the solution of problem $\mathbf{P}_i$ when the KBM is invoked with parameter k, including the ideal battery case $k = 0$. The corresponding node lifetimes are denoted by $T_i^*(\boldsymbol{w}^i, k)$. The robustness property we identify rests on the following lemma, which provides simple sufficient conditions under which $\boldsymbol{w}^i(0) = \boldsymbol{w}^i(k)$ for any $k > 0$.

Lemma 3: Consider the NLP $\mathbf{P}_i$ with solution $\boldsymbol{w}^i(k)$ under battery parameter $k \geq 0$. If the initial conditions for the node energies satisfy $B_j = R_j$ for all $j = 0, \ldots, N-1$, then

$$\boldsymbol{w}^i(0) = \boldsymbol{w}^i(k) \quad \text{for any } k > 0. \tag{27}$$

Proof: See Appendix.

The next theorem is a direct consequence of Lemma 3:

Theorem 2: If the initial conditions for all node energies satisfy $R_i = B_i$, $i = 0, \ldots, N-1$, then the optimal routing policy under an ideal battery model, $k = 0$, is unaffected when $k > 0$:

$$\boldsymbol{w}^*(0) = \boldsymbol{w}^*(k), \; k > 0.$$

Proof: See Appendix.

It should be noted that $B_j = R_j$ for all $j = 0, \ldots, N-1$ is a condition that is almost always automatically satisfied by Remark 1: when a battery is initialized at node j, it is normally "at rest"; therefore, $B_j = R_j$. From a practical standpoint, Theorem 2 implies that we can obtain $\boldsymbol{w}^*(0)$ under the ideal battery model assumption using a simple LP (see [21]) and still rely on this solution even if the batteries are in fact nonideal. Naturally, the resulting lifetimes are different, but the computational effort involved to derive an optimal routing policy is substantially reduced. Moreover, it makes the optimal routing policy *independent of the parameter* k, which is often difficult to estimate.

D. Simulation Examples

In order to illustrate the results of our analysis, let us consider a 7-node network as shown in Fig. 2 where node coordinates are

TABLE I
OPTIMAL ROUTING PROBS., 7-NODE NETWORK, IDEAL BATTERIES

w_{ij}	1	2	3	4	5	6
1	N/A	0.343073	0.656927	0	0	0
2	N/A	N/A	0.837081	0.000002	0	0.162917
3	N/A	N/A	N/A	0.971801	0	0.028199
4	N/A	N/A	N/A	N/A	0.929019	0.070981
5	N/A	N/A	N/A	N/A	N/A	1

TABLE II
LIFETIMES UNDER ROUTING POLICY GIVEN IN TABLE I

Node i	1	2	3	4	5	6
Lifetime	54.553	54.554	54.557	54.554	54.555	122.055

given next to each node. Nodes 1 and 7 are the source and base nodes, respectively, while the rest are relay nodes. We set $C_s = 0.0001$, $C_f = C_r = 0.05$, and $\beta = 2$ in the energy model. The total initial energy is $R = 100$ and we assume all nodes have the same initial energy, so that $R_i = 16.67, i = 1, \ldots, 6$. We also set initial conditions for the KBM at all nodes, so that $R_i = B_i$, $i = 1, \ldots, 6$. Table I shows the optimal routing probabilities for this network obtained through Algorithm A1 when ideal batteries are used (this can be recovered in our analysis by setting $k = 0$ in applying Algorithm A1). The optimal network lifetime in this case is 54.55 and Table II shows all node lifetimes under the optimal routing policy (we do not provide specific units in our examples, but based on standard known data, distance units in feet and time units in months or weeks are reasonable). Note that nodes 1–5 die virtually simultaneously, whereas the lifetime of node 6 is considerably longer. This is because energy consumption at each node depends on both the inflow rate to that node and the transmitting distances to other nodes. In this example, node 6 is located close to the base, hence using little energy in packet transmissions. In fact, by relocating node 6 to (120,120) and roughly doubling its distance from the base, it was observed that all 6 nodes die at the same time under the optimal policy. Another important observation in this example is that node 2 receives only 34% of the network inflow and this happens because there is no benefit in sending data packets to a relatively close relay node. The network topology in Fig. 2 and all energy model parameter values are taken from an example in [36] where the routing problem was solved for the ideal battery case. Our results under $k = 0$ recover almost the same routing probabilities and the exact same lifetimes as in this example. Moreover, [36] contains a comparison of the WSN lifetime obtained here with the one obtained using a locally greedy policy, random routing, and the EAR policy in [29]; it was shown that the former provides significant lifetime improvements over all three alternatives.

Next, we revisit the same problem with the KBM battery dynamics (11) and (12). Assuming $k = 0.001$ and using Algorithm A1, the optimal routing probabilities and node lifetimes are given in Tables III and IV, respectively. It is interesting to observe that even such a small value of k results in a lifetime improvement of approximately 3%, which is due to the recovery effect in the battery dynamics captured in (11) and (12). Tables V and VI provide the resulting optimal routing probabilities and node lifetimes for two additional larger values of k, showing considerable network lifetime improvements.

TABLE III
OPTIMAL ROUTING PROBS., 7-NODE NETWORK, *NONIDEAL* BATTERIES ($k = 0.001$)

w_{ij}	1	2	3	4	5	6
1	N/A	0.343110	0.656890	0	0	0
2	N/A	N/A	0.837114	0	0	0.162886
3	N/A	N/A	N/A	0.971793	0	0.028207
4	N/A	N/A	N/A	N/A	0.929022	0.070978
5	N/A	N/A	N/A	N/A	N/A	1

TABLE IV
LIFETIMES UNDER ROUTING POLICY GIVEN IN TABLE III AND $k = 0.001$

Node i	1	2	3	4	5	6
Lifetime	56.0697	56.0696	56.0695	56.0696	56.0697	129.795

TABLE V
OPTIMAL ROUTING PROBABILITIES AND NETWORK LIFETIME FOR A 7-NODE NETWORK (FIG. 2)

Routing probability	$k = 0$	$k = 0.002$	$k = 0.01$
w_{12}	0.343073	0.343110	0.343110
w_{13}	0.656927	0.656890	0.656890
w_{14}	0	0	0
w_{15}	0	0	0
w_{16}	0	0	0
w_{23}	0.837081	0.837114	0.837113
w_{24}	0.000002	0	0.000001
w_{25}	0	0	0
w_{26}	0.162917	0.162886	0.162886
w_{34}	0.971801	0.971793	0.971793
w_{35}	0	0	0
w_{36}	0.028199	0.028207	0.028207
w_{45}	0.929019	0.929022	0.929022
w_{46}	0.070981	0.070978	0.070978
w_{56}	1	1	1
Lifetime	54.554539	57.635541	71.157489
Improvement(%)	N/A	5.65	30.43

TABLE VI
LIFETIMES UNDER ROUTING POLICY GIVEN IN TABLE V

Node i	1	2	3	4	5	6
$k = 0.002$	57.636	57.635	57.635	57.636	57.636	138.038
$k = 0.01$	71.157	71.157	71.157	71.157	71.158	195.12

Comparing Tables I and III, note that the optimal routing probabilities for the ideal and nonideal battery cases are virtually identical, thus confirming our result in Theorem 2 (whose conditions are satisfied in this example). As a result, one can adopt in practice a simple ideal battery model, leading to a simple optimal routing solution through an LP as in [7] and [21]. Similar results are obtained for a symmetric network topology with the same positions for source and base nodes. As one would expect, all nodes die simultaneously due to this symmetry.

Next, we consider an example in which initial node energies are no longer identical, specifically: $R_1 = 20$, $R_2 = 17$, $R_3 = 14$, $R_4 = 11$, $R_5 = 8$, and $R_6 = 5$, while still maintaining the condition $R_i = B_i$. We use the same network shown in Fig. 2 and only shift the source node to the point $(-15, -15)$. Using Algorithm A1, the optimal routing probabilities and network lifetime for different values of k are shown in Tables VII and VIII, respectively. As expected, the robustness property identified in Theorem 2 still applies.

TABLE VII
OPTIMAL ROUTING PROBABILITIES AND NETWORK LIFETIME FOR A 7-NODE NETWORK WITH DIFFERENT INITIAL ENERGIES

Routing probability	$k = 0$	$k = 0.001$	$k = 0.01$
w_{12}	0.910030	0.910034	0.910034
w_{13}	0	0	0
w_{14}	0	0	0
w_{15}	0	0	0
w_{16}	0.089970	0.089966	0.089966
w_{23}	0.950300	0.950301	0.950301
w_{24}	0	0	0
w_{25}	0	0	0
w_{26}	0.049700	0.049699	0.049699
w_{34}	0.889337	0.889332	0.889332
w_{35}	0	0	0
w_{36}	0.110663	0.110668	0.110668
w_{45}	0.823208	0.823210	0.823210
w_{46}	0.176792	0.176790	0.176790
w_{56}	1	1	1
Lifetime	35.25	35.88	42.06
Improvement(%)	N/A	1.79	19.32

TABLE VIII
LIFETIMES UNDER ROUTING POLICY GIVEN BY TABLE VII

i	1	2	3	4	5	6
$k = 0$	35.252	35.253	35.254	35.253	35.254	37.295
$k = 0.001$	35.882	35.882	35.882	35.882	35.882	36.616
$k = 0.01$	42.064	42.065	42.065	42.065	42.065	43.98

IV. A JOINT OPTIMAL ROUTING AND INITIAL ENERGY ALLOCATION PROBLEM

In this section, we go a step beyond routing as a mechanism through which we can control the WSN resources by also controlling the allocation of initial energy over its nodes so as to maximize the lifetime. An application where this problem arises is in a network with rechargeable nodes. In this case, solving the joint optimal routing and initial energy allocation problem provides optimal recharging amounts maximizing the network lifetime, which may not correspond to full charges for all nodes.

Let us define the total initial energy available as $\bar{R}$ and let $\boldsymbol{R} = [R_0, \ldots, R_{N-1}]$. From Theorem 1, we know that the optimal routing policy is fixed unless the topology of the network changes. Then, we can formulate the following problem:

$$\max_{\substack{R_i, i=0,\ldots,N-1 \\ w_{ij}, j=1,\ldots,N-1}} T$$

$$\text{s.t.} \quad T \le T_i^*(\boldsymbol{w}, R_i), \quad i = 0, \ldots, N-1$$

$$\sum_{i \prec j, j < N} w_{i,j} \le 1, \quad 0 \le w_{i,j} \le 1, \quad i, j = 0, \ldots, N, \quad i \prec j$$

$$0 < R_i < \min(B_i, \bar{R}), \quad \sum_{i=0}^{N-1} R_i = \bar{R}. \tag{28}$$

This is an NLP problem where the control variables are both the routing probabilities $w_{i,j}$ and the initial energies R_i. In this case, $T_i^*(\boldsymbol{w}, R_i)$ is the solution of (19) for all $i = 0, \ldots, N-1$, which is now dependent on both $\boldsymbol{w}$ and R_i. Differentiating (19) with respect to R_i, we get

$$\frac{1}{2} - \frac{G_i(\mathbf{w})g_i(\mathbf{w})}{2}\frac{\partial T}{\partial R_i} + k\left[B_i - \frac{G_i(\mathbf{w})g_i(\mathbf{w})}{2k}\right]e^{-2kT}\frac{\partial T}{\partial R_i}$$
$$+\frac{1}{2}e^{-2kT} - kR_i e^{-2kT}\frac{\partial T}{\partial R_i} = 0$$

which yields:

$$\frac{\partial T}{\partial R_i} = \frac{1}{2}(1 + e^{-2kT})$$
$$\times \left[\frac{G_i(\mathbf{w})g_i(\mathbf{w})}{2}(1 + e^{-2kT}) - k(B_i - R_i)e^{-2kT}\right]^{-1}.$$

Observe that $\frac{\partial T}{\partial R_i} > 0$ if and only if

$$B_i - R_i < \frac{G_i(\mathbf{w})g_i(\mathbf{w})}{2k}(1 + e^{2kT}). \tag{29}$$

Recalling Remark 1, we may assume that $B_i = R_i$ since all batteries are normally initialized at an equilibrium state. In this case, (29) holds. Otherwise, (29) becomes a condition we need to impose so as to ensure that $\frac{\partial T}{\partial R_i} > 0$, which will be used in the result that follows.

If the solution of problem (28) is $(\boldsymbol{w}^*, \boldsymbol{R}^*)$, then $T_i^*(\boldsymbol{w}^*, R_i^*)$ is the solution of (19) under this routing vector and initial energy at node i. The following theorem establishes the fact that this optimal solution is such that all nodes deplete their energy at the same time.

Theorem 3: If condition (29) holds, the solution of problem (28) satisfies

$$T^* = T_0^*(\boldsymbol{w}^*, R_0^*) = T_1^*(\boldsymbol{w}^*, R_1^*) = \cdots = T_{N-1}^*(\boldsymbol{w}^*, R_{N-1}^*). \tag{30}$$

Proof: See Appendix.

Remark 3: In order to guarantee (30), it is necessary that $T_i^*(\boldsymbol{w}^*, R_i^*) < \infty$. Looking at (19) and recalling that $g_i(\boldsymbol{w}) > 0$, this is equivalent to assuming that $G_i(\boldsymbol{w}) > 0$, i.e., no node is left unutilized.

Based on Theorem 3, we can simplify the NLP problem (28). In particular, we solve it in two steps. In Step 1, assuming a fixed routing policy $\boldsymbol{w}$, we determine the corresponding optimal initial energy distribution policy by solving the set of equations:

$$T_0^*(\boldsymbol{w}, R_0) = T_1^*(\boldsymbol{w}, R_1) = \cdots = T_{N-1}^*(\boldsymbol{w}, R_{N-1})$$
$$\text{s.t.} \sum_{i=0}^{N-1} R_i = \bar{R}. \tag{31}$$

Its solution is defined to be $\boldsymbol{R}^*(\boldsymbol{w})$ with an associated lifetime $T^*(\boldsymbol{w})$. Then, in Step 2, we search over the feasible set of $\boldsymbol{w}$ given by (15) to determine the optimal $T^*(\boldsymbol{w})$ by using a standard nonlinear optimization solution procedure. We should point out, however, that solving problem (31) to obtain parametric solutions for $T^*(\boldsymbol{w})$ and $\boldsymbol{R}^*(\boldsymbol{w})$ is not a simple task and common solvers fail to accomplish it. Instead, we can proceed by selecting one of the parametric equations for $T_i^*(\mathbf{w}, R_i)$ as an objective function and add (31) as constraints to a new NLP problem below, whose solution we can obtain with standard optimization solvers:

$$\begin{aligned} &\max_{R_i, w_{ij}, j=1,\ldots,N-1} T_i^*(\mathbf{w}, R_i) \\ &\text{s.t.} \quad T_i^*(\mathbf{w}, R_i) - T_j^*(\mathbf{w}, R_j) = 0, \quad i,j = 0,\ldots,N-1, \quad i \neq j \\ &\sum_{i \prec j, j < N} w_{i,j} \le 1, \quad 0 \le w_{i,j} \le 1, \quad i,j = 0,\ldots,N, \quad i \prec j \\ &0 < R_i < \min(B_i, \bar{R}), \quad \sum_{i=0}^{N-1} R_i = \bar{R}. \end{aligned} \tag{32}$$

Remark 4: As in Section III, our analysis can recover the ideal battery case by setting $k = 0$ in (11) and (12), which implies that $T_i^*(\mathbf{w}) = R_i[G_i(\mathbf{w})g_i(\mathbf{w})]^{-1}$. This simplifies the solution of (31) as follows. Setting $K_i(\mathbf{w}) = [G_i(\mathbf{w})g_i(\mathbf{w})]^{-1}$, (31) implies that

$$R_i = \frac{K_0(\mathbf{w})}{K_i(\mathbf{w})} R_0, \quad i = 1,\ldots,N-1$$

$$R_0 = \bar{R}\left[1 + \sum_{i=1}^{N-1} \frac{K_0(\mathbf{w})}{K_i(\mathbf{w})}\right]^{-1} = \frac{\bar{R}}{K_0(\mathbf{w})}\left[\sum_{i=0}^{N-1} \frac{1}{K_i(\mathbf{w})}\right]^{-1}$$

and it follows that

$$R_i^*(\mathbf{w}) = \frac{\bar{R}}{K_i(\mathbf{w})}\left[\sum_{j=0}^{N-1} \frac{1}{K_j(\mathbf{w})}\right]^{-1}, \quad i = 1,\ldots,N-1.$$

Then, the lifetime $T^*(\mathbf{w})$ is given by

$$\begin{aligned} T^*(\mathbf{w}) &= K_0(\mathbf{w})R_0 = \bar{R}\left[\sum_{j=0}^{N-1} \frac{1}{K_j(\mathbf{w})}\right]^{-1} \\ &= \bar{R}\left[\sum_{i=0}^{N-1} G_i(\mathbf{w})\left(\sum_{i \prec j, j < N} w_{i,j}k_{ij} + k_{i,N}\right)\right]^{-1}. \end{aligned}$$

Consequently, the solution of problem (28) is the same as that of the NLP problem:

$$\begin{aligned} &\min_{\mathbf{w}} \sum_{i=0}^{N-1} G_i(\mathbf{w})\left(\sum_{i \prec j, j < N} w_{i,j}k_{ij} + k_{i,N}\right) \\ &\text{s.t.} 0 \le w_{i,j} \le 1, 0 \le i,j \le N \,\text{and}\, i \prec j \\ &\qquad \sum_{i \prec j, j < N} w_{i,j} \le 1, \quad G_i(\mathbf{w}) > 0. \end{aligned}$$

A. Simulation Examples

In this section, we consider a numerical example for the joint optimal routing and initial energy allocation problem. As in Section III-D, first the problem is solved for a network with ideal node batteries and then using the KBM dynamics (11) and (12). Let us consider the same network as in Fig. 2 except we relocate the source node to $(-15, -15)$. Table IX shows the optimal routing probabilities and initial energies of all nodes under different vaklues of k, including the ideal battery case where $k = 0$ in (11) and (12). Note that the WSN lifetime with $k = 0$ is 63.33, which considerably exceeds the value 54.55 seen in Section III-D, even though the distance between the source and base nodes is larger in this case. Moreover, once again we observe that both optimal initial energies and routing probabilities are the same over different values of k. Finally, note the fact that the network lifetime coincides with all individual node lifetimes, which are the same by Theorem 3, and provides a strong justification for the definition of network lifetime being that of the first node to deplete its energy.

TABLE IX
OPTIMAL ROUTING PROBABILITIES, INITIAL BATTERY ENERGY, AND NETWORK LIFETIME FOR A 7-NODE NETWORK

Routing probability	$k = 0.001$	$k = 0.002$	$k = 0.01$
w_{12}	1	1	1
w_{23}	1	1	1
w_{34}	1	1	1
w_{45}	1	1	1
w_{56}	1	1	1
R_1	9.57	9.57	9.57
R_2	23.53	23.53	23.53
R_3	17.55	17.55	17.55
R_4	18	18	18
R_5	22.7	22.7	22.7
R_6	8.65	8.65	8.65
Lifetime	65.3752	67.501	85.6695
Improvement(%)	3.23	6.59	35.27

V. CONCLUSIONS AND FUTURE WORK

We have shown that an optimal routing policy for maximizing a fixed topology sensor network's lifetime is time invariant even when the batteries used as energy sources for the nodes are modeled so as to consider "nonideal" phenomena such as the rate capacity effect and the recovery effect. The associated fixed routing probabilities may be obtained by solving a set of relatively simple NLP problems. In addition, under very mild conditions, this optimal policy is independent of the battery parameter k, where $k = 0$ for ideal batteries. Therefore, one can resort to the case of ideal batteries where the optimal routing problem is much simpler to solve and can be reduced to an LP problem. We have also considered a joint routing and initial energy allocation problem over the network nodes with the same network lifetime maximization objective. In this case, the solution to this problem is given by a policy that depletes all node energies at the same time and the corresponding energy allocation and routing probabilities are obtained by solving an NLP problem.

Extensions of our analysis to networks with multiple sources and base stations are expected to be straightforward. The robustness property we have identified for the optimal routing policy with respect to the battery dynamics assumed may no longer hold if different nodes use different battery characteristics (i.e., different parameters k_i). In addition, it remains to investigate whether different battery models used can still preserve the time-invariant nature of the optimal routing policy and the robustness property identified in Theorem 2. It is also interesting to explore how an optimal routing policy may depend on a changing network topology. Finally, the solutions we have obtained so far are centralized and require global location information, so that an obvious direction to pursue is one seeking distributed versions of the same optimal routing and energy allocation problem approaches.

Appendix

Proof of Theorem 1: Since $r_i(t) \geq 0$ for all i and $t \in [0,T]$, the optimal control problem under S_i is state-unconstrained except for $r_i(T) = 0$. Thus, the terminal state constraint function $\Phi(r(T), b(T))$ is reduced to $r_i(T)$ and the costate boundary conditions [6] are given by

$$\begin{cases} \lambda_{i1}(T) = \nu \frac{\partial \Phi(r(T),b(T))}{\partial r_i} = \nu \\ \lambda_{i2}(T) = 0 \end{cases} \quad \begin{cases} \lambda_{j1}(T) = 0 \\ \lambda_{j2}(T) = 0, \quad j \neq i \end{cases}$$

where ν is an unspecified scalar constant. This allows us to solve the costate equations in (18) to obtain for $t \in [0,T]$:

$$\begin{matrix} \lambda_{i1}(t) = \frac{\nu}{2}(1 + e^{2k(t-T)}) & \lambda_{j1}(t) = 0 \\ \lambda_{i2}(t) = \frac{\nu}{2}(1 - e^{2k(t-T)}) & \lambda_{j2}(t) = 0 \end{matrix}, \quad j \neq i. \tag{33}$$

Using (33) in (17), we can simplify the Hamiltonian as follows:

$$H(\mathbf{w},t,\lambda) = -1 + \lambda_{i1}(t)[-G_i(\mathbf{w}(t))g_i(\mathbf{w}(t)) + k(b_i(t) - r_i(t))] - \lambda_{i2}(t)[k(b_i(t) - r_i(t))]. \tag{34}$$

Observe that the control variables $w_{i,j}(t)$ appear only in $G_i(\mathbf{w}(t))$ and $g_i(\mathbf{w}(t))$ in the problem formulation (10)–(16). Thus, we can set $U_i(t) = G_i(\mathbf{w}(t))g_i(\mathbf{w}(t)),\ i = 0, \ldots, N-1$ to be the effective control variables with $U_l \leq U_i(t) \leq U_u$, where $U_l \geq 0$ and U_u are, respectively, the lower bound and upper bound of $U_i(t)$ for all $t \in [0,T]$. Note that both are constant since their determination depends exclusively on (13), (14) subject to (15), independent of the states $r_i(t)$ and $b_i(t)$. In particular, they depend on the fixed network topology and the values of the energy parameters $k_{i,j},\ k_{i,N}$ in (14). Applying the Pontryagin minimum principle to (34):

$$U_i^*(t) = \arg \min_{U_l \leq U_i(t) \leq U_u} H(U_i, t, \lambda^*)$$

implies that the optimal control is of bang-bang type:

$$U_i^*(t) = \begin{cases} U_u & \text{if } \lambda_{i1}(t) > 0 \\ U_l & \text{if } \lambda_{i1}(t) < 0 \end{cases} \tag{35}$$

with the possibility that there is a singular arc on the optimal trajectory if $\lambda_{i1}(t) = 0$. Moreover, the optimal solution must satisfy the transversality condition [6] $(\lambda^* \frac{d\Phi}{dt} + L)_{t=T} = 0$, where $L = -1$ and we have seen that $\Phi(r(T), b(T)) = r_i(T)$. Therefore

$$-1 + \nu \dot{r}_i(T) = -1 + \nu - G_i(\mathbf{w}(T))g_i(\mathbf{w}(T)) + kb_i(T)] = 0$$

and it follows that

$$\nu = \frac{1}{-G_i(\mathbf{w}(T))g_i(\mathbf{w}(T)) + kb_i(T)}. \tag{36}$$

Observing that $\nu \neq 0$ and looking at (33), we can immediately exclude the singular case $\lambda_{i1}(t) = 0$. Moreover, since $r_i(T) = 0$ and $r_i(t) > 0$ for all $t \in [0,T)$, it follows that $\dot{r}_i(T) < 0$ and (36) implies that $\nu < 0$. Therefore, from (33), $\lambda_{i1}(t) < 0$ throughout $[0,T)$. Consequently, $U_i^*(t) = U_l$ for $t \in [0,T]$ by (35). We conclude that the optimal control problem under S_i is reduced to the following optimization problem:

$$\min_{\mathbf{w}(t)} G_i(\mathbf{w}(t))g_i(\mathbf{w}(t))$$
$$\text{s.t.} \quad (13)\text{–}(15) \text{ and } 0 = r_i(T) \leq r_j(T), \quad j \neq i. \tag{37}$$

When $t = T$, the solution of this problem is $\mathbf{w}^*(T)$ and depends only on $r_j(T),\ j \neq i$, and, as already argued, the fixed network topology and the values of the fixed energy parameters $k_{i,j},\ k_{i,N}$ in (14). The same applies to any other $t \in [0,T)$; therefore, there exists a time-invariant optimal control policy $\mathbf{w}^*(t) = \mathbf{w}^*(T)$, which minimizes the Hamiltonian and proves the theorem. ■

Proof of Lemma 1: We begin with a lower bound for $T_i^*(\mathbf{w})$, $i = 0, \ldots, N-1$. Recalling the state equation (11) and observing that $k(b_i(t) - r_i(t)) \geq 0$, it follows that a lower bound for $T_i^*(\mathbf{w})$, when $r_i(t)$ first reaches zero, is given by the value of $\mathbf{w}$ that maximizes $[G_i(\mathbf{w}) \sum_{i \prec j, j < N} w_{i,j}k_{i,j} + k_{i,N}]$, i.e.,

$$T_i^*(\mathbf{w}) \geq R_i \left[G_i(\mathbf{w}) \sum_{i \prec j, j < N} w_{i,j}k_{i,j} + k_{i,N} \right]^{-1}. \tag{38}$$

The inflow rate $G_i(\mathbf{w})$ is upper-bounded by the sending rate of the source $G_0(\mathbf{w}) = 1$; therefore, $G_i(\mathbf{w}) \leq 1$. Thus

$$G_i(\mathbf{w}) \left(\sum_{i \prec j, j < N} w_{i,j}k_{ij} + k_{iN} \right) \leq \sum_{i \prec j, j < N} w_{i,j}k_{ij} + k_{iN}. \tag{39}$$

Next, consider $\sum_{i \prec j, j < N} w_{i,j}k_{i,j} + k_{i,N}$. In view of (20) and $k_{i,N} > 0$, setting $w_{i,j} = 0$ for all $j < N$ and $i \prec j$ attains the maximal value of this expression, i.e.,

$$\sum_{i \prec j, j < N} w_{i,j}k_{i,j} + k_{i,N} \leq k_{i,N}. \tag{40}$$

Combining (39) and (40), we have

$$G_i(\mathbf{w}) \left(\sum_{i \prec j, j < N} w_{i,j}k_{i,j} + k_{i,N} \right) \leq k_{i,N}$$

and it follows from (38) that

$$T_i^*(\mathbf{w}) \geq \frac{R_i}{k_{i,N}}. \tag{41}$$

Regarding an upper bound for $T_i^*(\mathbf{w})$, if $i \neq 0$, it is possible to have $G_i(\mathbf{w}) = 0$, while the upper bound for the term

$k(b_i(t) - r_i(t))$ in (11) is $k\bar{B}_i$, where $\bar{B}_i$ is the battery capacity. Hence, we can only write $T_i^*(\mathbf{w}) \leq \infty$ and this establishes (22).

If $i = 0$, we have $G_0(\mathbf{w}) = 1$ and it follows from (11) that

$$\dot{r}_0(t) \leq -\left(\sum_{0 \prec j, j < N} w_{0,j} k_{0,j} + k_{0,N}\right) + k\bar{B}_0.$$

Therefore, an upper bound for $T_0^*(\mathbf{w})$ is obtained by minimizing $[\sum_{0 \prec j, j<N} w_{0,j}k_{0,j} + k_{0,N} - k\bar{B}_0]$. This entails solving an LP problem as follows:

$$\min_{w} \sum_{0 \prec j, j < N} w_{0,j} k_{0,j} + k_{0,N} - k\bar{B}_0$$
$$\text{s.t.} \sum_{0 \prec j, j < N} w_{0,j} \leq 1, \quad 0 \leq w_{0,j} \leq 1.$$

For this problem, one of the extreme points of the feasible set will be an optimal solution. There are N extreme points $[w_{0,1}, \ldots, w_{0,N-1}]$, such that:

$$w_{0,j} = \begin{cases} 1 & \text{if } j = m \\ 0 & \text{otherwise} \end{cases}, \quad m = 1, \ldots, N-1 \tag{42}$$

and the point $[0, \ldots, 0]$. The latter cannot minimize the objective function, since, from (20), we know that $k_{0,j} < 0$. Thus, the optimal extreme point must be one of the $N-1$ extreme points in (42). In this case, the objective function becomes

$$\sum_{0 \prec j, j < N} w_{0,j} k_{0,j} + k_{0,N} - k\bar{B}_0 = w_{0,m} k_{0,m} + k_{0,N} - k\bar{B}_0$$
$$= k_{0,m} + k_{0,N} - k\bar{B}_0$$

for some $m = 1, \ldots, N-1$. Thus, in order to minimize the objective function, we need to find the smallest $k_{0,m}$. It follows from (21) that the optimal extreme point is such that

$$w_{0,j} = \begin{cases} 1 & \text{if } j = \gamma(0) \\ 0 & \text{otherwise} \end{cases}$$

and the optimal value is $k_{0,\gamma(0)} + k_{0,N} - k\bar{B}_0$. It follows that

$$T_0^*(\mathbf{w}) \leq \frac{R_0}{k_{0,\gamma(0)} + k_{0,N} - k\bar{B}_0} \tag{43}$$

which along with (41) proves (23) and completes the proof. ■

Proof of Lemma 2: There are two cases to consider:

Case 1: If $w_{0,N}^* = 1$, then it is obvious that $T_0^*(\mathbf{w}^*) \leq T_i^*(\mathbf{w}^*)$ for all $i = 1, \ldots, N-1$ since none of the nonsource nodes is used.

Case 2: If $w_{0,N}^* < 1$, we use a contradiction argument. Let us assume that under the optimal routing vector $\mathbf{w}^*$ there exists a node, say $p > 0$, which dies first in the network, i.e., $T_p^*(\mathbf{w}^*) = T^* < T_0^*(\mathbf{w}^*)$. Next, let us introduce the following perturbation to the optimal routing vector:

$$\mathbf{w'}_{m,n} = \begin{cases} w_{m,n}^* + K\epsilon, & \text{if } m = 0,\ n = N \\ w_{mn}^* - \epsilon, & \text{if } m = 0,\ 1 \leq n \leq N-1,\ w_{m,n}^* > 0 \\ w_{m,n}^*, & \text{otherwise} \end{cases}$$

where $\epsilon > 0$ is sufficiently small, so that the new routing policy $\mathbf{w'}$ is still feasible, and $K = \sum_{j=1}^{N} 1\,[w_{0,j} > 0]$ where $1\,[w_{0,j} > 0]$ is the usual indicator function (this is always possible for $w_{0,n}^* > 0$). In other words, we only perturb routing probabilities from the source node 0 to other nodes. Consequently, we increase the flow rate from the source to the sink node, and decrease flow rates into other nodes so as to maintain the same total flow out of node 0. It follows that the source node's life must decrease since it sends more traffic through the longest link. At the same time, the lifetimes of all other nodes receiving positive flows from node 0 must increase since the inflow rates into all of them decrease. Therefore, letting T_i' denote the node i lifetime under the perturbed routing vector $\mathbf{w'}$, we have

$$T_0' = T_0^*(\mathbf{w}^*) - f_0(\epsilon),\ T_j' = T_j^*(\mathbf{w}^*) + f_j(\epsilon), \quad j = 1, \ldots, N-1$$

where $f_k(x)$, $k = 0, \ldots, N-1$ is a continuous function, such that $f_k(x) \geq 0$ and $f_k(0) = 0$. Since $f_k(x)$ is continuous, we can find a small enough $\epsilon > 0$ such that $T_P' < T_0'$, so that the source node 0 cannot die first under $\mathbf{w'}$. Therefore, the lifetime under routing policy $\mathbf{w'}$ is $T' = \min_{j \neq 0} T_j'$.

Since the lifetimes of all nonsource nodes increase under $\mathbf{w'}$, it follows that $T' = \min_{j \neq 0} T_j' > T^*$. In other words, $\mathbf{w'}$ provides a longer network lifetime that $\mathbf{w}^*$ contradicting the assumption that $\mathbf{w}^*$ is optimal. ■

Proof of Lemma 3: Let $r_i^k(t)$, $b_i^k(t)$ denote the node i battery state variables under $k \geq 0$. When $k = 0$, (11) becomes $\dot{r}_i^0(t) = -G_i(\mathbf{w}^i(0)) g_i(\mathbf{w}^i(0))$. Therefore, for any $j \neq i$, we have

$$\frac{\dot{r}_i^0(t)}{\dot{r}_j^0(t)} = \frac{G_i(\mathbf{w}^i(0)) g_i(\mathbf{w}^i(0))}{G_j(\mathbf{w}^i(0)) g_j(\mathbf{w}^i(0))}. \tag{44}$$

When $k > 0$, let $z_i^k(t) = b_i^k(t) - r_i^k(t)$ and note that by subtracting (11) from (12), we have

$$\dot{z}_i^k(t) = G_i(\mathbf{w}(t)) g_i(\mathbf{w}(t)) - 2kz_i^k(t).$$

Fixing the routing vector $\mathbf{w}(t)$ to $\mathbf{w}^i(0)$ and solving the differential equation above with initial condition $z_i^k(0) = B_i - R_i = 0$ by assumption, we get $z_i^k(t) = \frac{G_i(\mathbf{w}^i(0)) g_i(\mathbf{w}^i(0))}{2k}(1 - e^{-2kt})$.

Using this in (11), we have

$$\dot{r}_i^k(t) = -G_i(\mathbf{w}^i(0)) g_i(\mathbf{w}^i(0)) + \frac{G_i(\mathbf{w}^i(0)) g_i(\mathbf{w}^i(0))}{2}(1 - e^{-2kt})$$
$$= -G_i(\mathbf{w}^i(0)) g_i(\mathbf{w}^i(0)) \frac{1 + e^{-2kt}}{2}. \tag{45}$$

Therefore

$$\frac{\dot{r}_i^k(t)}{\dot{r}_j^k(t)} = \frac{G_i(\mathbf{w}^i(0)) g_i(\mathbf{w}^i(0))}{G_j(\mathbf{w}^i(0)) g_j(\mathbf{w}^i(0))}, k > 0$$

which is identical to (44). Thus, under $k > 0$, the inequalities $T_i^*(\mathbf{w}^i, k) \leq T_j^*(\mathbf{w}^i, k)$ remain just as valid as $T_i^*(\mathbf{w}^i, 0) \leq T_j^*(\mathbf{w}^i, 0)$ under $k = 0$ and it follows that the solution $\mathbf{w}^i(k)$ is unaffected relative to $\mathbf{w}^i(0)$, completing the proof. ■

Proof of Theorem 2: By assumption, Lemma 3 applies to all nodes $i = 0, \ldots, N-1$, i.e., $\mathbf{w}^i(0) = \mathbf{w}^i(k)$. Algorithm A1 gives $\mathbf{w}^*(k)$ as the solution of the NLP $\mathbf{P}_i$ such that $\max_i \{T_i^*(\mathbf{w})\} = T_i^*(\mathbf{w}^i(k))$ for some i for any $k \geq 0$. It then follows from Lemma 3 that $\mathbf{w}^*(0) = \mathbf{w}^i(0) = \mathbf{w}^i(k) = \mathbf{w}^*(k)$. ■

Proof of Theorem 3: We use a contradiction argument. Let us assume that under the optimal policy $(\mathbf{w}^*, \boldsymbol{R}^*)$, not all nodes die together. We then define the following two index sets:

$$S_1 = \{i : T_i^*(\mathbf{w}^*, R_i^*) = T^*\}, \quad S_2 = \{i : T_i^*(\mathbf{w}^*, R_i^*) > T^*\}.$$

According to our assumption, S_2 is not empty and let $j = \arg\min_{i \in S_2}\{T_i^*(\mathbf{w}^*, R_i^*)\}$, i.e., node j is the first one to die after time T^* and for all $i \in S_1$, we have $T^* = T_i^*(\mathbf{w}^*, R_i^*) < T_j^*(\mathbf{w}^*, R_j^*)$ (if there are two or more nodes with the same value $T_j^*(\mathbf{w}^*, R_j^*)$, then we select any one of them). Keeping the routing vector to its optimal value $\mathbf{w}^*$, we then perturb the energy allocation vector $\boldsymbol{R}^*$ to a new vector $\boldsymbol{R}'$ as follows:

$$\begin{aligned} R_i' &= R_i^* + \epsilon \quad \text{for all } i \in S_1 \\ R_j' &= R_j^* - |S_1|\epsilon \\ R_k' &= R_k \quad \text{for all } k \in S_2,\ k \neq j \end{aligned}$$

where $\epsilon > 0$ is sufficiently small to ensure $R_j' > 0$. Since $\sum_{i=0}^{N-1} R_i' = \bar{R}$, it follows that $(\mathbf{w}^*, \boldsymbol{R}')$ is a feasible policy. Under this policy, the node lifetimes are given by $T_i' = T_i^*(\mathbf{w}^*, R_i')$, the solution of (19) under $(\mathbf{w}^*, \boldsymbol{R}')$. Since we have shown that $\frac{\partial T}{\partial R_i} > 0$ under (29), we have

$$T_i' = \begin{cases} T_i^*(\mathbf{w}^*, R_i^*) + f_i(\epsilon) & \text{if } i \in S_1 \\ T_i^*(\mathbf{w}^*, R_i^*) - f_i(\epsilon)|S_1| & \text{if } i = j \\ T_i^*(\mathbf{w}^*, R_i^*) & \text{otherwise} \end{cases}$$

where $f_k(x)$ is a continuous function such that $f_k(x) \geq 0$ and $f_k(0) = 0$. Since $f_k(x)$ is continuous, we can find a small enough $\epsilon > 0$ and hence $f_i(\epsilon)$ to guarantee that

$$T^* = T_i^*(\mathbf{w}^*, R_i^*) < T_i' < T_j', \quad \text{for all } i \in S_1$$

and the lifetime under $(\mathbf{w}^*, \boldsymbol{R}')$ is $T' = \min_{i \in S_1}\{T_i'\} > T^*$.

Thus, by choosing a small enough $\epsilon > 0$ the network lifetime under $(\mathbf{w}^*, \boldsymbol{R}')$ is larger than under $(\mathbf{w}^*, \boldsymbol{R}^*)$ which contradicts the optimality of $(\mathbf{w}^*, \boldsymbol{R}^*)$. Therefore, we conclude that S_2 must be empty, which implies (30). ■

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