

*A VIEW OF
COOPERATIVE CONTROL
FOR
AUTONOMOUS VEHICLES
AND SENSOR NETWORKS*



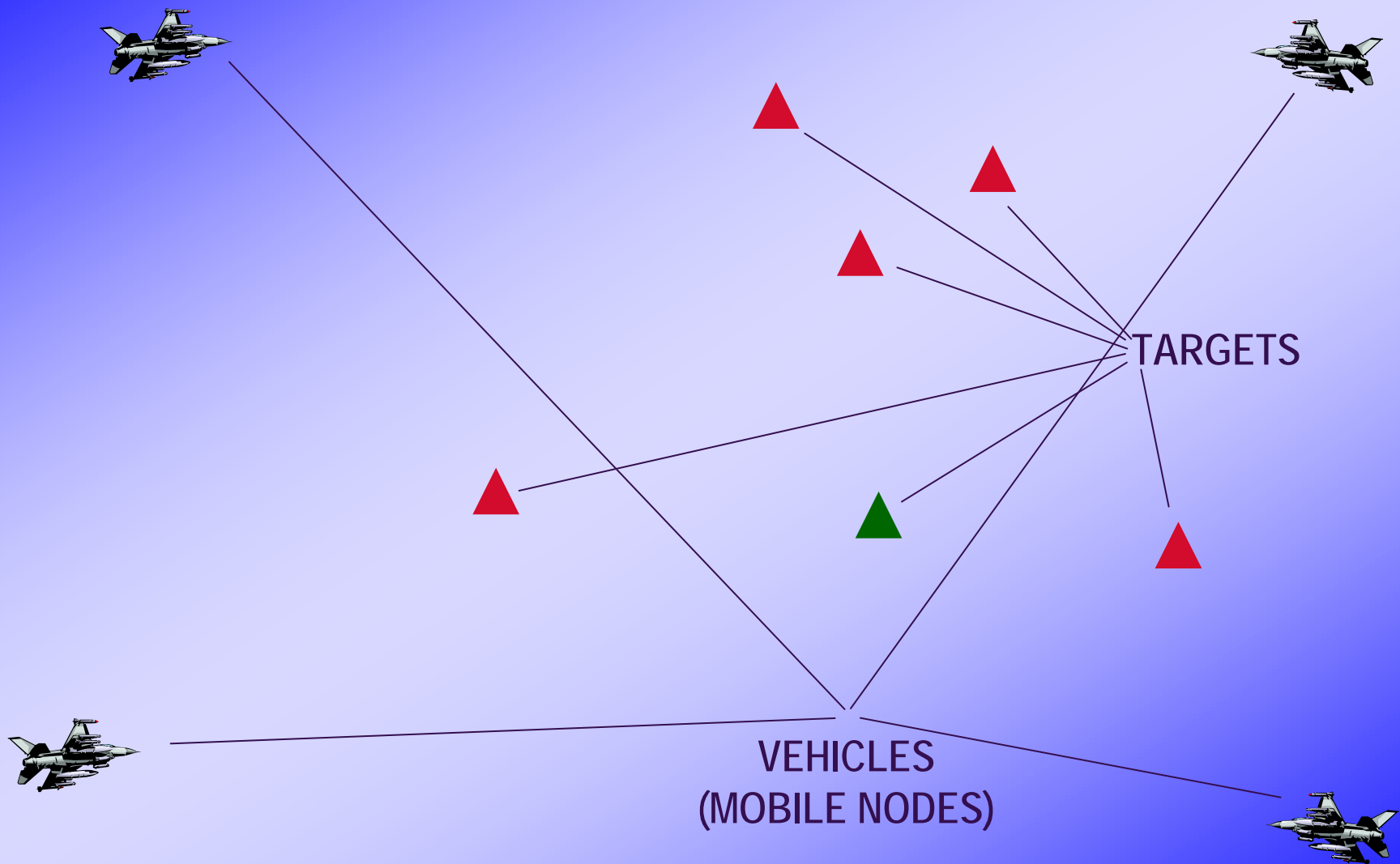
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Boston University*

OUTLINE

- COOPERATIVE CONTROL SETTING
- COOPERATIVE "MISSION" TYPES
- *REWARD MAXIMIZATION MISSIONS*
- **COOPERATIVE RECEDING HORIZON (CRH) CONTROL**
- *SENSOR NETWORKS, COVERAGE CONTROL MISSIONS*
- DEMOS: Applets and Movies

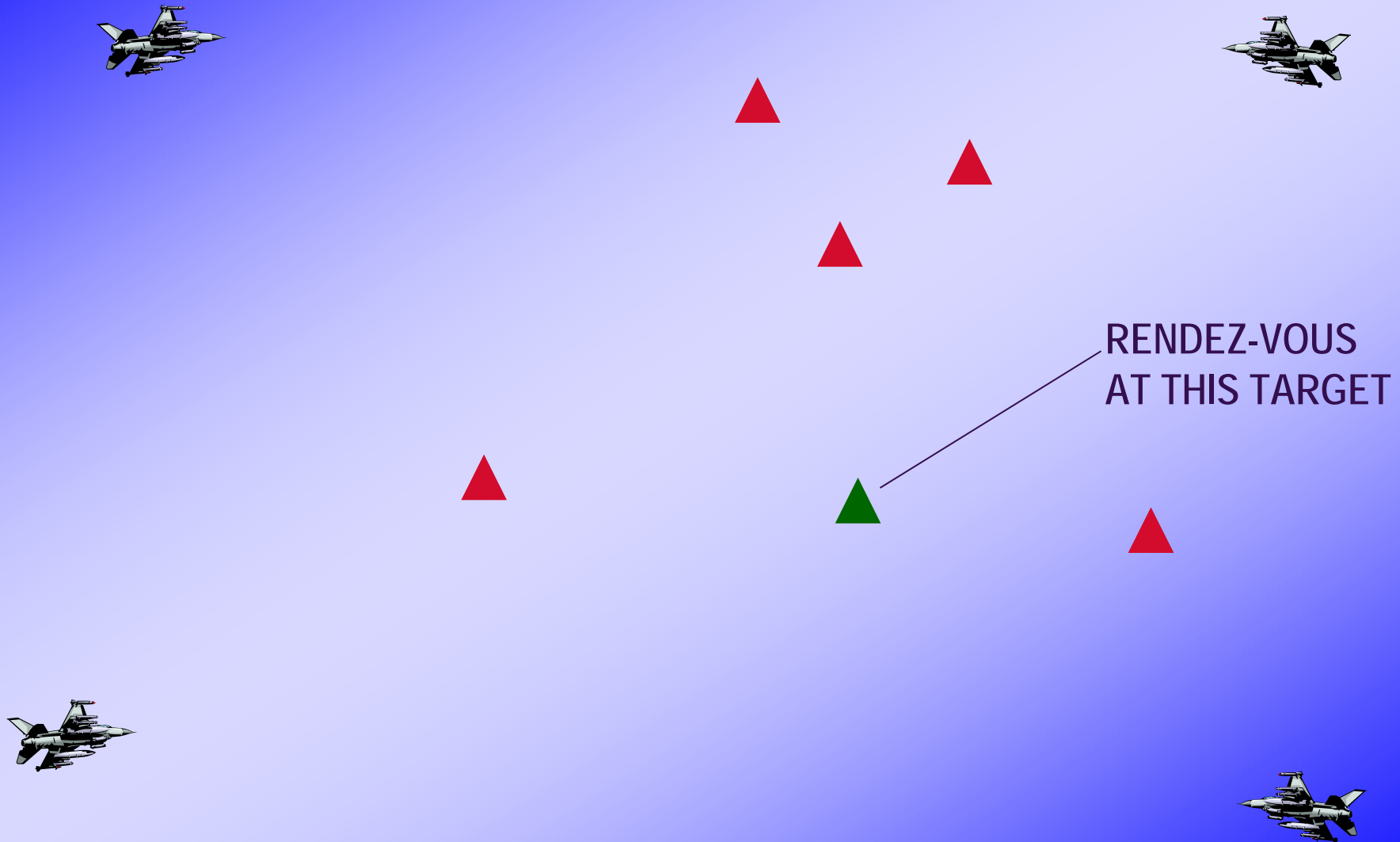
COOPERATIVE MISSION SETTING



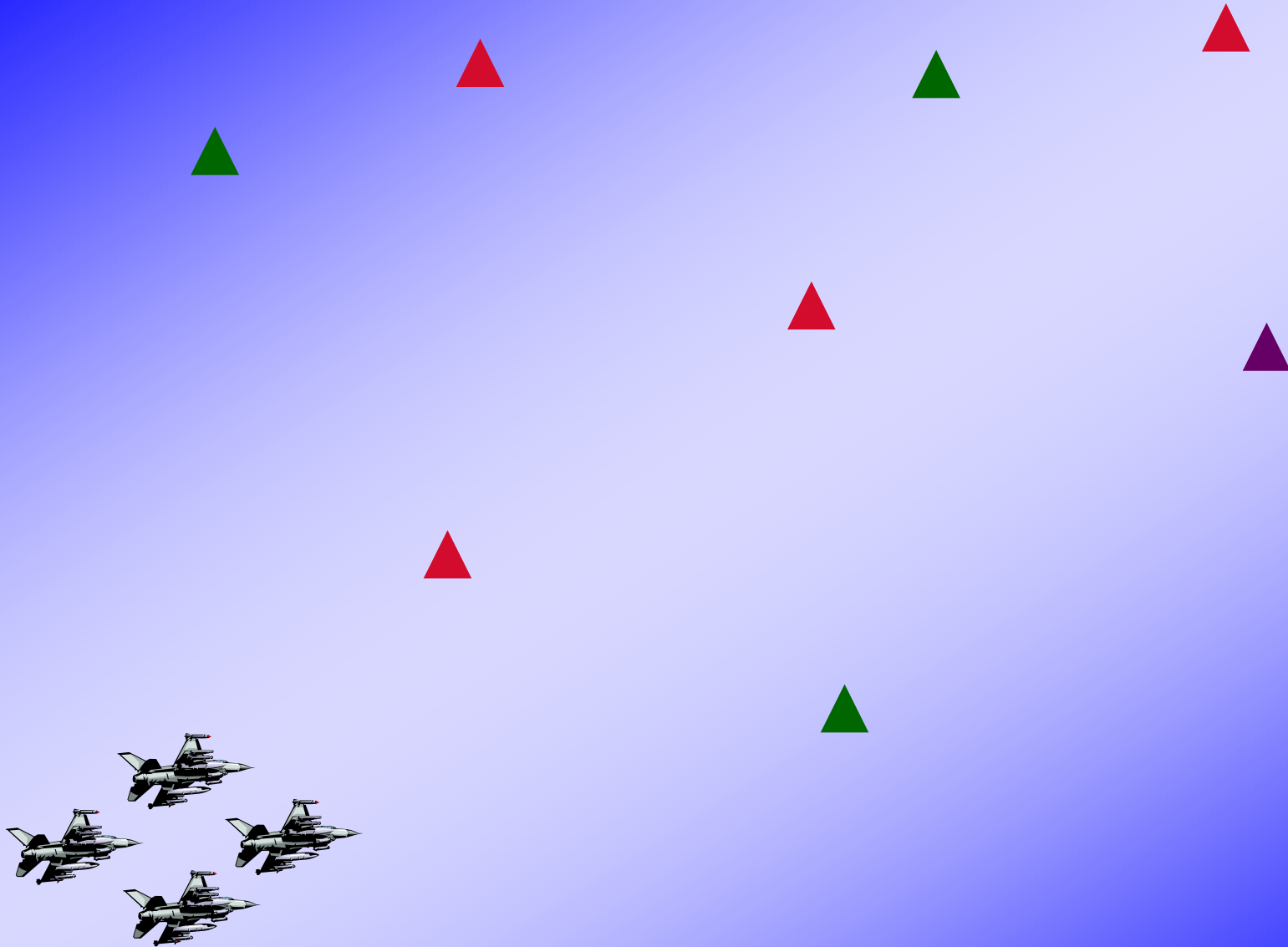
DIFFERENT COOPERATIVE MISSION TYPES

- RENDEZ-VOUS AT SOME TARGET POINT
- FORMATION MAINTENANCE
- REWARD MAXIMIZATION
- COVERAGE CONTROL

RENDEZ-VOUS MISSION



FORMATION MAINTAINING MISSION



REWARD MAXIMIZATION MISSION

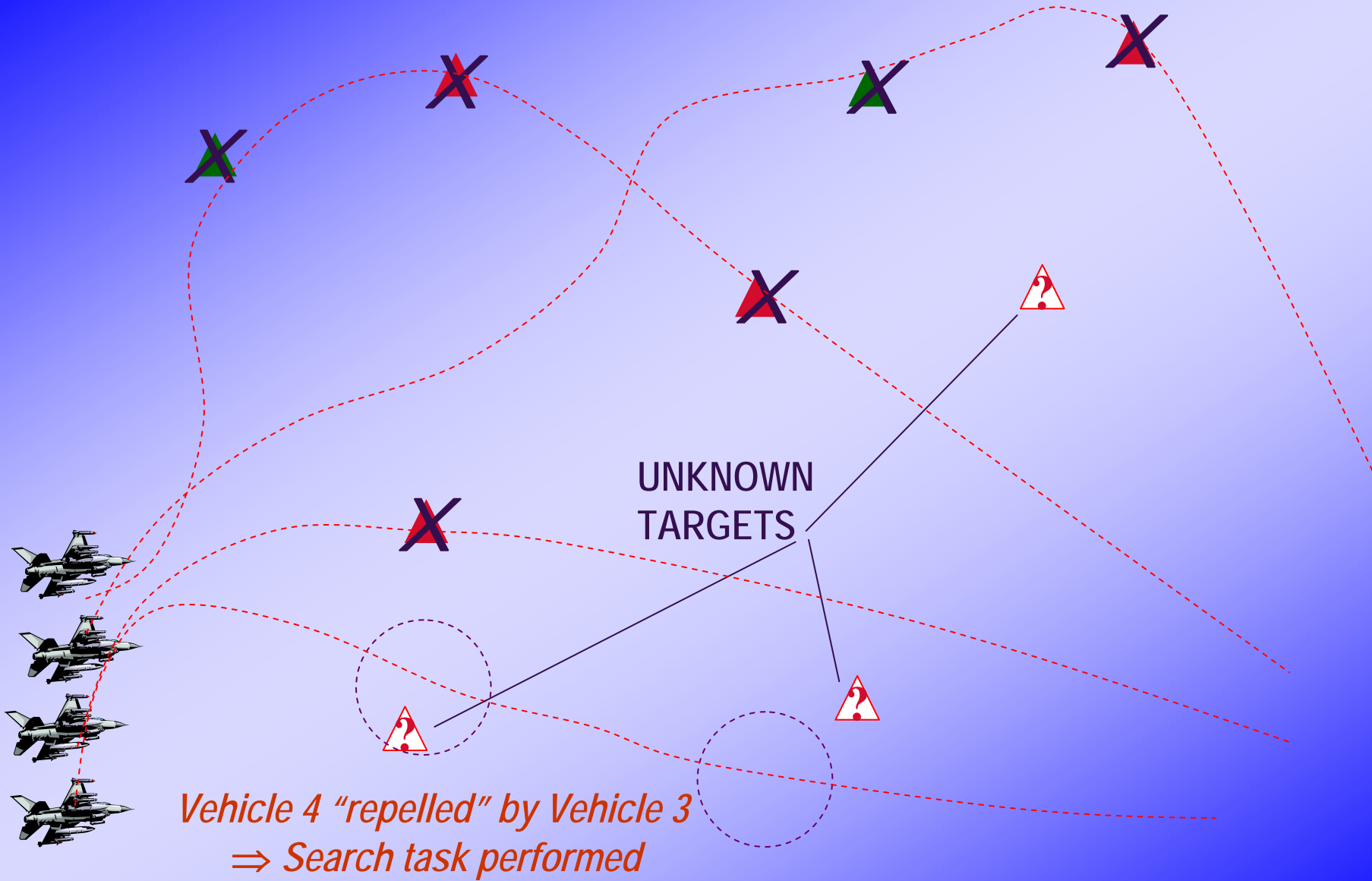


MISSION OBJECTIVE: *MAXIMIZE TOTAL REWARD COLLECTED BY VISITING TARGETS BEFORE THEIR "DEADLINES" EXPIRE*



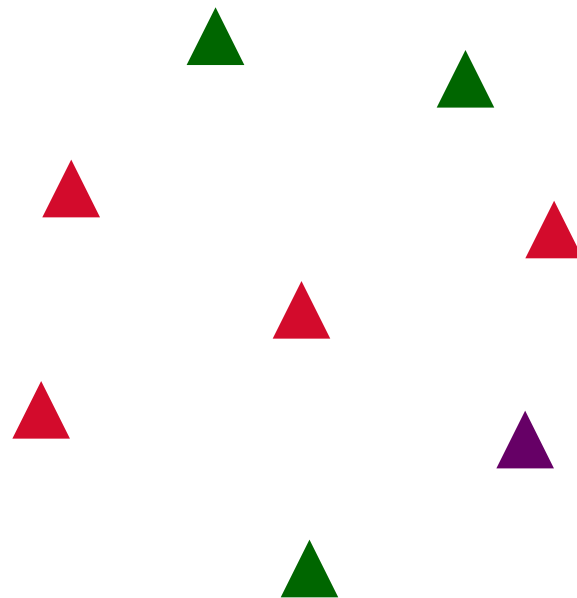
REWARD MAXIMIZATION MISSION

CONTINUED

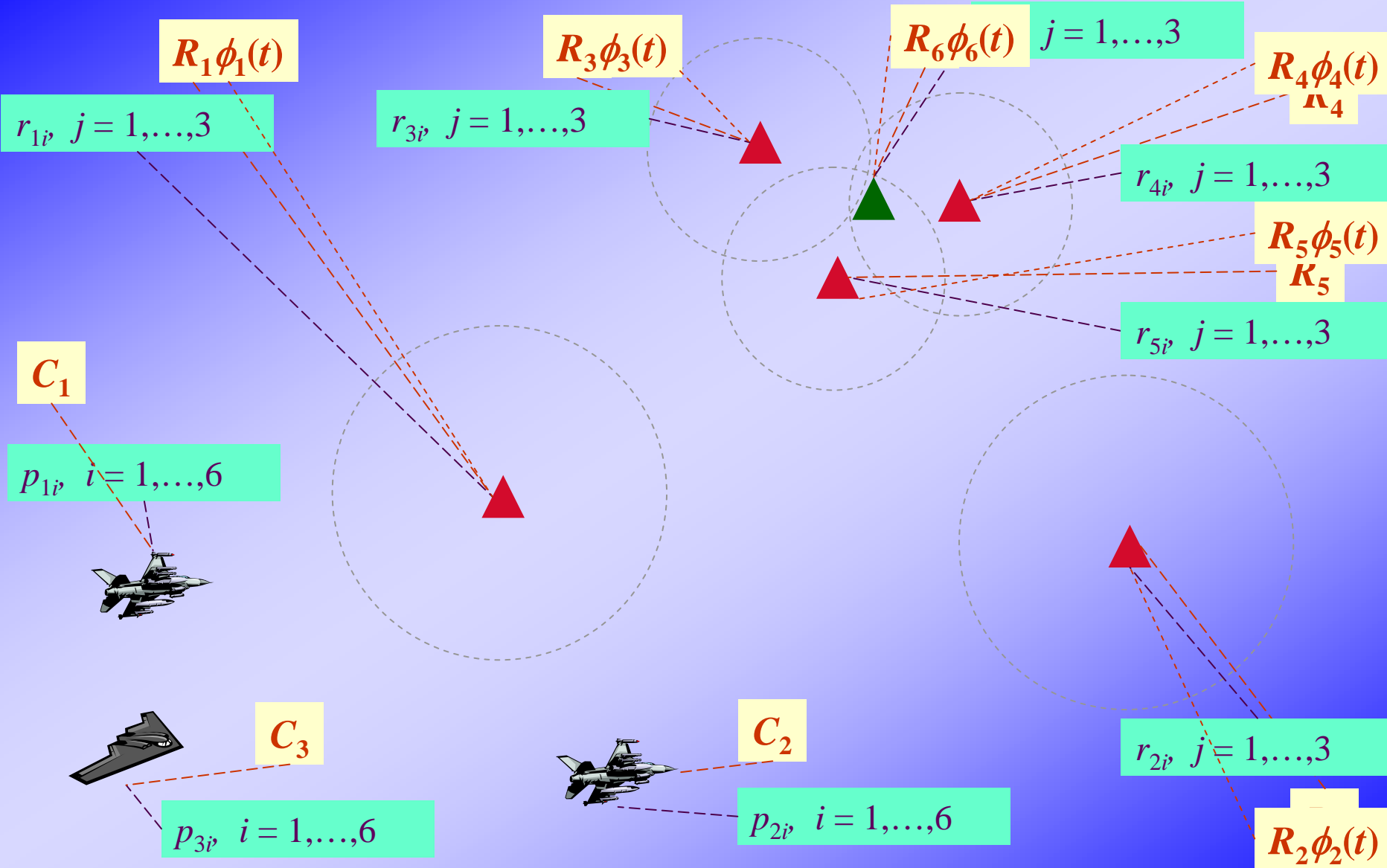


*Vehicle 4 "repelled" by Vehicle 3
⇒ Search task performed*

COVERAGE CONTROL MISSION



COOPERATIVE *REWARD MAXIMIZATION* MISSION



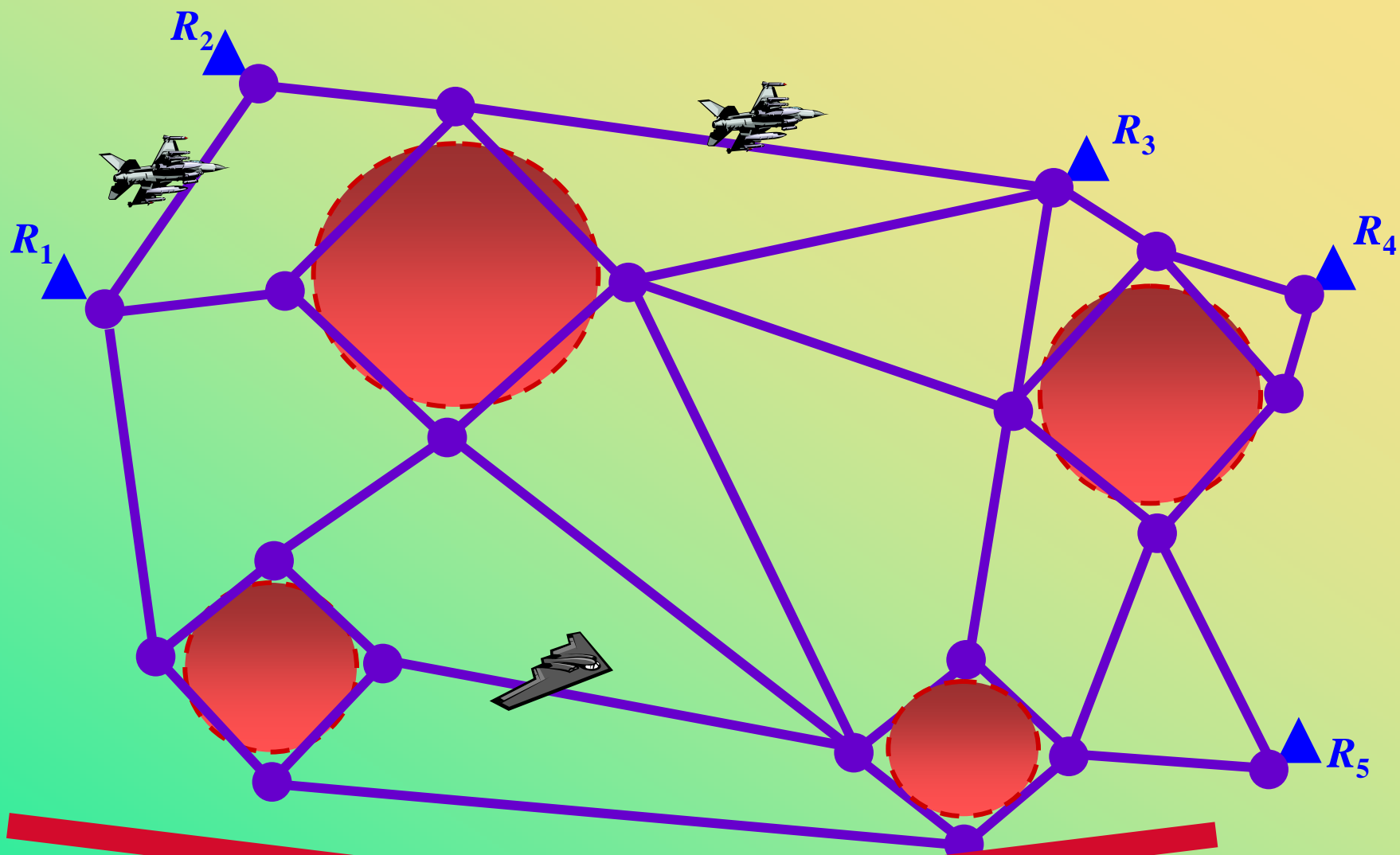
This is like the notorious TRAVELING SALESMAN problem, except that...

- ... there are multiple (cooperating) salesmen
- ... there are deadlines + time-varying costs
- ... environment is stochastic
(vehicles may fail, threats damage vehicles, etc.)

SOLUTION APPROACHES

- Stochastic Dynamic Programming – *Wohletz et al, 2001*
Extremely complex...
- Functional Decomposition
 - Dynamic Resource Allocation – *Castanon and Wohletz, 2002*
 - Assignment Problems through Mixed Integer Linear Programming – *Bellingham et al, 2002*
Combinatorially complex...
 - Path Planning – *Hu and Sastry, 2001, Lian and Murray 2002, Gazi and Passino, 2002, Bachmayer and Leonard, 2002*

COMBINATORIAL + STOCHASTIC COMPLEXITY



~~1. Target Assignment → 2. Routing → 3. Path Control~~

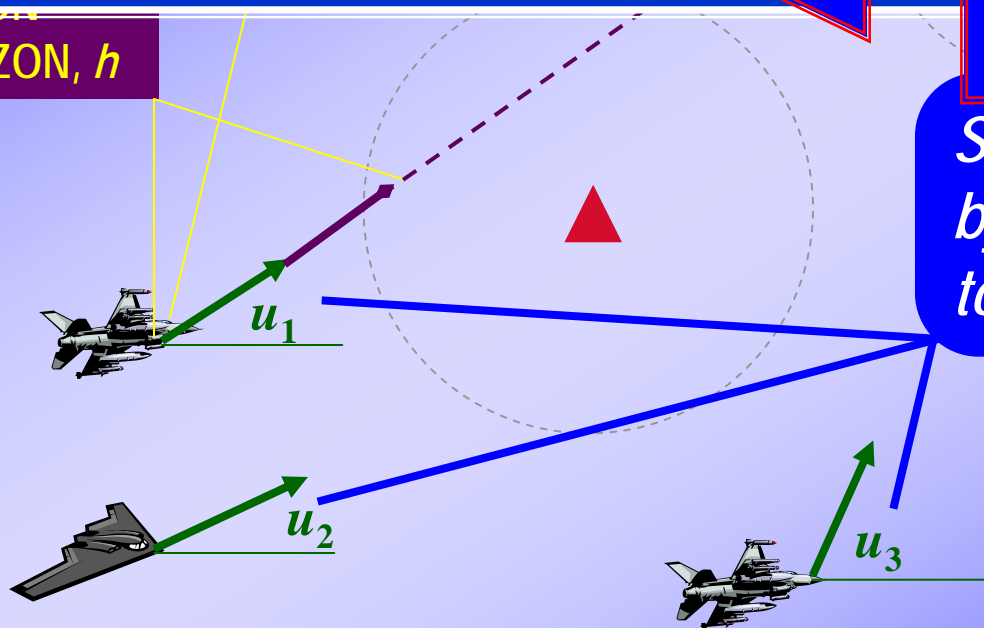
RECEDING HORIZON (RH) CONTROL: *MAIN IDEA*

- Do not attempt to assign vehicles to targets
- Cooperatively steer vehicles towards “high expected reward” regions
- Repeat process periodically/on-event
- Worry about final vehicle-target assignment at the last possible instant

Turns out vehicles converge to targets on their own!

*Solve optimization problem by selecting all u_i to maximize total **expected** rewards over H*

HORIZON, h



See also Franco, Parisini, Polycarpou 04; Dunbar, Murray, 04; Richards, How, 04

CRH CONTROL PROBLEM FORMULATION

- Target positions ($i = 1, \dots, N$): $y_i \in \mathbb{R}^2$
- Vehicle dynamics ($j = 1, \dots, M$):
 - State: $x_j(t) \in \mathbb{R}^2$ position of j th vehicle at time t
 - Control: $u_j(t)$ Vehicle heading at time t

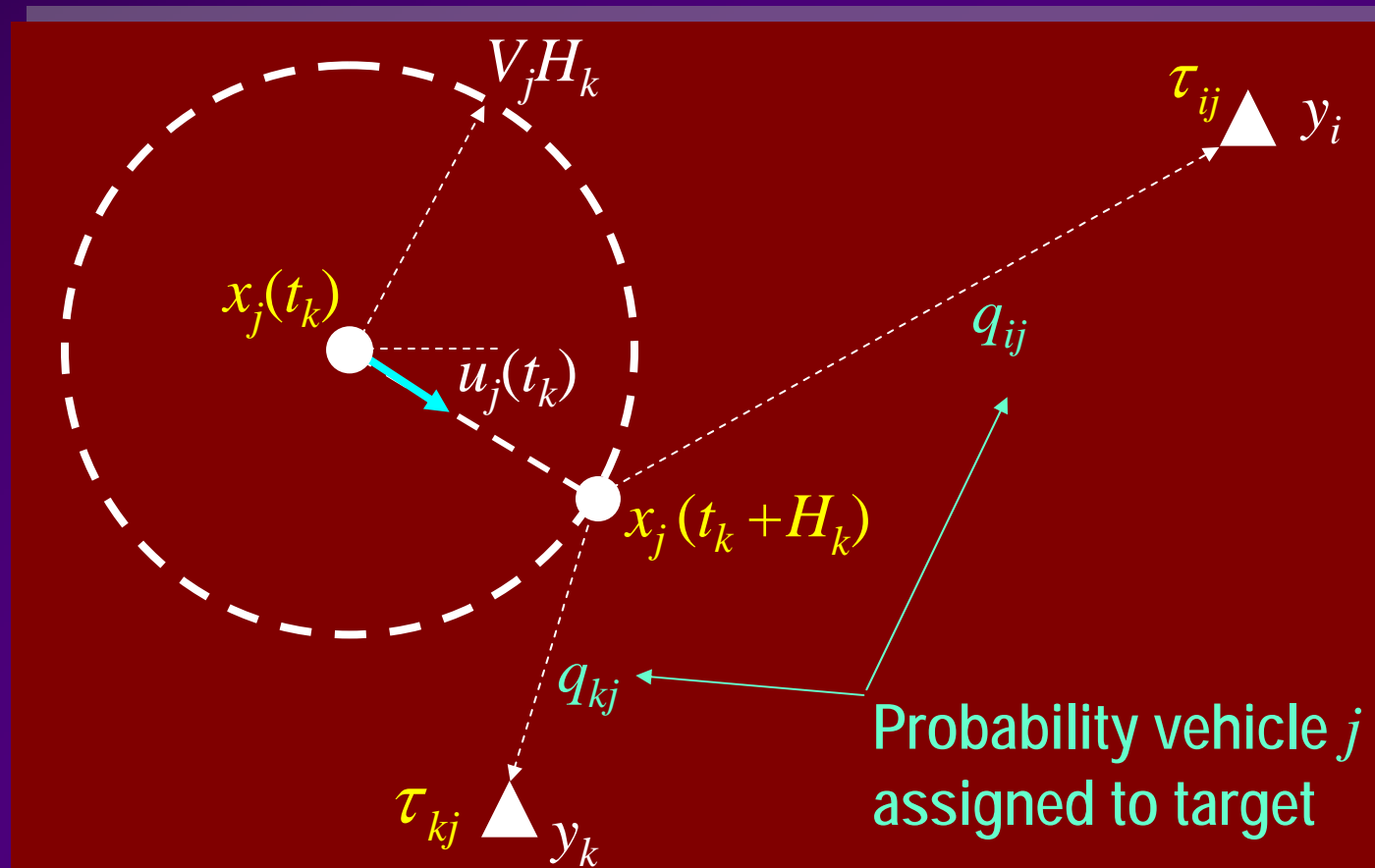
$$\dot{x}_j(t) = V_j \begin{bmatrix} \cos u_j(t) \\ \sin u_j(t) \end{bmatrix}, \quad x_j(0) = x_j^0$$

- At k th iteration, time t_k ($k=1,2,\dots$):
 - Planning Horizon: H_k
 - Vehicle position at time $t_k + H_k$: $x_j(t_k + H_k) = x_j(t_k) + \dot{x}_j(t_k)H_k$

- At k th iteration ($k=1,2,\dots$):

Earliest time vehicle j can reach target i under control $u_j(t_k)$:

$$\tau_{ij}(u_j(t_k), t_k) = (t_k + H_k) + \|x_j(t_k + H_k) - y_i\| / V_j$$



- Objective at k th iteration:

Maximize **EXPECTED REWARD** over horizon H_k

$$\max_{\mathbf{u}} \sum_{i=1}^M \sum_{j=1}^N R_i \phi_i(\tau_{ij}) p_{ij}(\tau_{ij}) q_{ij}(t_k + H_k) - \sum_{i=1}^M \sum_{j=1}^N C_j r_{ij}(t_k + H_k)$$

Target i value attainable by veh. j
[depends on $u_j(t)$]

veh. j value

Control vehicle headings

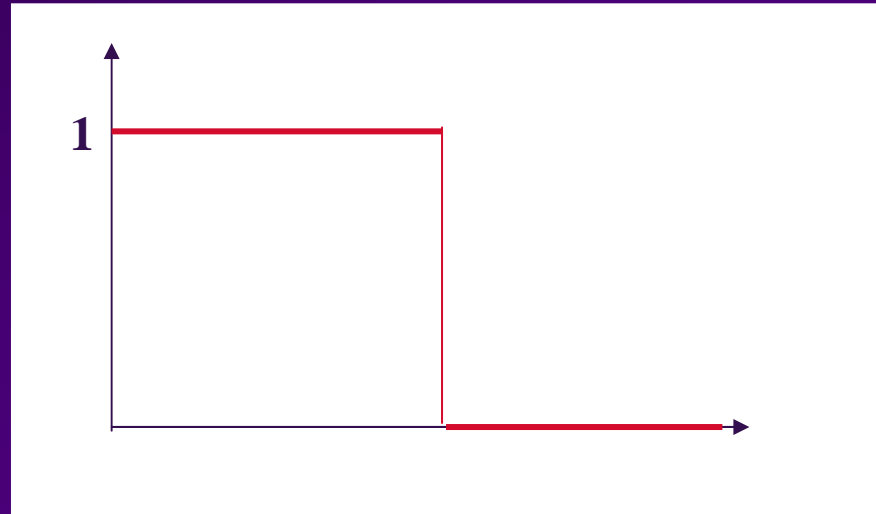
Prob. vehicle j collects target i reward
Earliest time when vehicle j can collect reward from target i
[depends on $u_j(t)$]

Prob. vehicle j assigned to target i
[depends on $u_j(t)$]

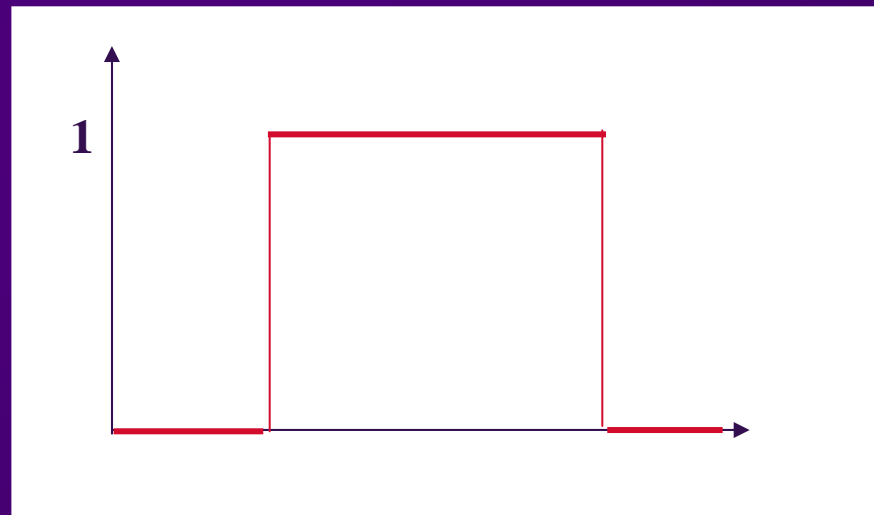
Prob. vehicle j destroyed by target i
[depends on $u_j(t)$]

THE FUNCTION $\phi_i(t)$ [REWARD DISCOUNTING FUNCTION]

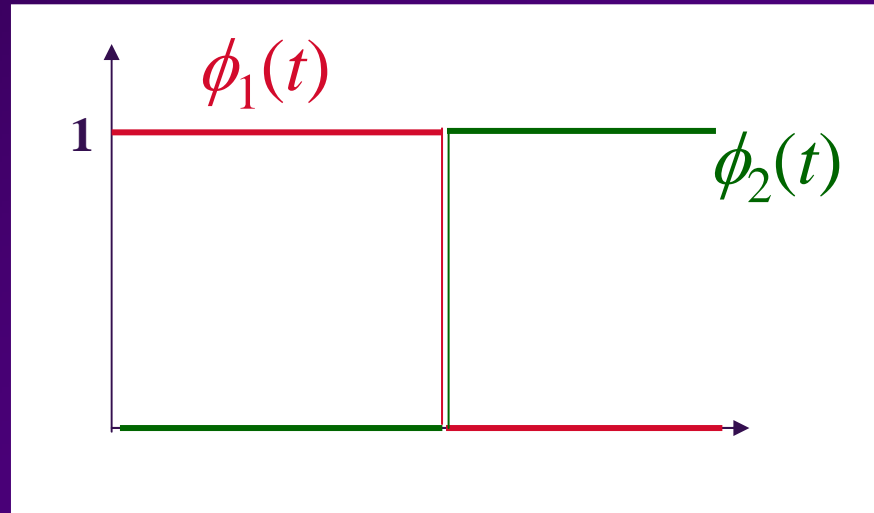
- Targets with deadlines:



- Targets with time windows:

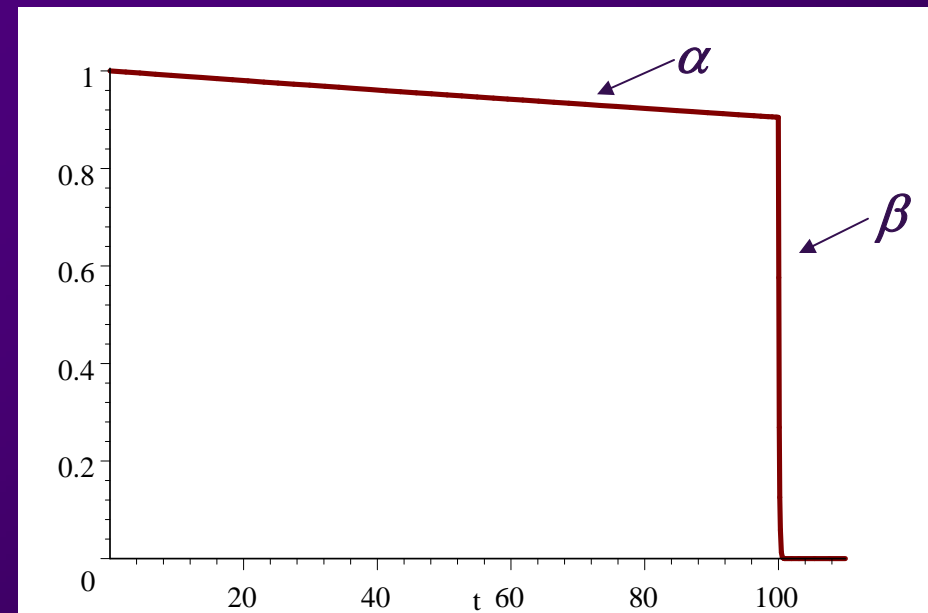


- Sequencing targets:



- A general purpose ϕ -function:

$$\phi_i(t) = \begin{cases} e^{\frac{\ln(1-\alpha)}{D_i}t} & \text{if } t \leq D_i \\ e^{\frac{\ln(1-\alpha)}{D_i}t} e^{-\beta(t-D_i)} & \text{if } t > D_i \end{cases}$$



THE FUNCTION q_{ij} [TARGET ASSIGNMENT FUNCTION]

- Vehicle-to-target distance: $d_{ij} = \|x_j - y_i\|$

- Relative distance:

$$\delta_{ij} = \frac{d_{ij}}{\sum_{m=1}^M d_{im}}$$

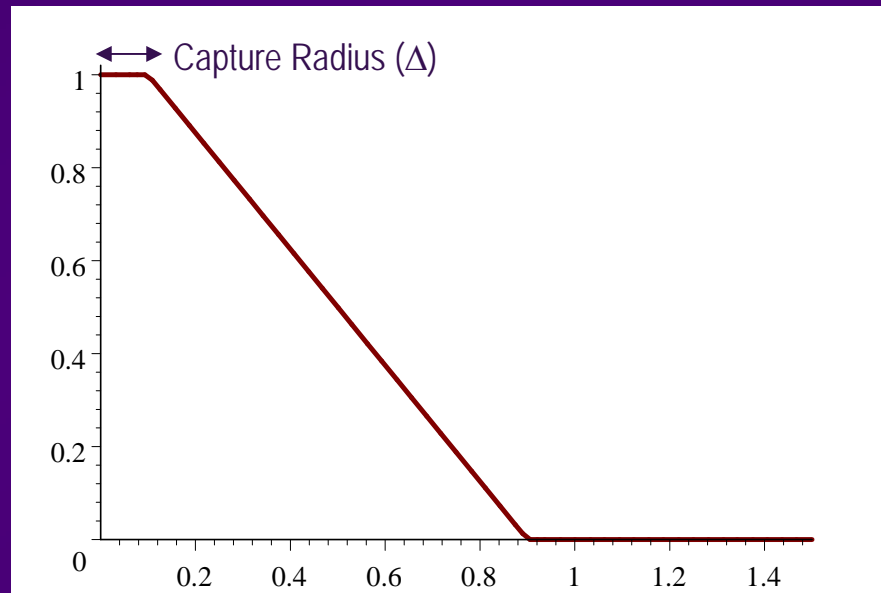
- Target assignment function $q_{ij}(\delta_{ij})$:

Monotonically non-increasing and s.t.

$$q_{ij}(0) = 1, \quad q_{ij}(1) = 0$$

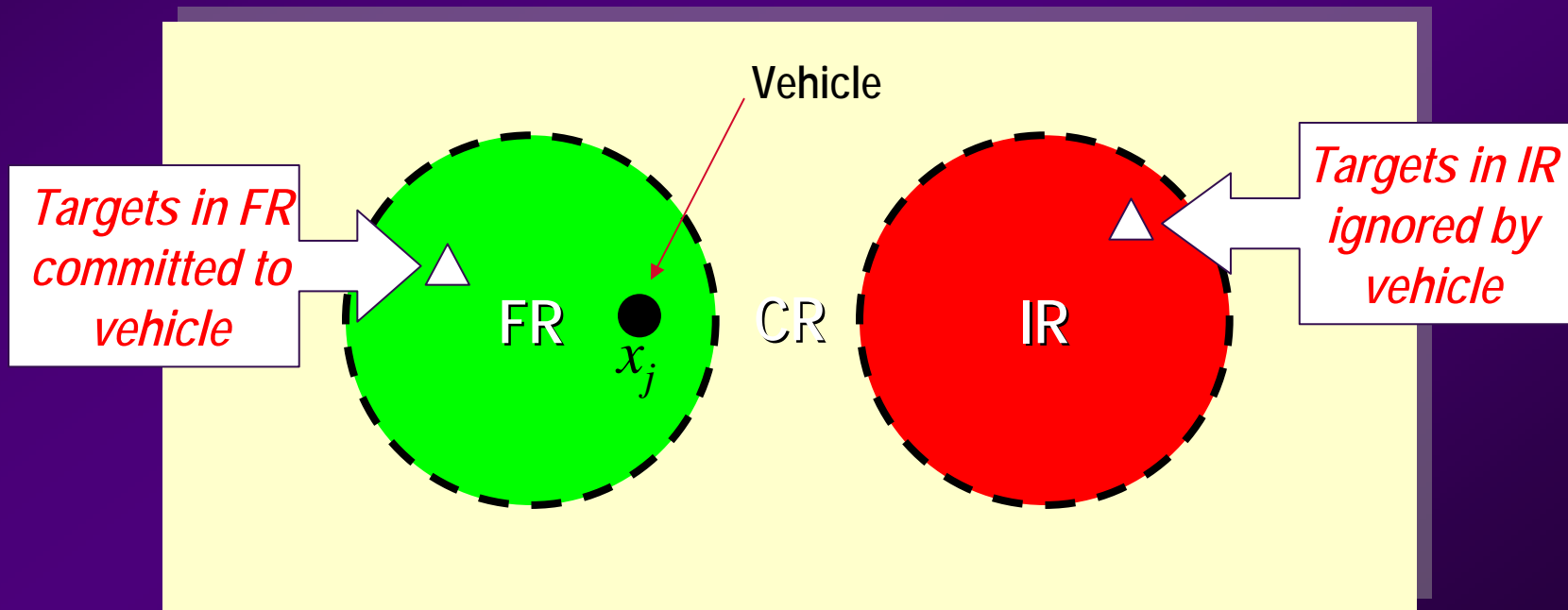
- A example of q_{ij} function ($M=2$):

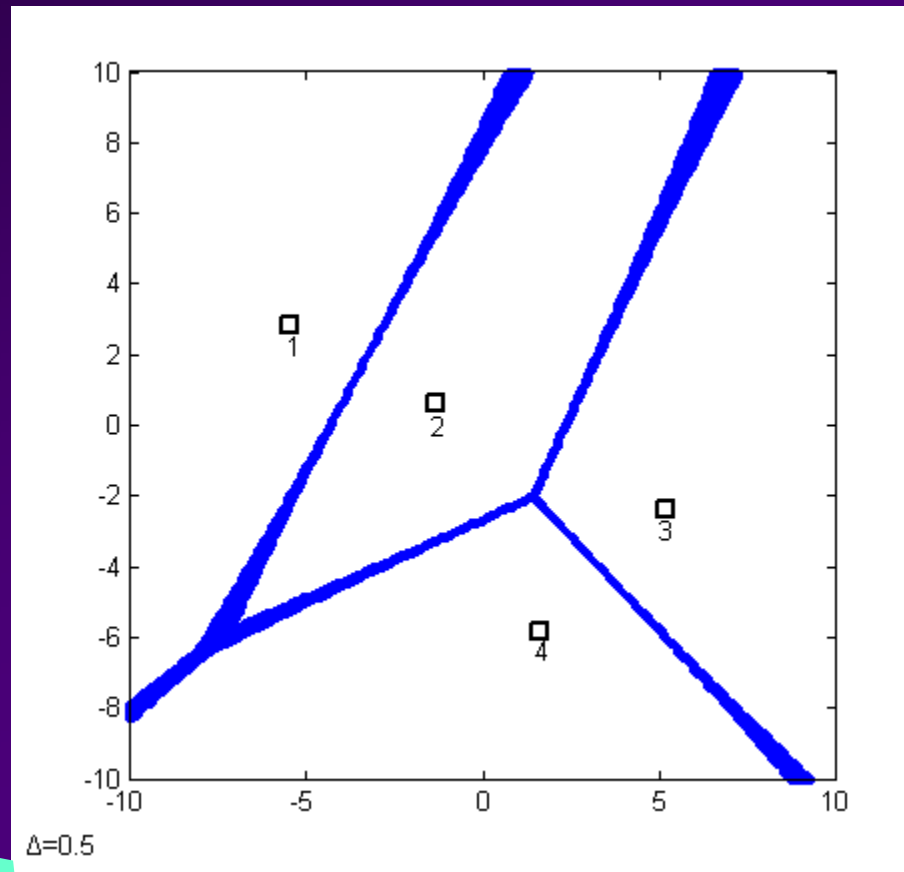
$$q_{ij}(\delta_{ij}) = \begin{cases} 1 & \text{if } \delta_{ij} \leq \Delta \\ \frac{1}{1-2\Delta} [(1-\Delta) - \delta_{ij}] & \text{if } \Delta < \delta_{ij} \leq 1-\Delta \\ 0 & \text{otherwise} \end{cases}$$



$q_{ij}(t)$ defines *DYNAMIC RESPONSIBILITY REGIONS* for vehicle j

- S_j – Full Responsibility Region (FR) $\delta_{ij} \leq \Delta$
- C_j – Cooperative Region (CR) $\Delta < \delta_{ij} \leq 1 - \Delta$
- I_j – Invisibility Region (IR) $\delta_{ij} > 1 - \Delta$





Voronoi partition

What happens as parameter Δ increases?

Partition of a plane with into n convex polygons such that each polygon contains *exactly one* point and every point in a given polygon is closer to its central point than to any other.

2-VEHICLE CASE – DYNAMIC PARTITIONING



II: Only vehicle 1 goes to target

III: Both vehicles go to target

IV: Only vehicle 2 goes to target (1 is repelled !)

PLANNING AND ACTION HORIZONS

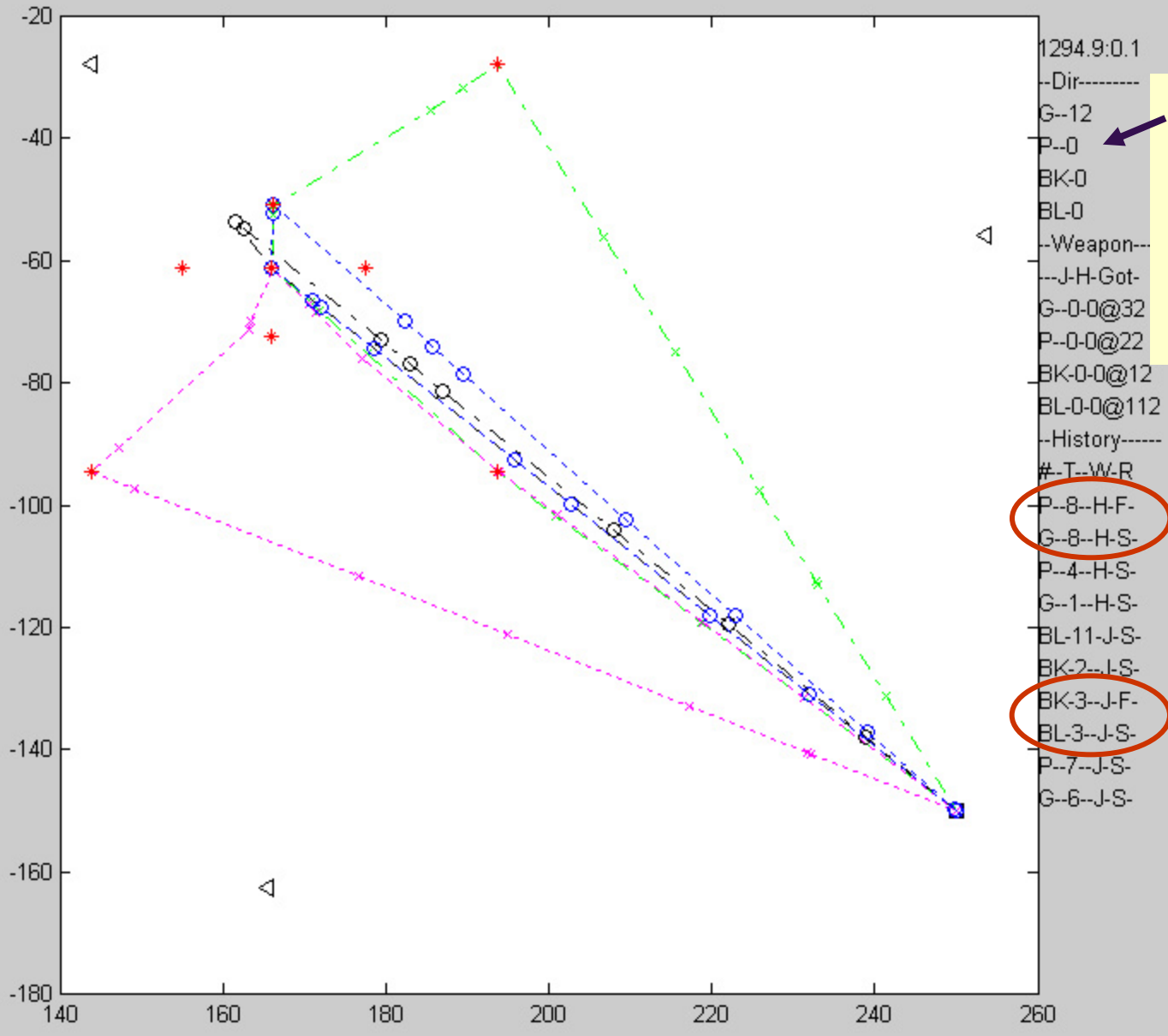
PLANNING Horizon $H(t)$:

$$H(t) = d_{\min}(t) \equiv \min_{i,j} d_{ij}(t)$$

ACTION Horizon $h(t)$:

$$h(t) = \alpha_H + \beta_H H(t), \quad \alpha_H \geq 0, \quad 0 \leq \beta_H \leq 1$$

OR: Whenever next EVENT occurs



FOUR veh's:
 Green
 Purple
 Black
 Blue

TARGET ASSIGNMENT

MAIN IDEA IN **CRH** APPROACH:

Replace complex *Discrete Stochastic Optimization* problem by a sequence of simpler *Continuous Optimization* problems

But how do we guarantee that vehicles ultimately head for the desired DISCRETE TARGET POINTS?

STABILITY ANALYSIS

• TARGETS: y_i

• UAVs: x_j

DEFINITION: Vehicle trajectory $\mathbf{x}(t) = [x_1(t), \dots, x_M(t)]$ generated by a controller is *stationary*, if there exists some $t_v < \infty$, such that $\|x_j(t_v) - y_i\| \leq s_i$ for some $i = 1, \dots, N, j = 1, \dots, M$.

Target Size

QUESTION:

Under what conditions is a CRH-generated trajectory stationary?

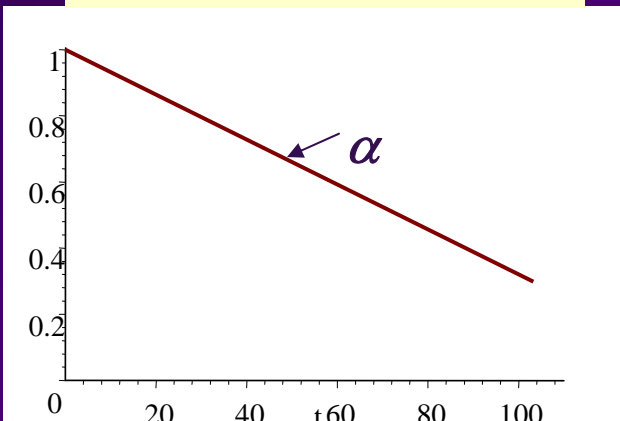
Recall objective function:

$$\max_{\mathbf{u}} \sum_{i=1}^M \sum_{j=1}^N R_i \phi_i(\tau_{ij}) p_{ij}(\tau_{ij}) q_{ij}(t_k + H_k) - \sum_{i=1}^M \sum_{j=1}^N C_j r_{ij}(t_k + H_k)$$

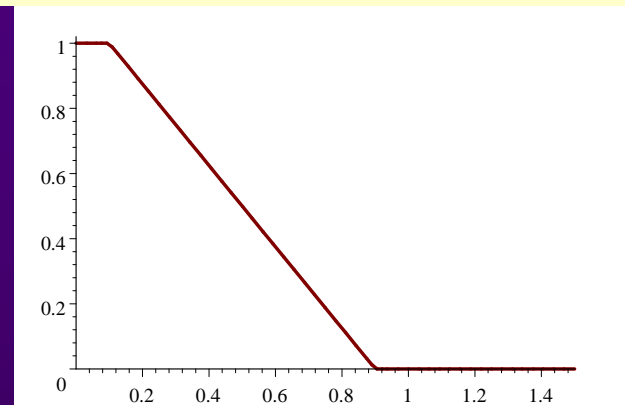
$$\phi_i(t) = 1 - \frac{\alpha}{T} t \quad t \in [0, T]$$

$$p_{ij} = 1$$

$$r_{ij} = 0$$



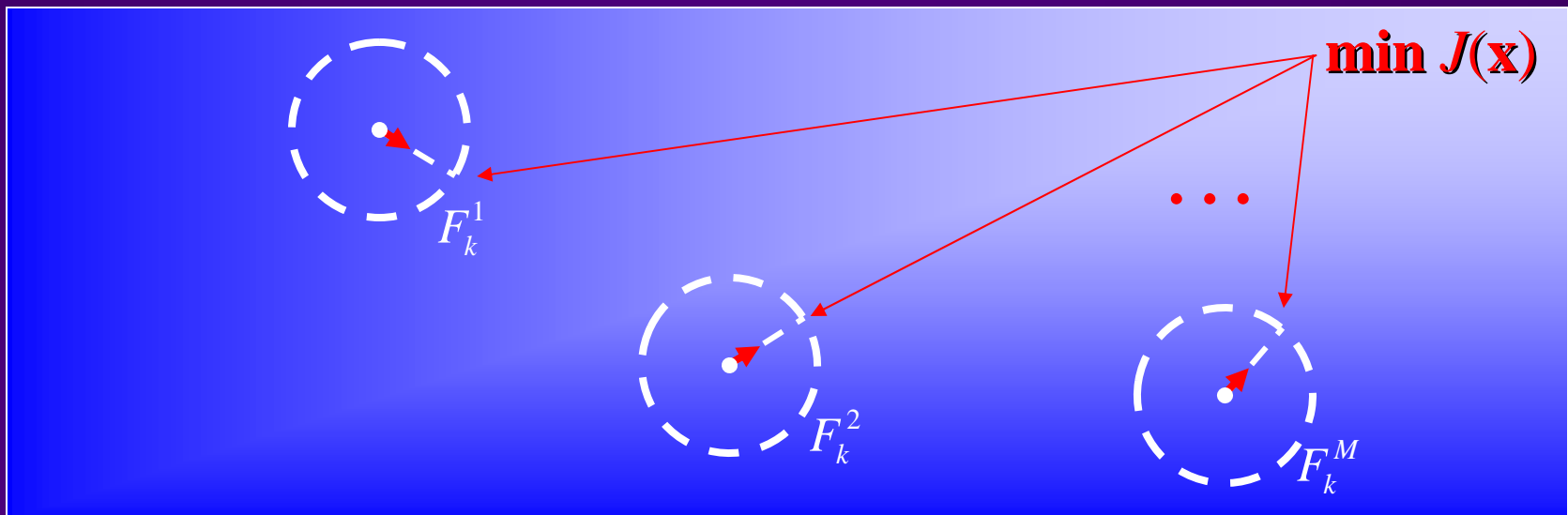
$$q_{ij}(\delta_{ij}) = \begin{cases} 1 & \text{if } j \in B_i, \delta_{ij} \leq \Delta \\ \frac{1}{1-2\Delta} [(1-\Delta) - \delta_{ij}] & \text{if } j \in B_i, \Delta < \delta_{ij} \leq 1-\Delta \\ 0 & \text{otherwise} \end{cases}$$



Objective function reduces to:
$$J(\mathbf{x}) = \sum_{i=1}^N \sum_{j=1}^M R_i \|x_j - y_i\| q_{ij}$$

CRH controller solves optimization problem:
$$\begin{cases} \min_{\mathbf{x} \in F_k} J(\mathbf{x}) \\ F_k = \left\{ \mathbf{w} : \left\| w_j - x_j(t_k) \right\| = V H_k \right\} \end{cases}$$

i.e., minimize the potential function $J(x)$ over a set of M circles:



MAIN STABILITY RESULT

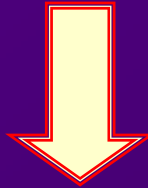
Local minima of $J(x)$: $x^l = (x_1^l, \dots, x_M^l) \in \mathbf{R}^{2M}$, $l = 1, \dots, L$

Vector of vehicle positions

at k th iteration of CRH controller: \mathbf{x}_k

Theorem: Suppose $H_k = \min_{i,j} d_{ij}(t_k)$.

If, for all $l = 1, \dots, L$, $x_j^l = y_i$ for some $i = 1, \dots, N$, $j = 1, \dots, M$,
then $J(\mathbf{x}_k) - J(\mathbf{x}_{k+1}) > b$ ($b > 0$ is a constant).



*If all local minima coincide with targets,
the CRH-generated trajectory is stationary*

MAIN STABILITY RESULT

QUESTION:

When do all local minima coincide with target points?

1 Vehicle, N targets



If there exists a y_i s.t. $R_i - \left\| \sum_{j=1, j \neq i}^N R_j \frac{y_i - y_j}{\|y_i - y_j\|} \right\| > 0$

2 Vehicles, 1 target



2 Vehicles, 2 targets

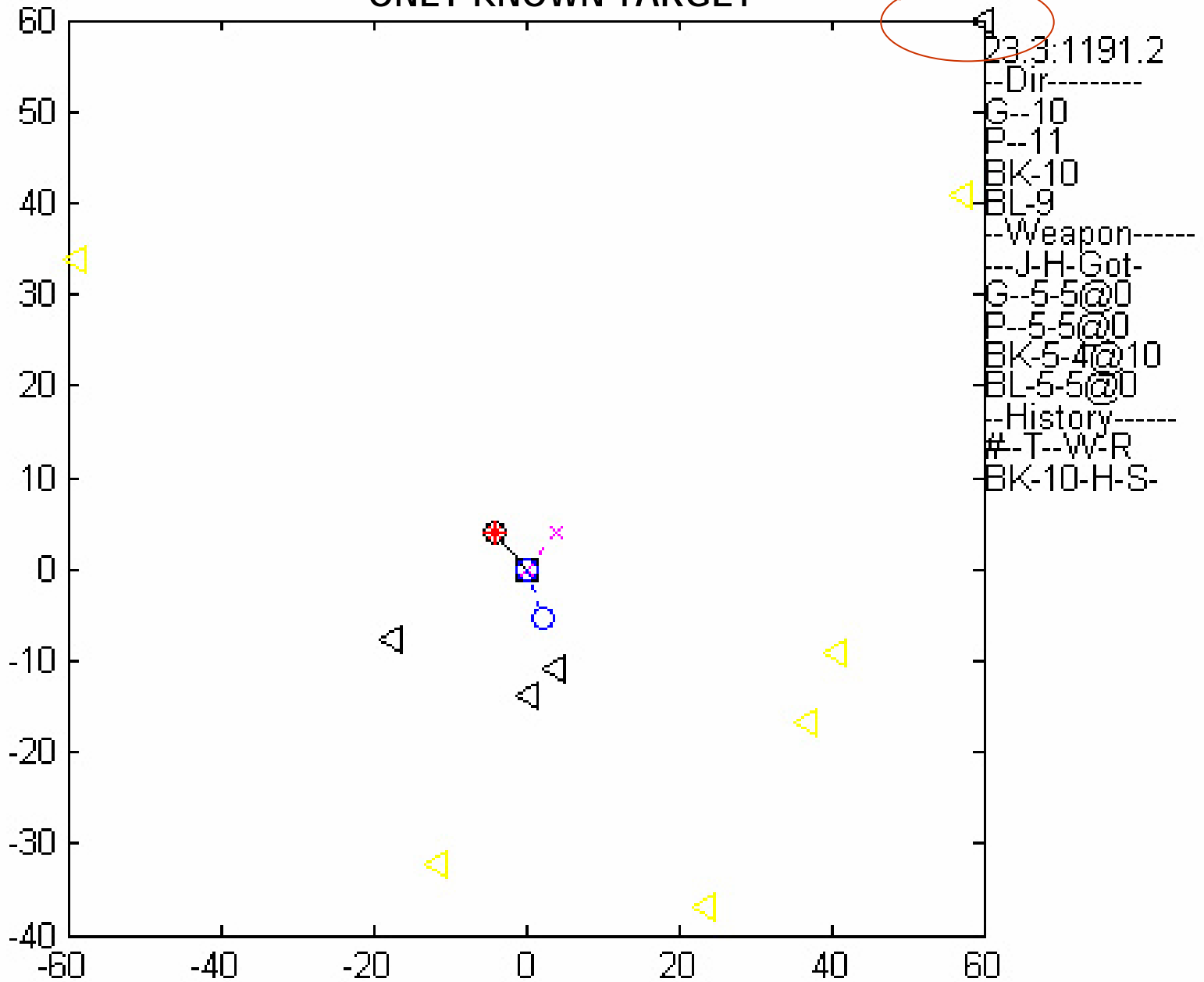


TO RECAP...

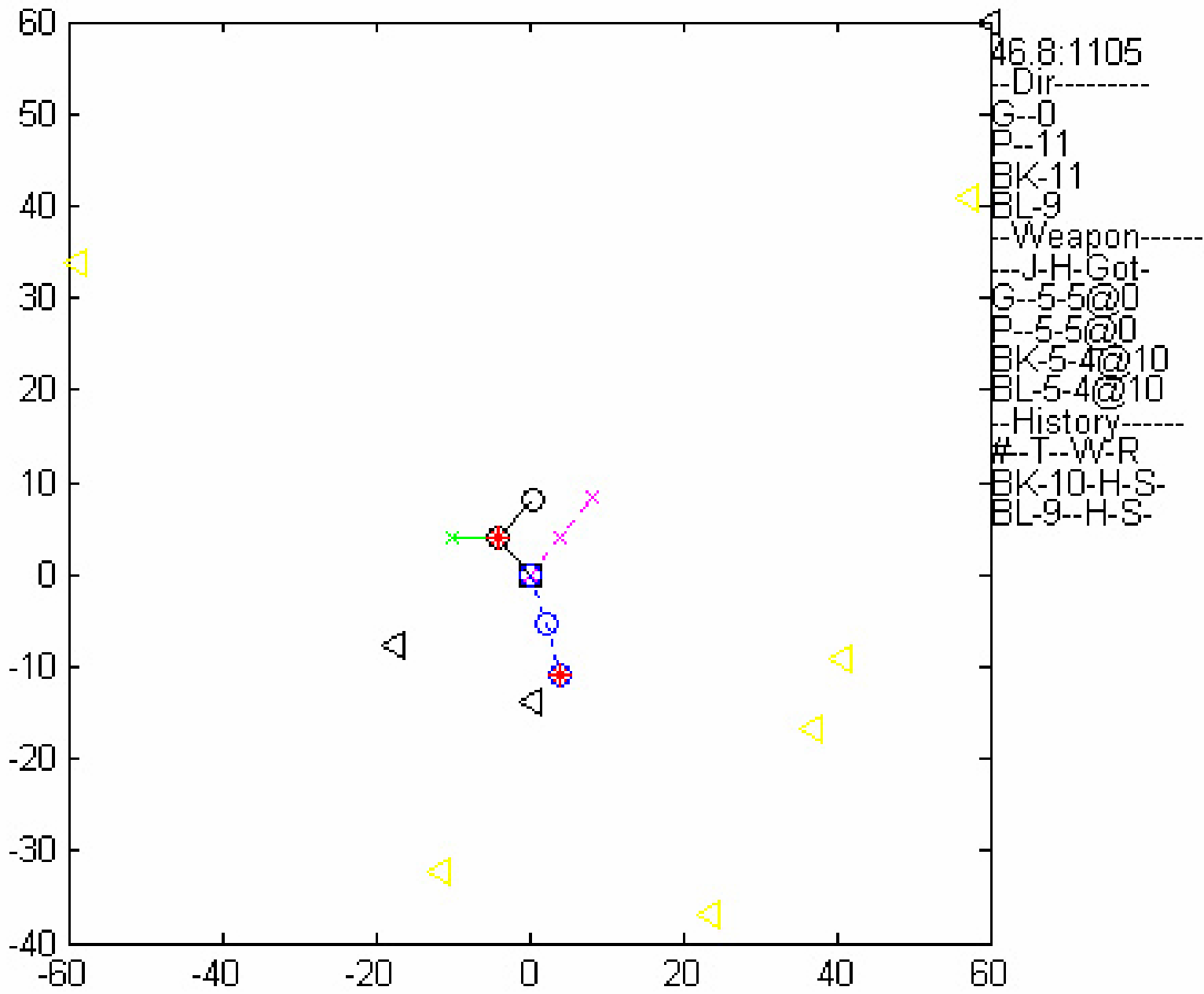
- **Limited look-ahead** – control optimizes expectation over “**planning horizon**”
- **Control updates** – *event-driven* (events are deterministic or random) or *time-driven* (for a given “**action horizon**”)
- **Target assignment** – done implicitly, not explicitly:
No combinatorial problem involved
- **Assignment + Routing + Path Control** – all done together

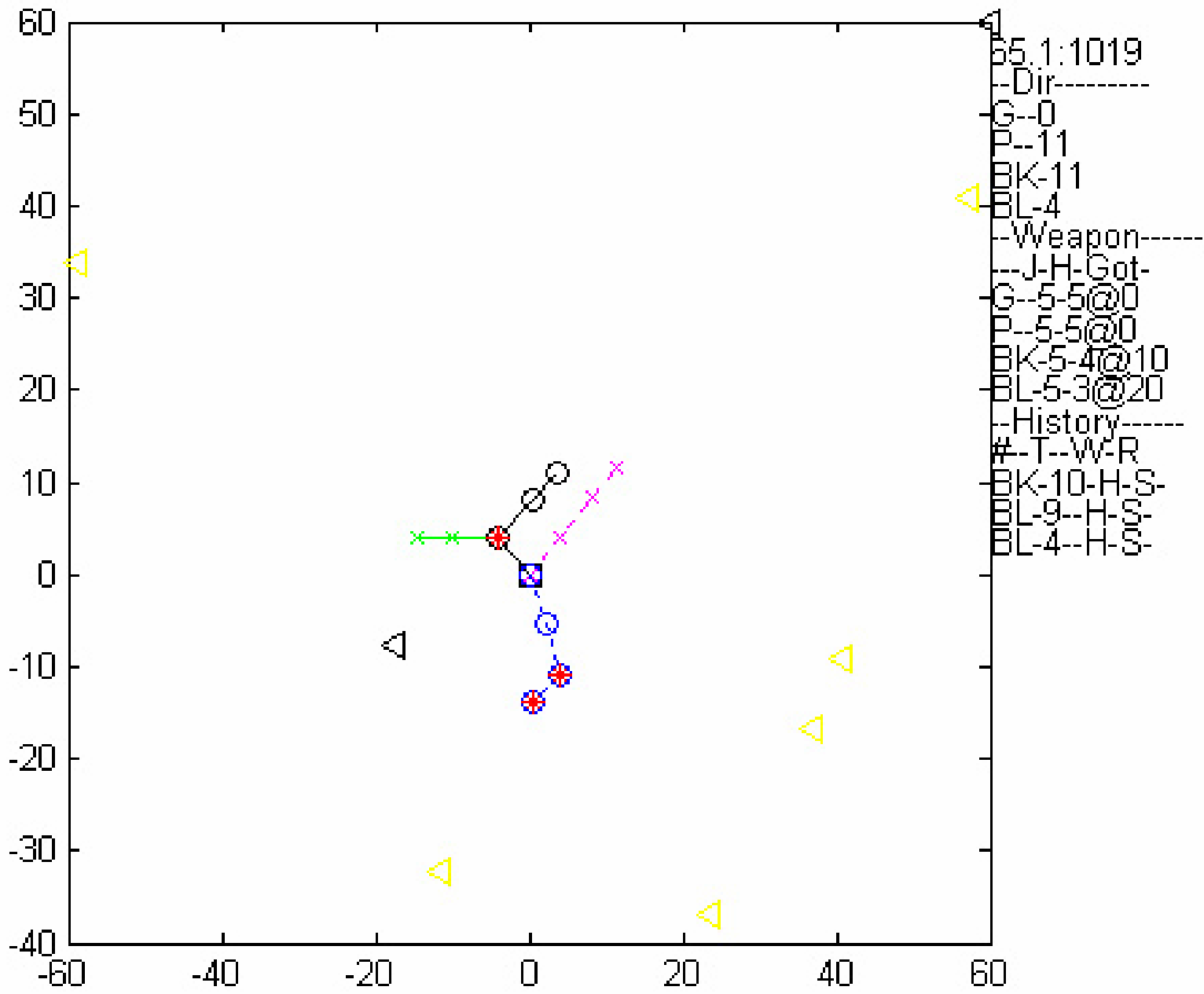
- Target values **change** – deadlines, target sequencing, return to base
- Vehicle capabilities **change** - resource depletion, failures, damage
- Threat capabilities **change** - radar on/off, threat damage
- Target locations **change** - new targets, moving targets
- **Obstacle avoidance** - targets with negative values
- **Randomness** - new control actions in response to random events
- **Constraints** – heading change, heading-dependent costs, sensing tasks

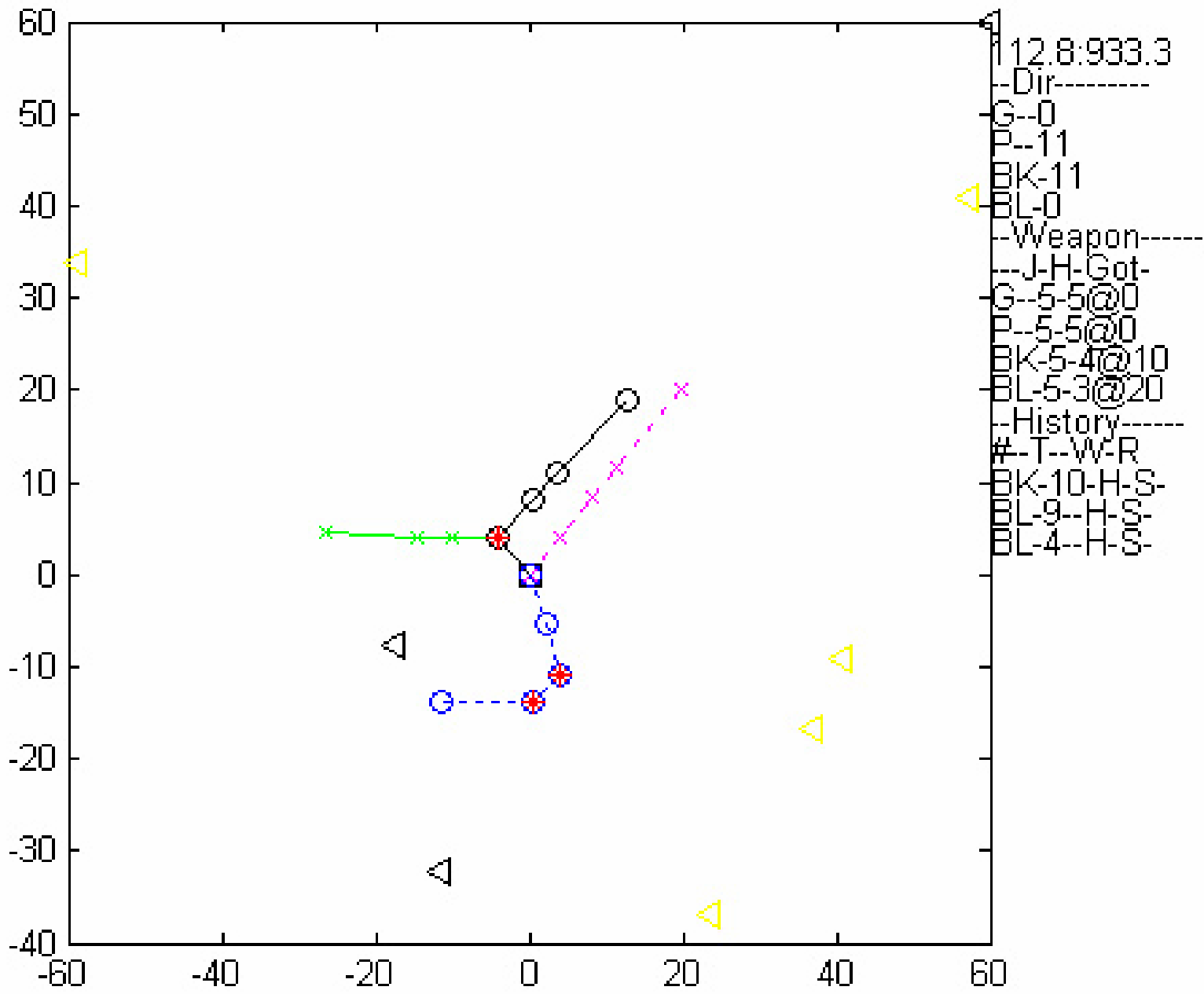
ONLY KNOWN TARGET

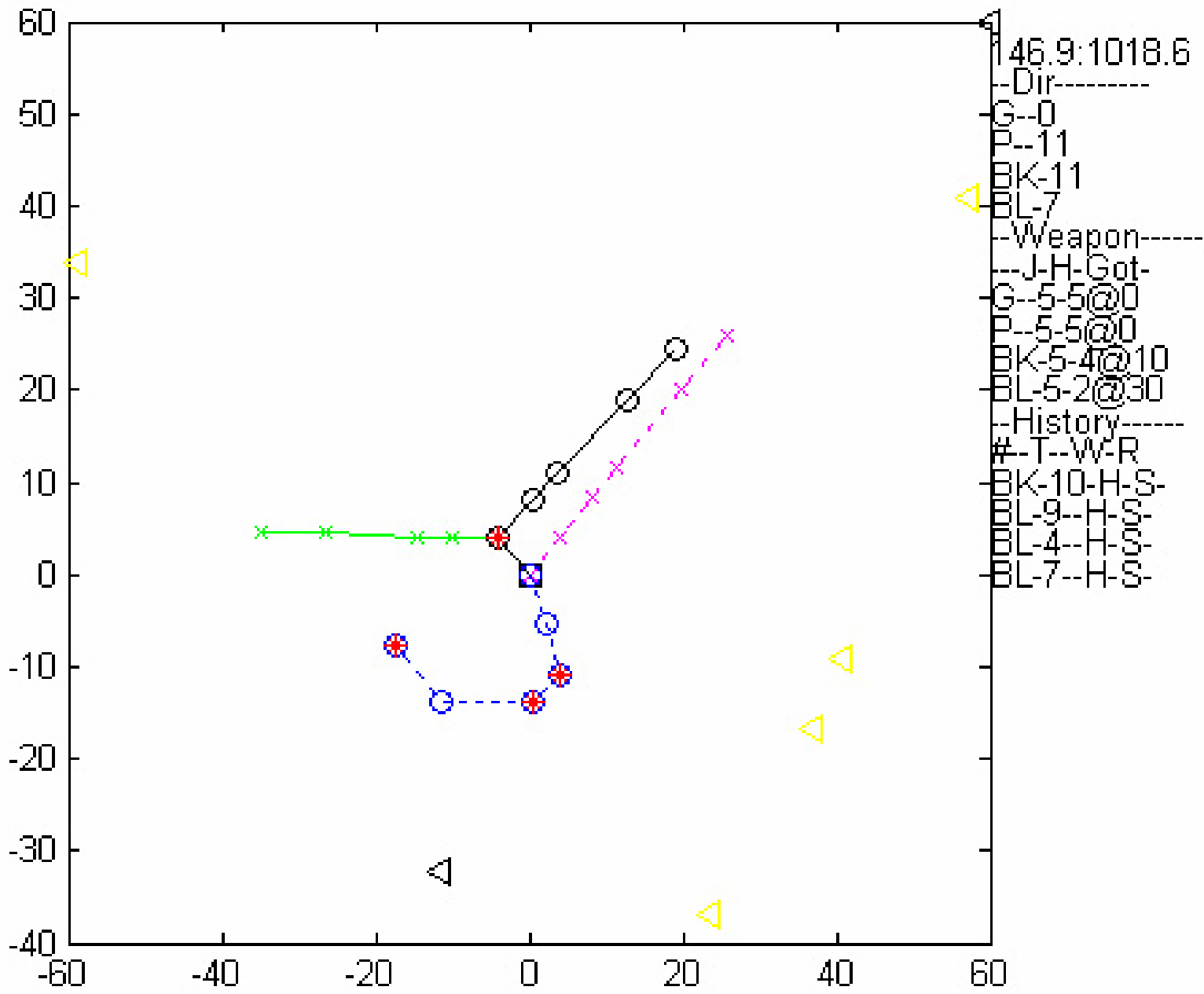


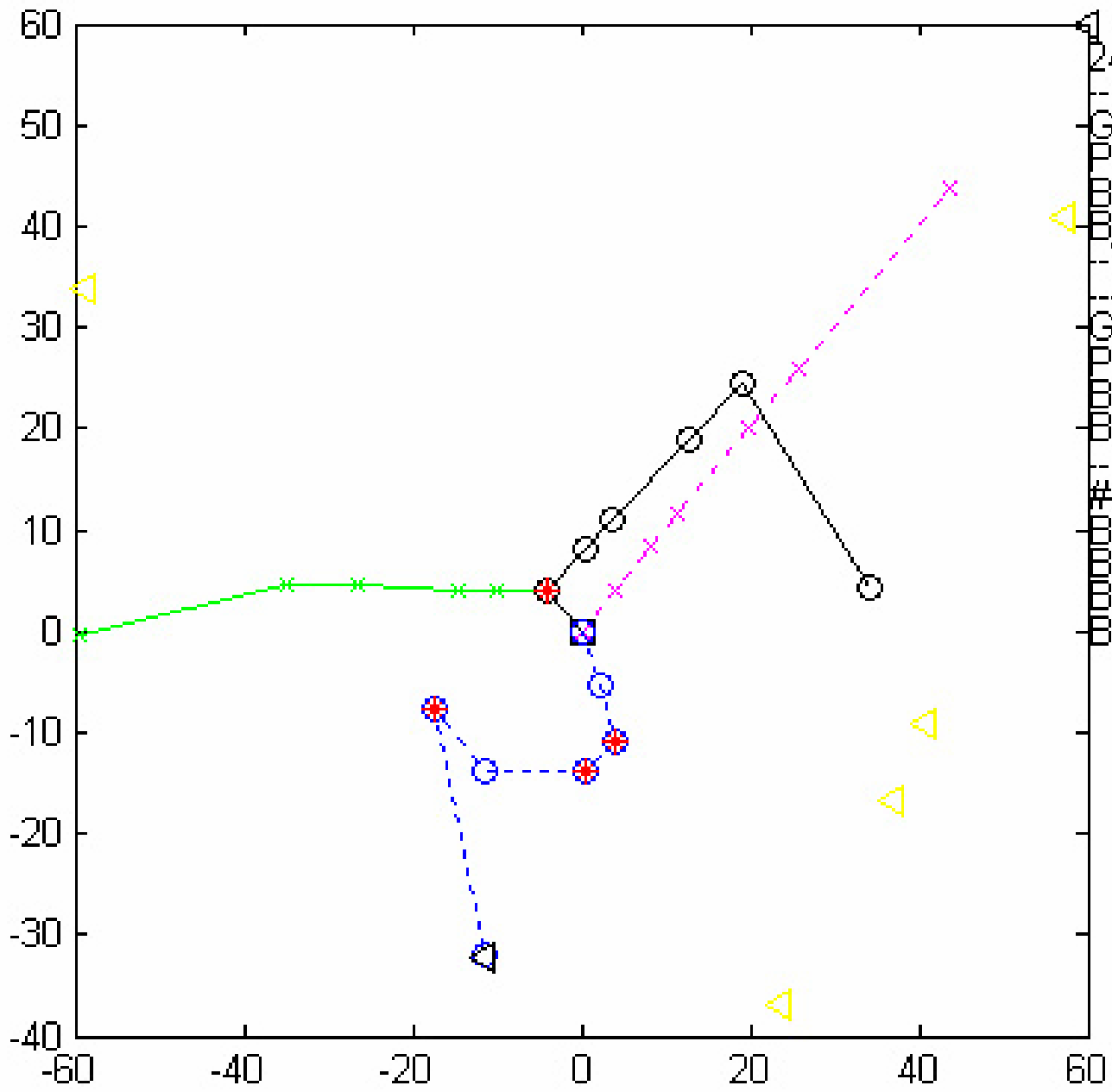
DETECTION RADIUS: 20



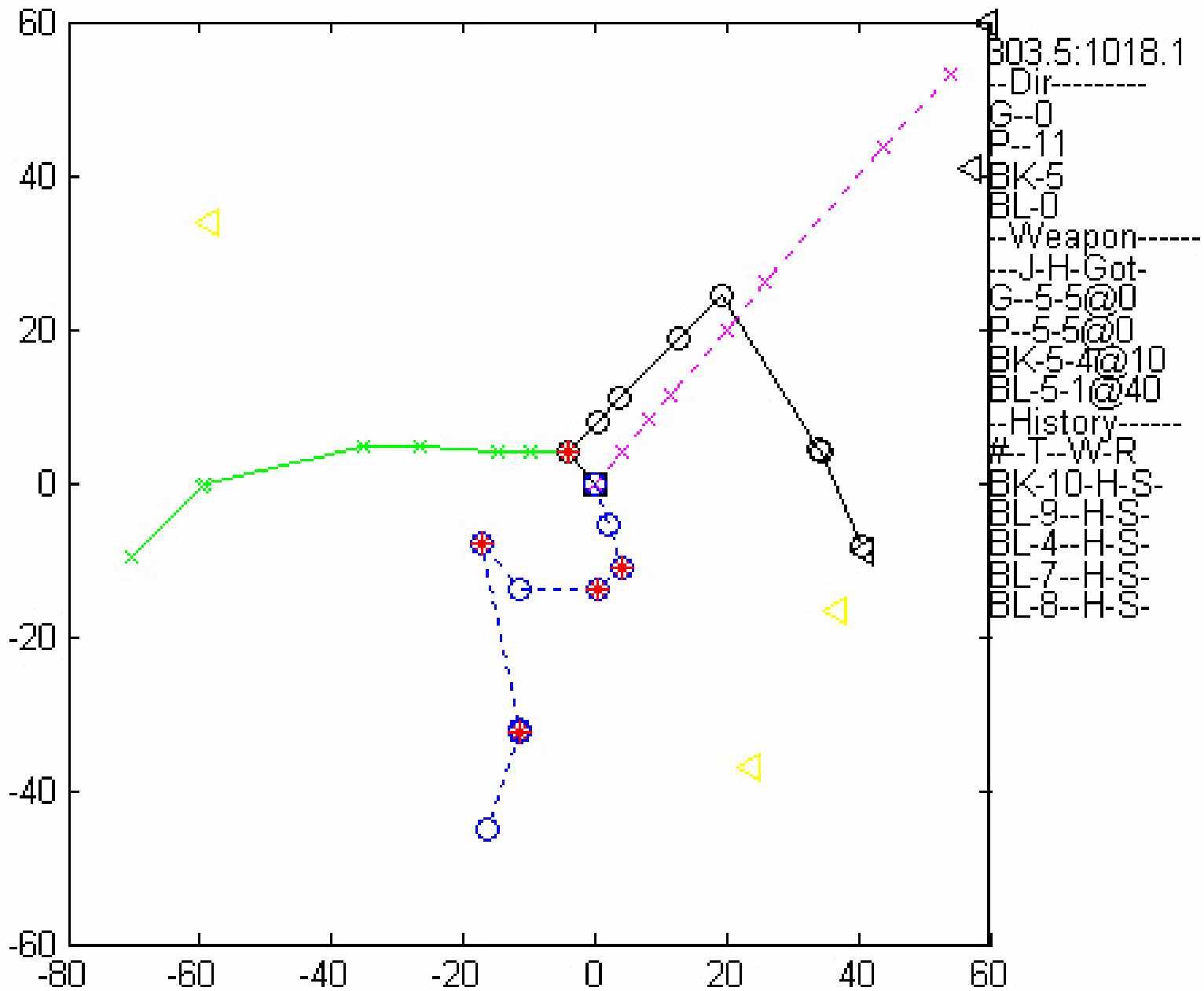


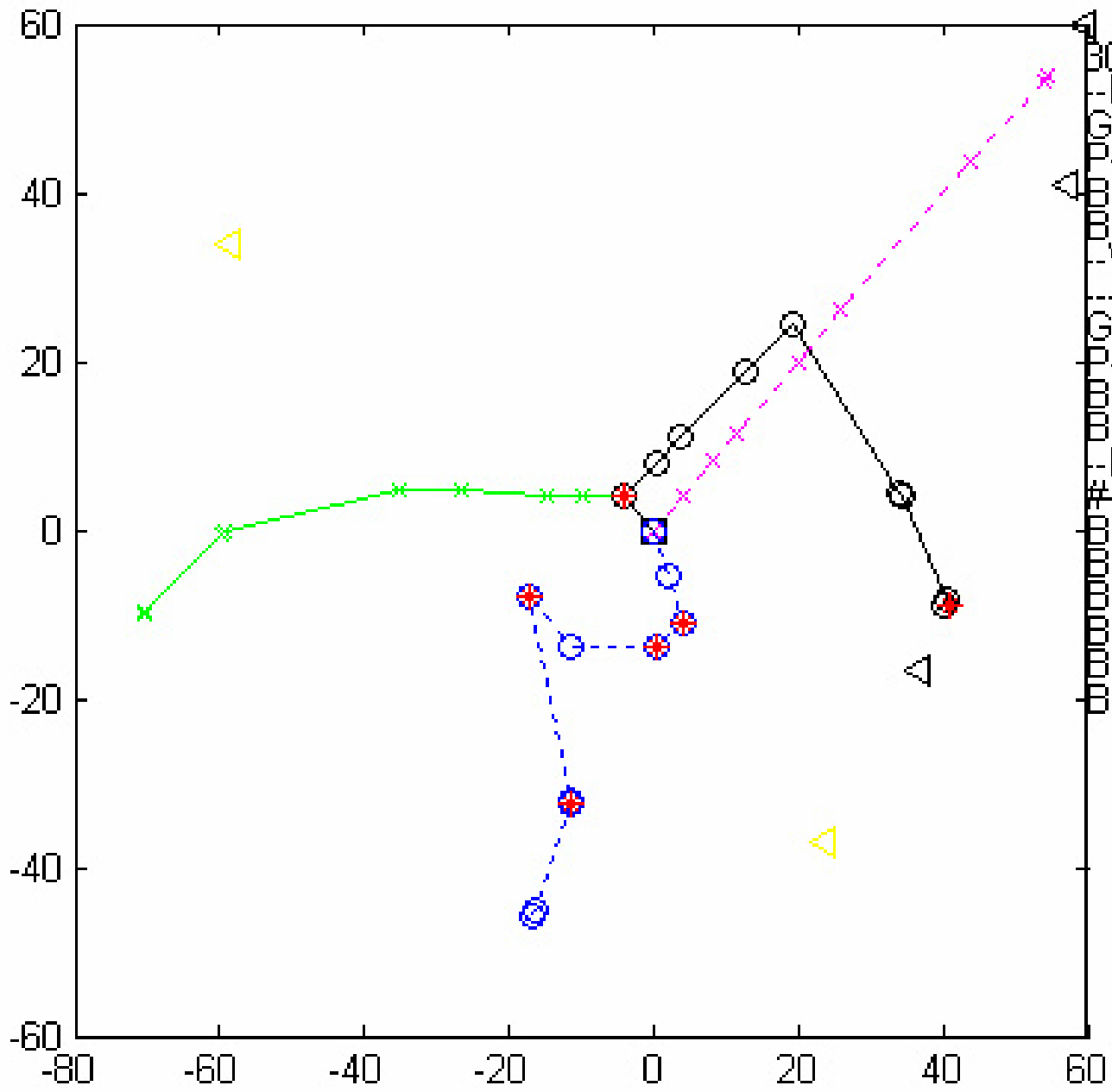




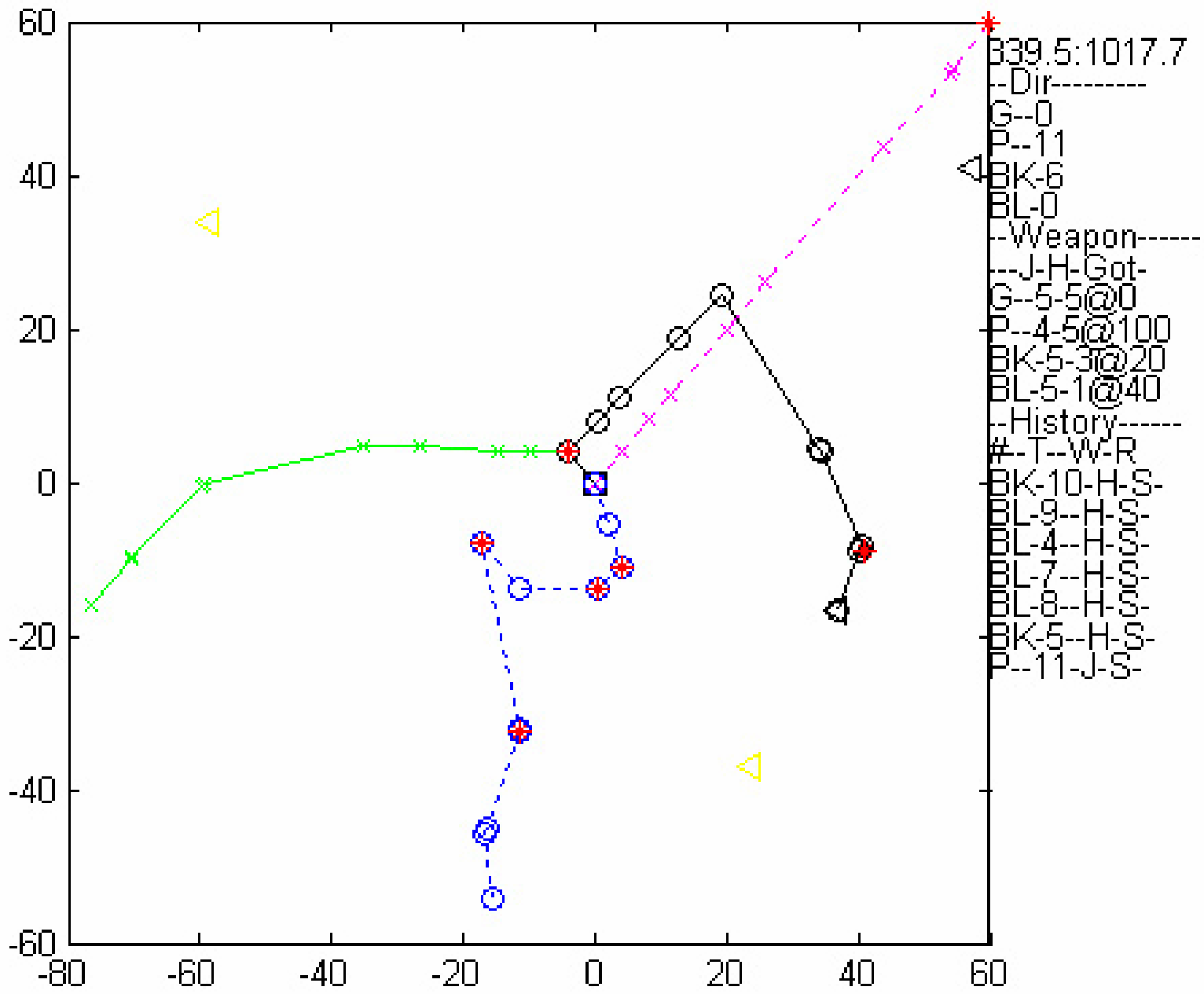


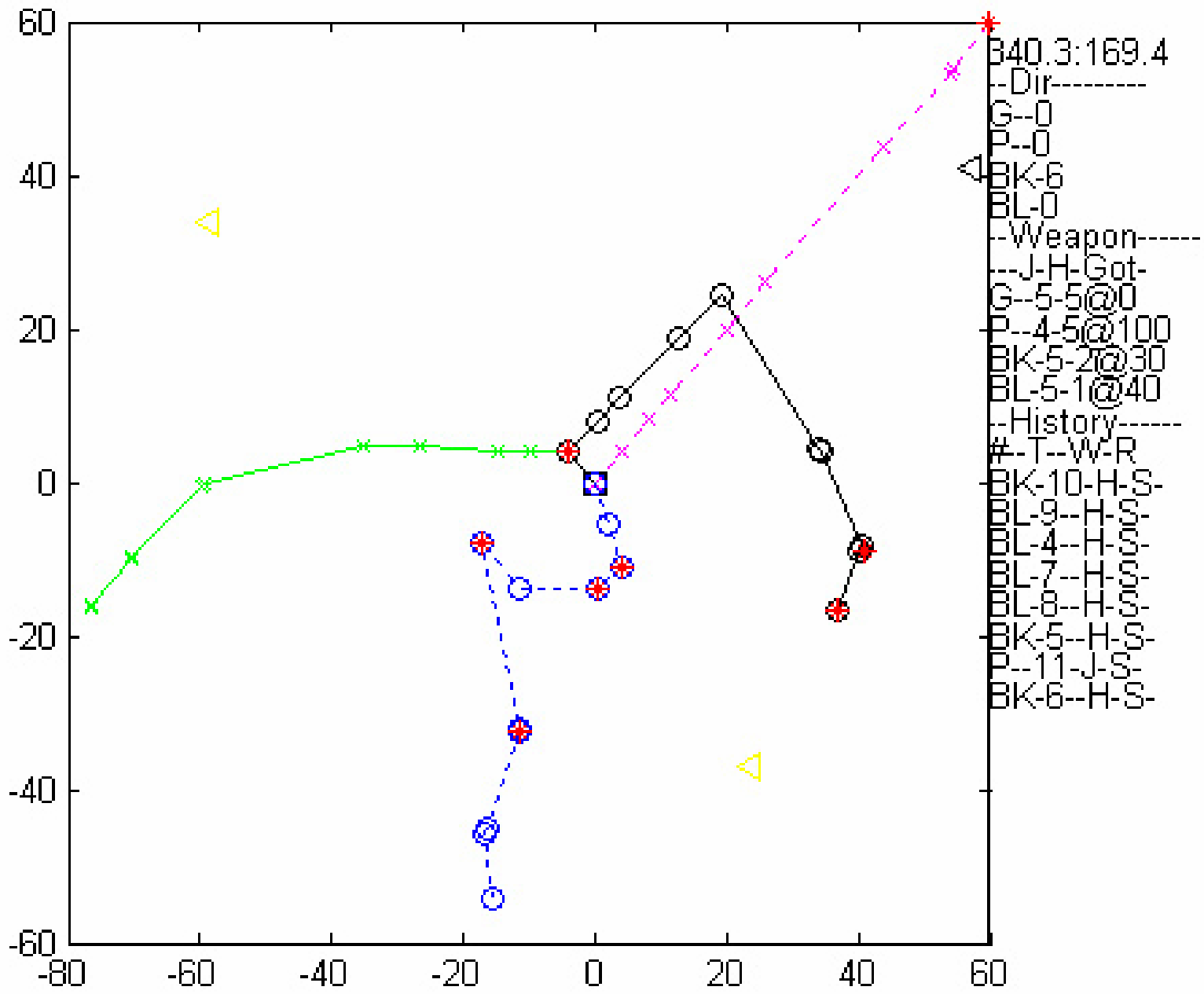
- 246.9.933.5
- Dir-----
- G-0
- P-11
- BK-0
- BL-8
- Weapon-----
- J-H-Got-----
- G-5-5@0
- P-5-5@0
- BK-5-4@10
- BL-5-2@30
- History-----
- #-T-VW-R
- BK-10-H-S-
- BL-9--H-S-
- BL-4--H-S-
- BL-7--H-S-

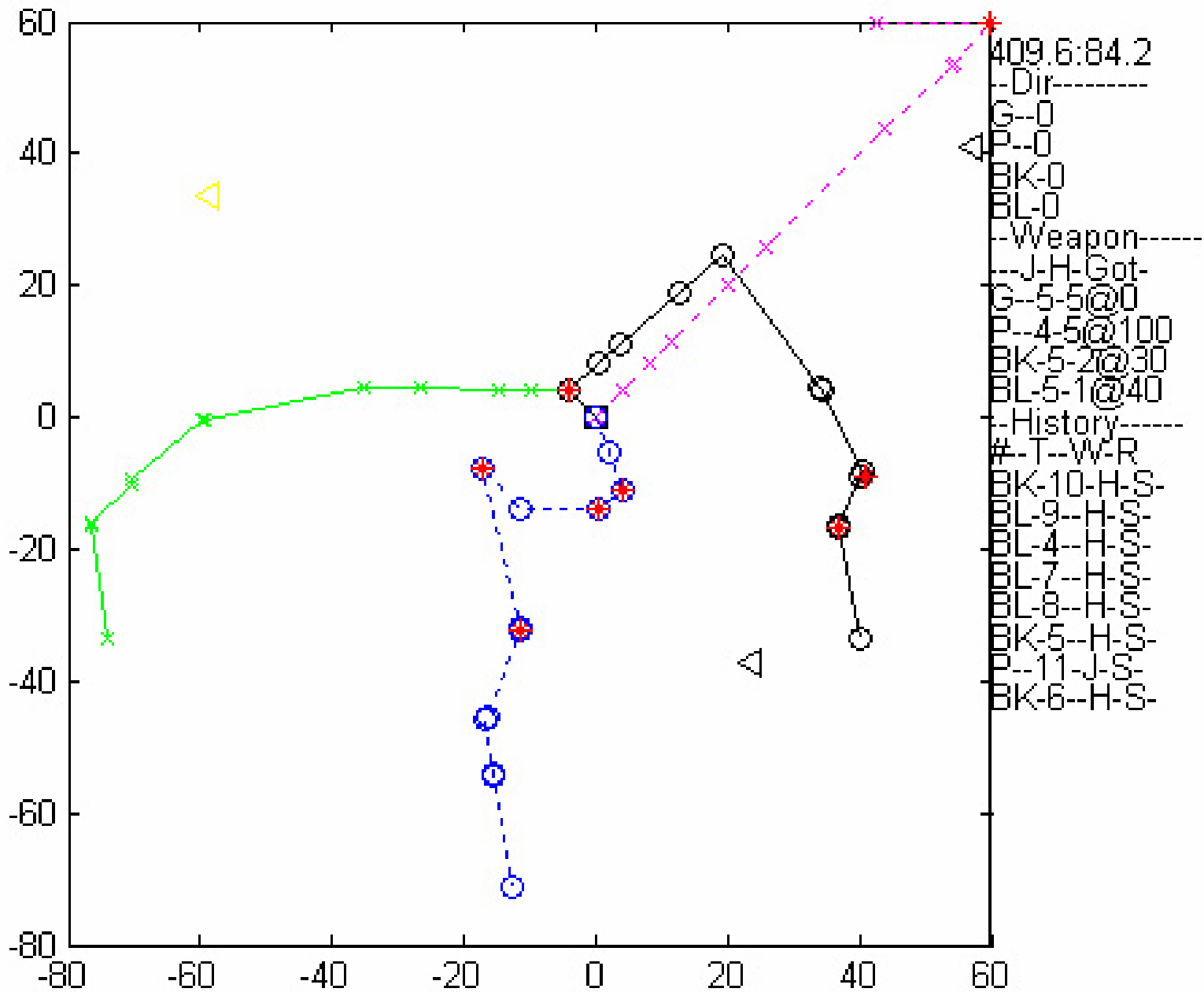


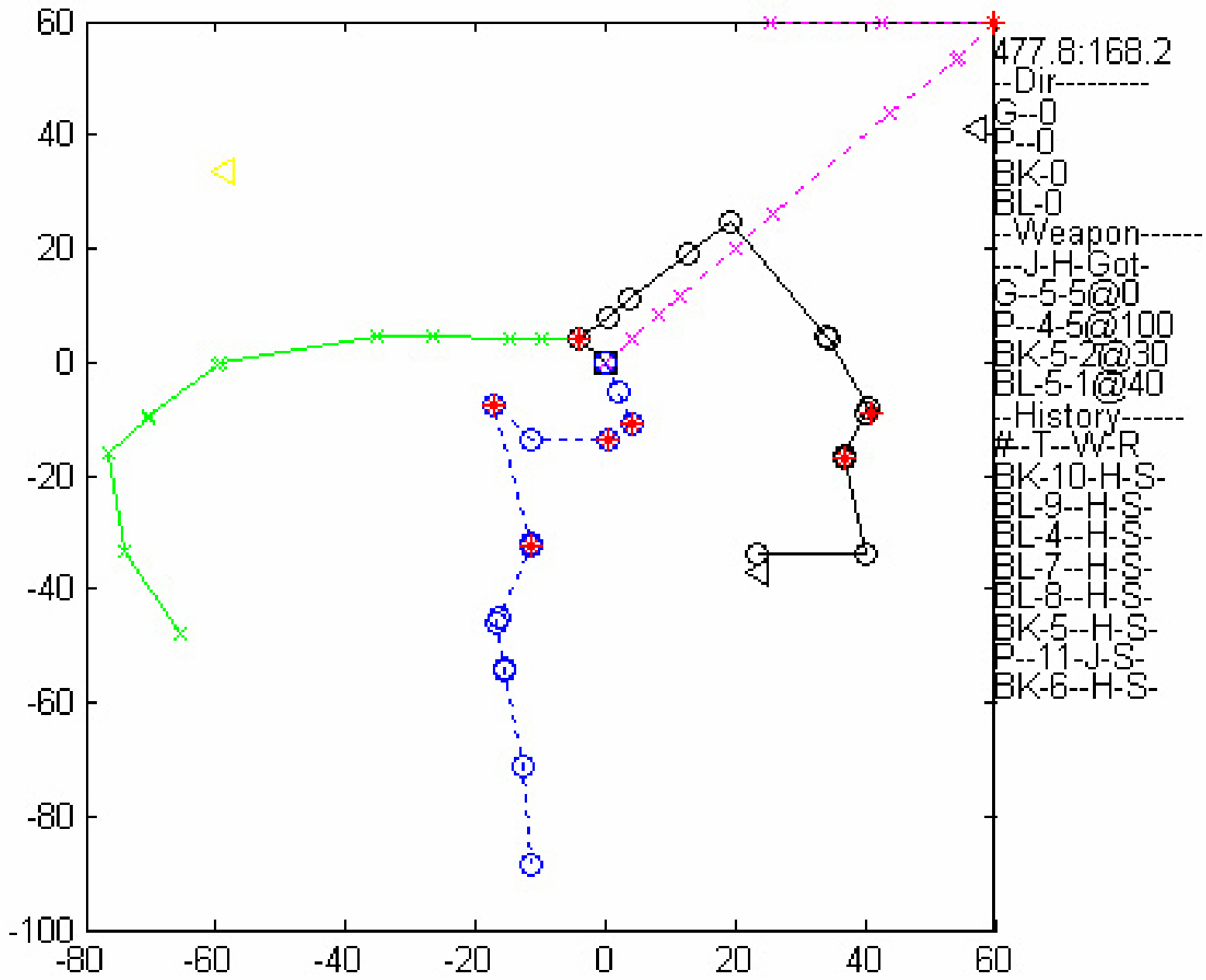


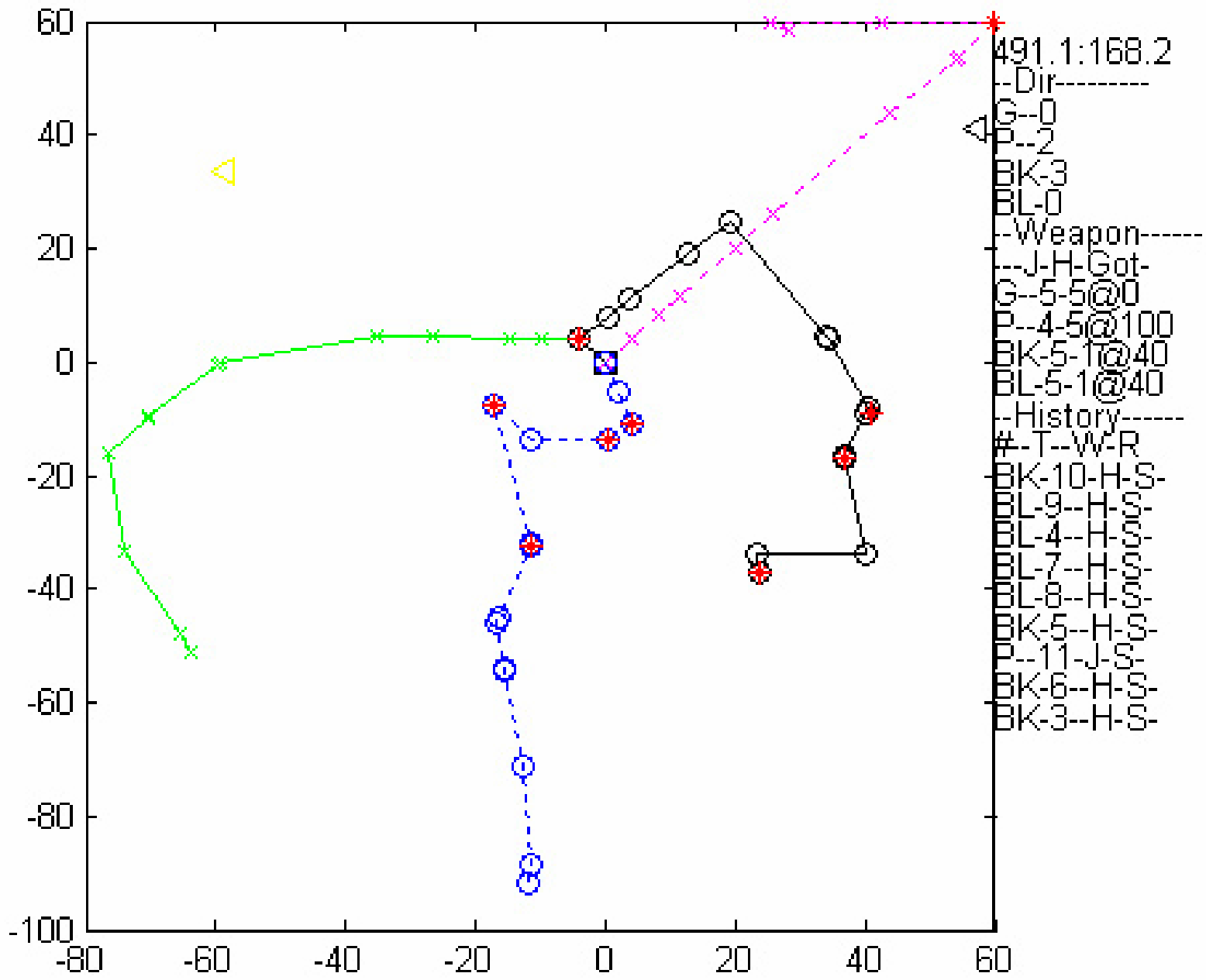
305.9:1102.9
 Dir-----
 G-0
 G-11
 BK-5
 BL-0
 Weapon-----
 J-H-Got-
 G-5-5@0
 G-5-5@0
 BK-5-3@20
 BL-5-1@40
 History-----
 #-T-VW-R
 BK-10-H-S-
 BL-9-H-S-
 BL-4-H-S-
 BL-7-H-S-
 BL-8-H-S-
 BK-5-H-S-

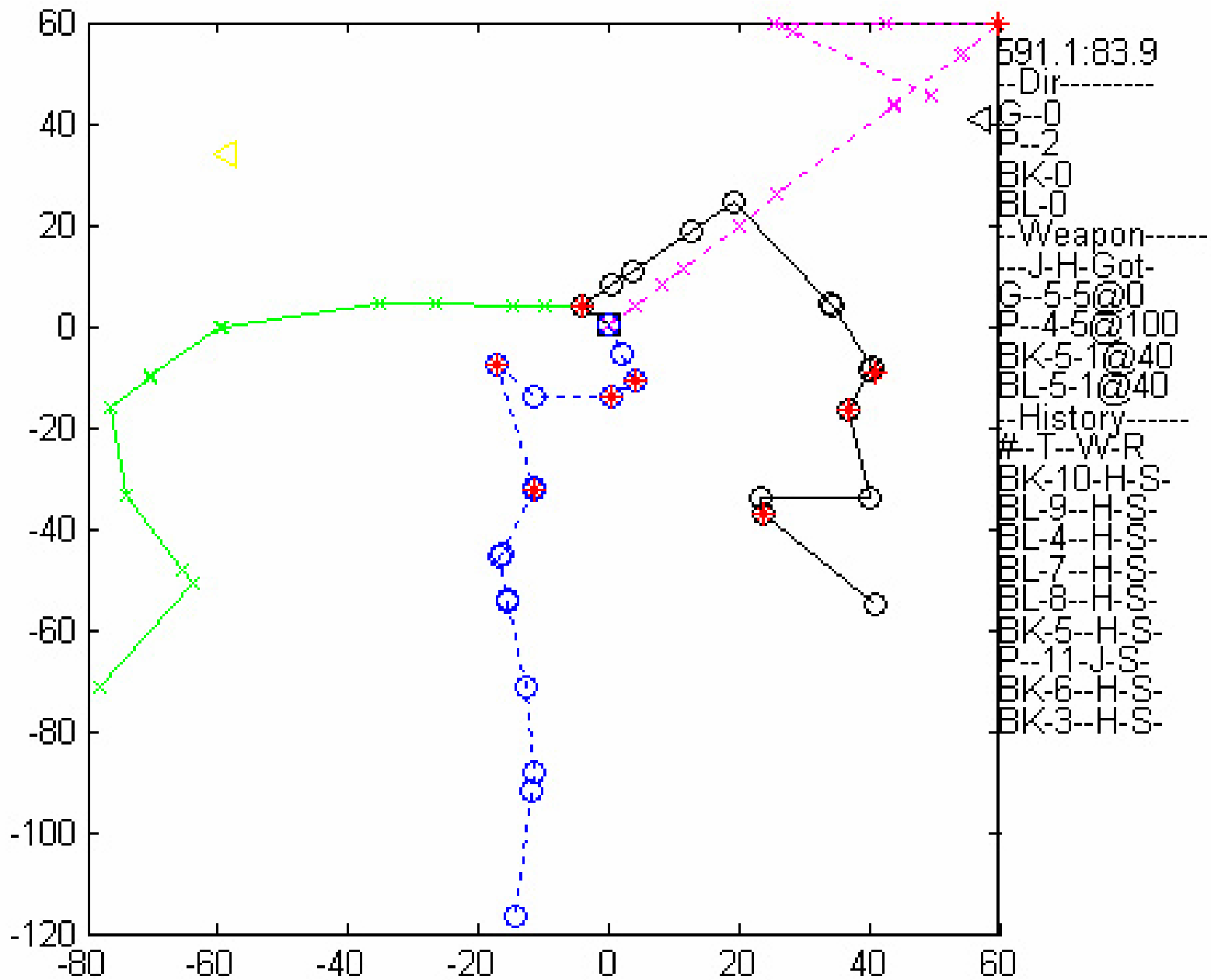


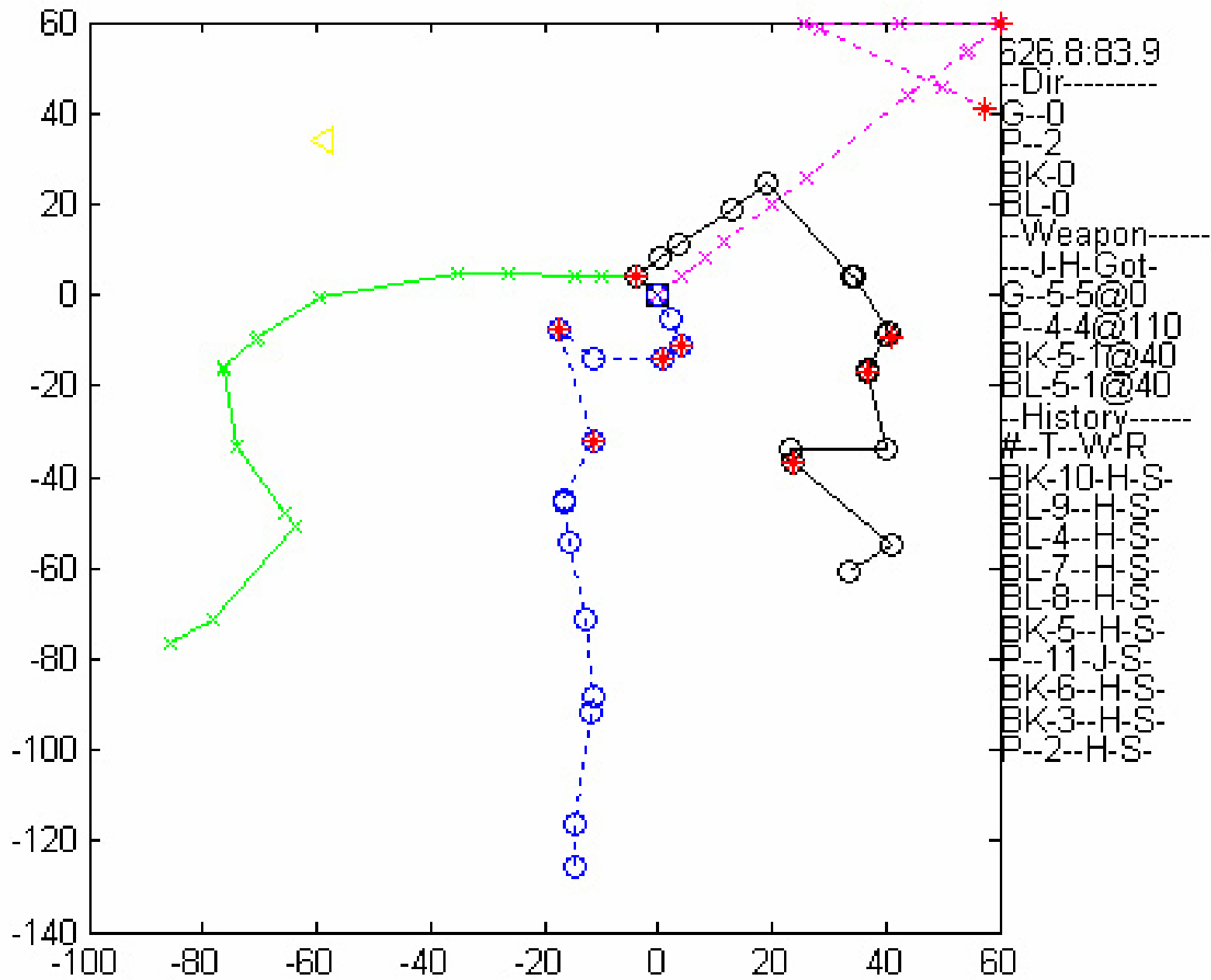


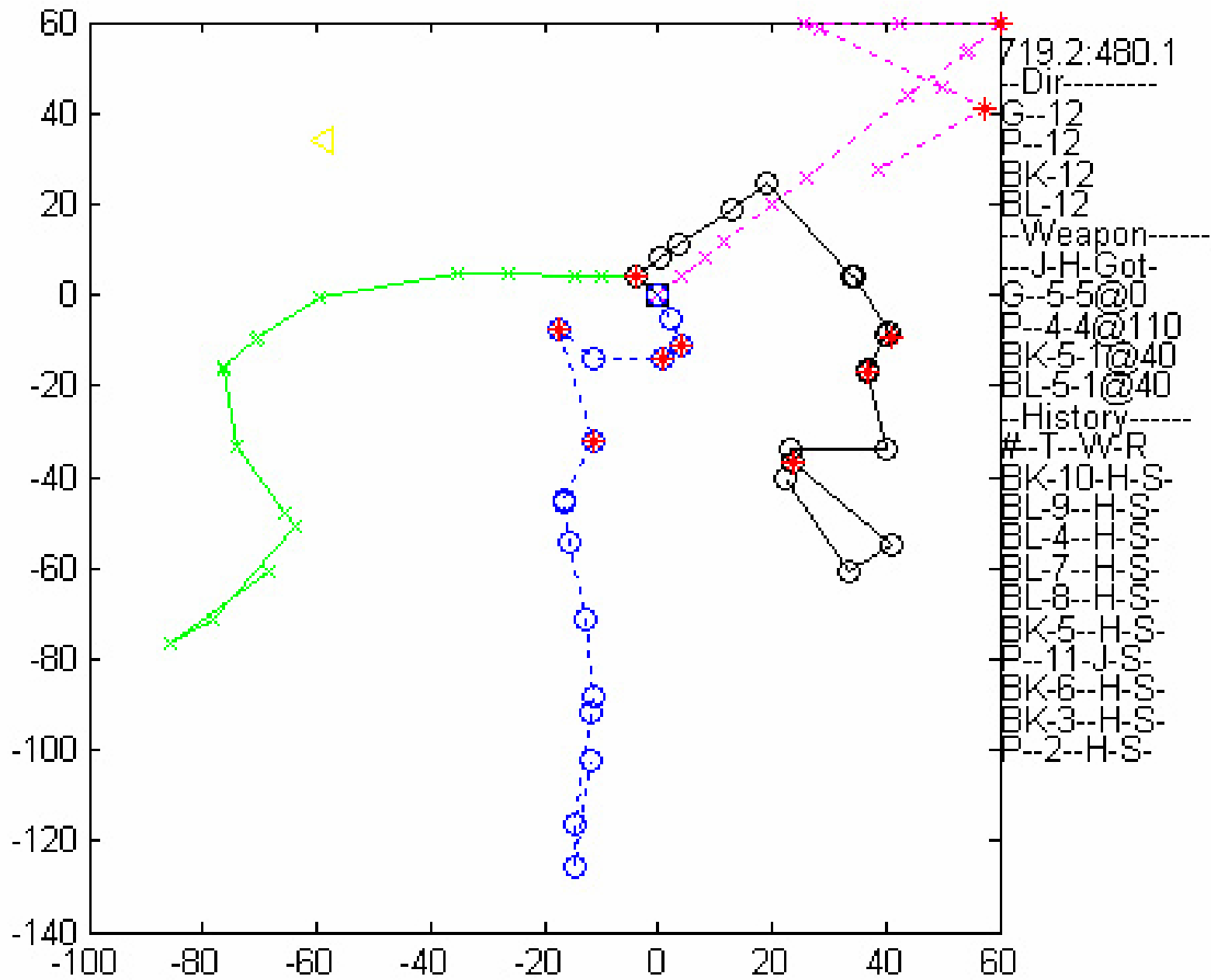


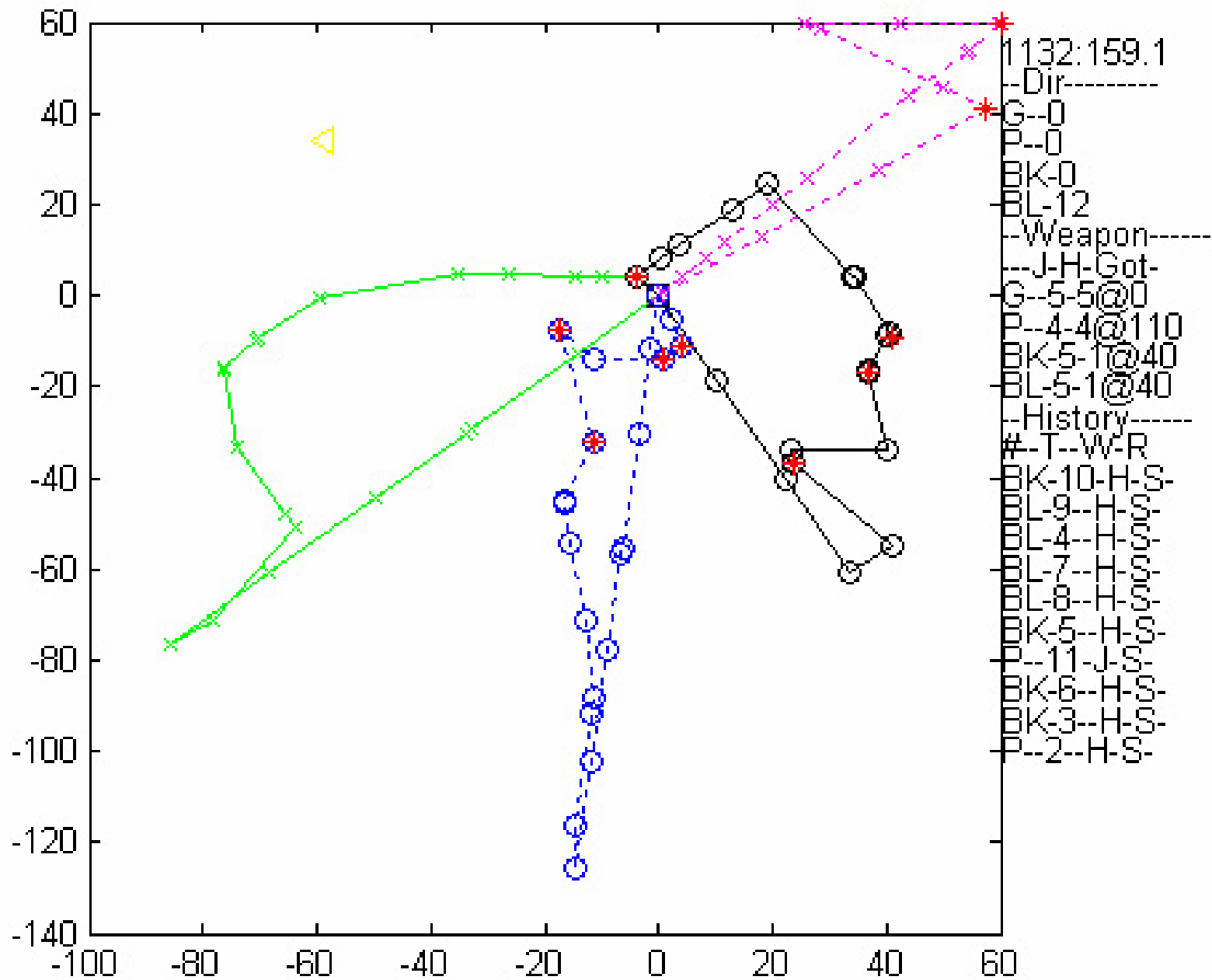












DISTRIBUTED COOPERATIVE CONTROL

Construct **GRADIENT FIELD** instead of artificial potential field

$$F_{ij} = \frac{\partial J_i}{\partial x_j} = c_i(x_j) \cdot f_i^0(x_j)$$

$$c_i(x_j) = \begin{cases} 1 & \text{if } y_i \in S_j \\ q_{ij} - \frac{(1 - \delta_{ij})(2\delta_{ij} - 1)}{(1 - 2\Delta)} & \text{if } y_i \in C_j \\ 0 & \text{if } y_i \in I_j \end{cases}$$

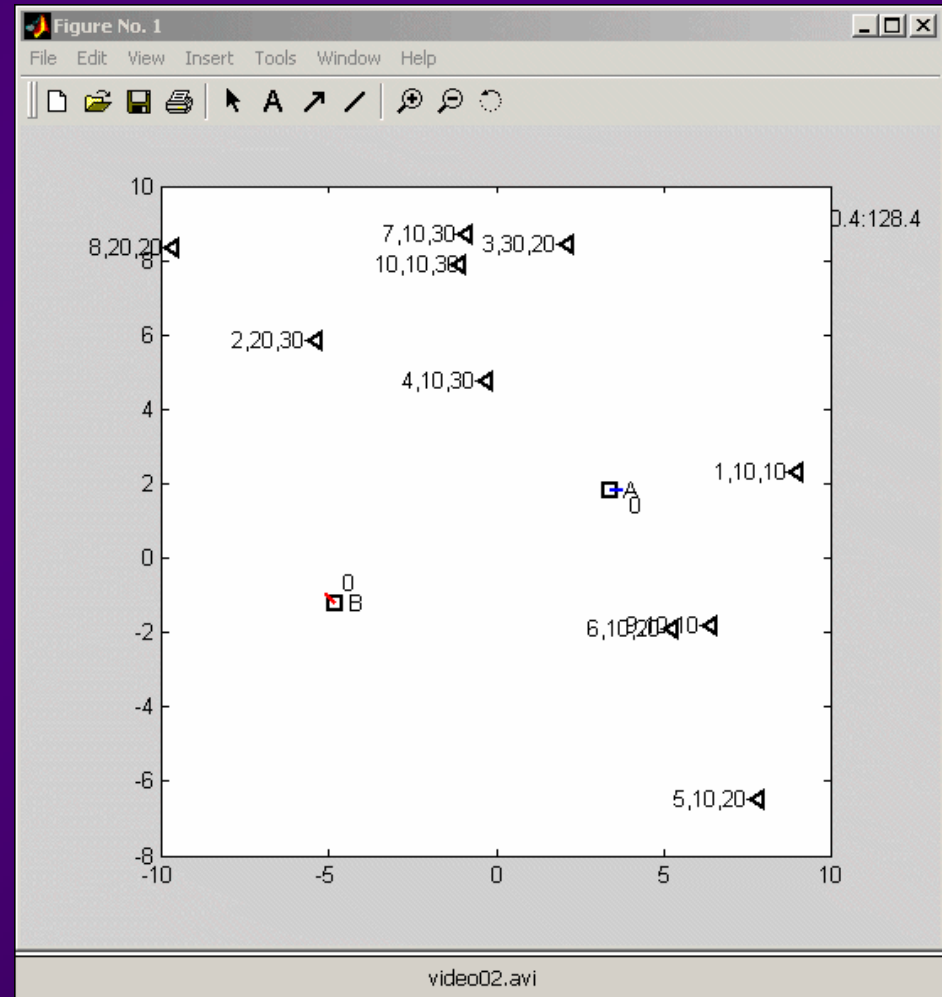
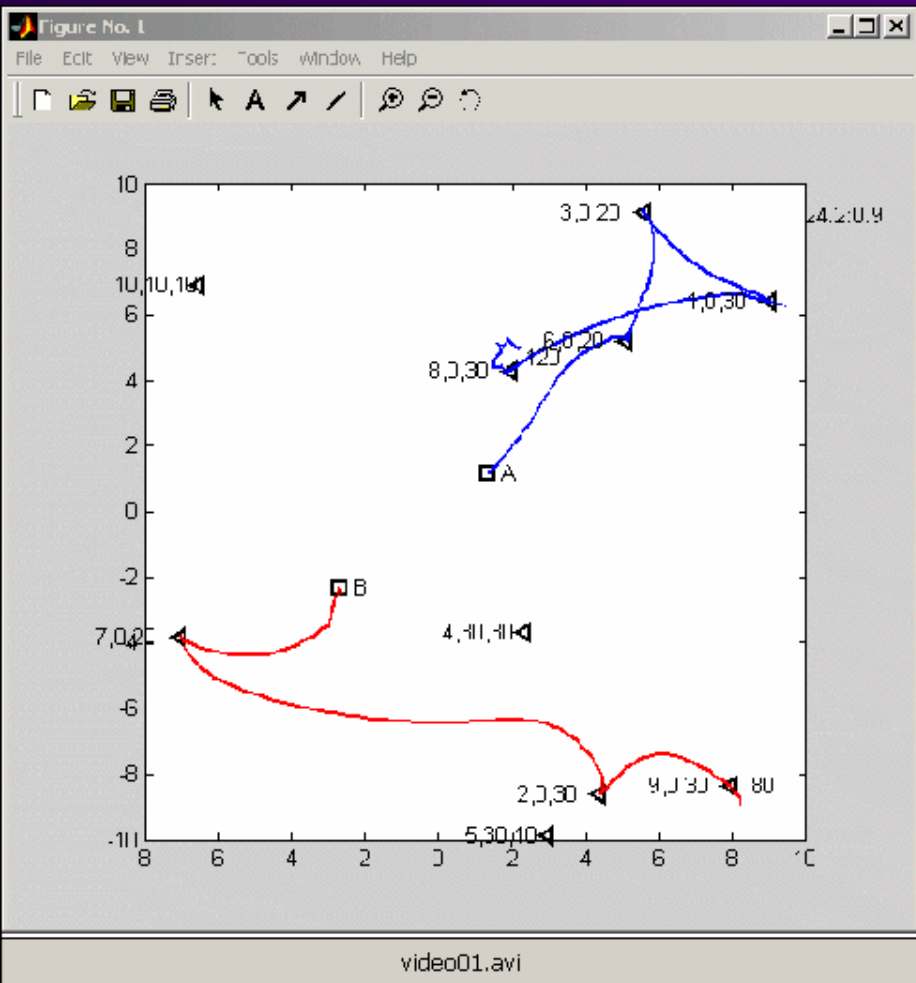
Cooperation coefficient

$$f_i^0(x_j) = \begin{cases} -\frac{R_i}{D_i} \frac{x_j - y_i}{\|x_j - y_i\|} & \text{if } x_j \neq y_i \\ 0 & \text{otherwise} \end{cases}$$

Force exerted by target i on vehicle j given that it is the only vehicle in the mission space

DISTRIBUTED COOPERATIVE CONTROL

- 2 examples ($M=2, N=10$)



OTHER ISSUES

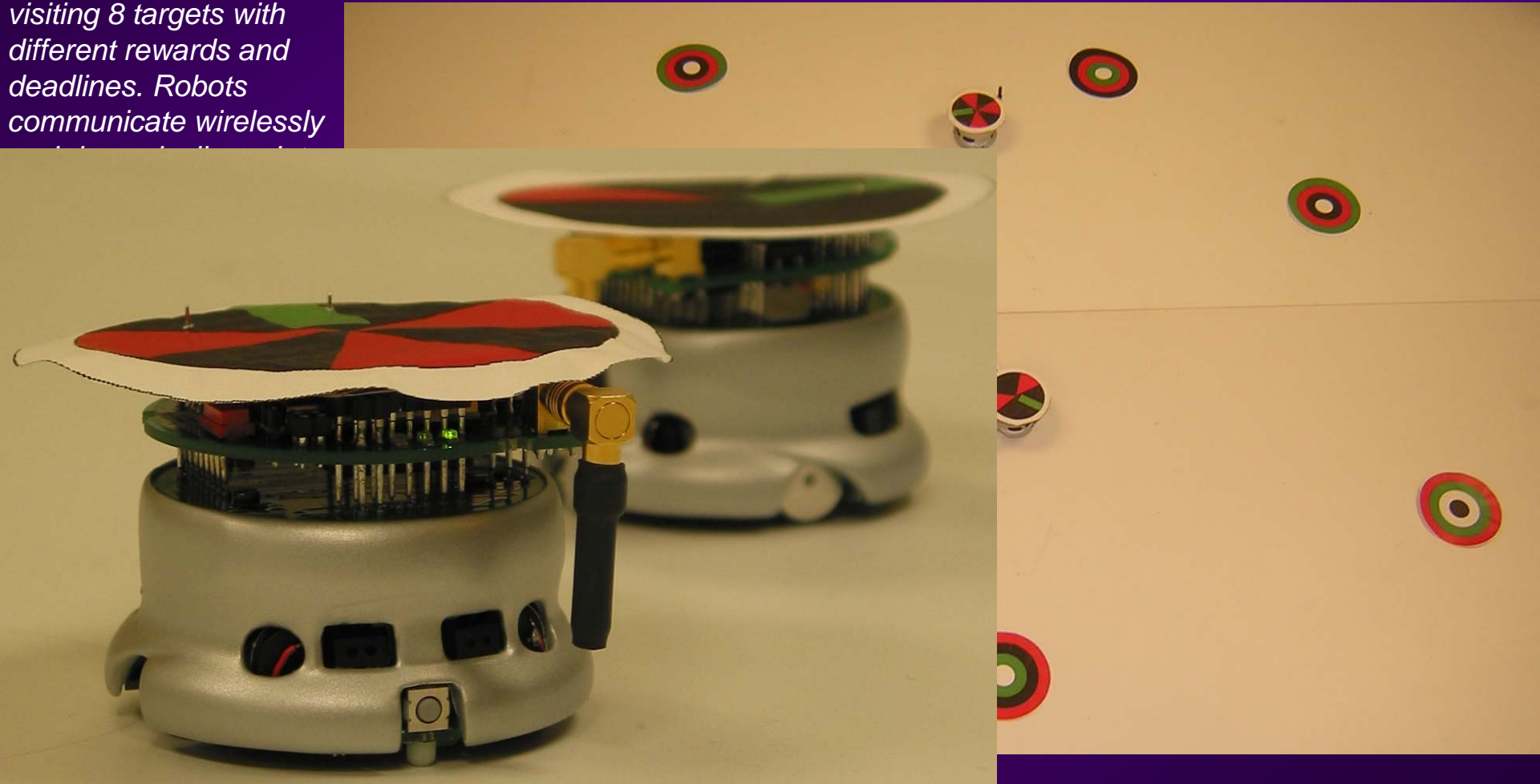
- Local optima in the CRH optimization problem
- Oscillatory vehicle behavior (*instabilities*)
- Additional path constraints
- Does CRH control generate optimal assignments?

REWARD MAXIMIZATION MISSION DEMO

MOVIES OF SUCH MISSIONS WITH SMALL ROBOTS:

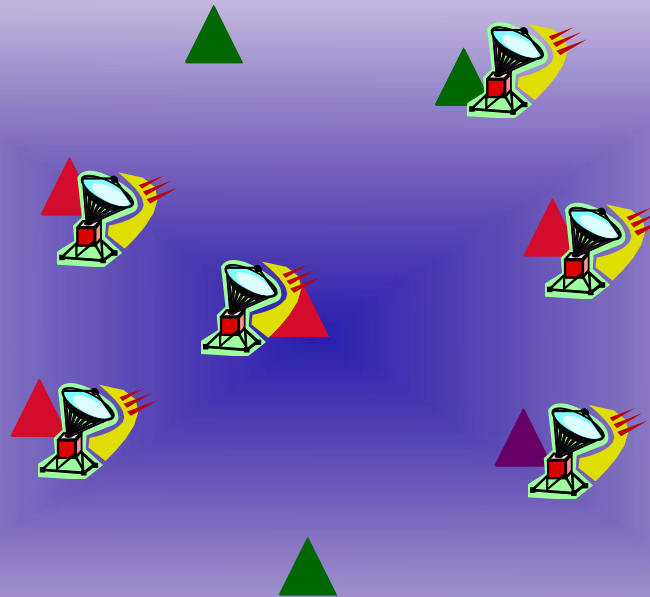
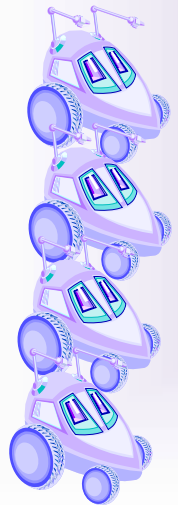
3 Khepera robots
executing mission:
visiting 8 targets with
different rewards and
deadlines. Robots
communicate wirelessly

<http://frontera.bu.edu/CoopCtrl.html>



COVERAGE CONTROL MISSION

SENSOR FIELD WITH
UNKNOWN DATA SOURCES
- ONLY DENSITY
FUNCTION ASSUMED

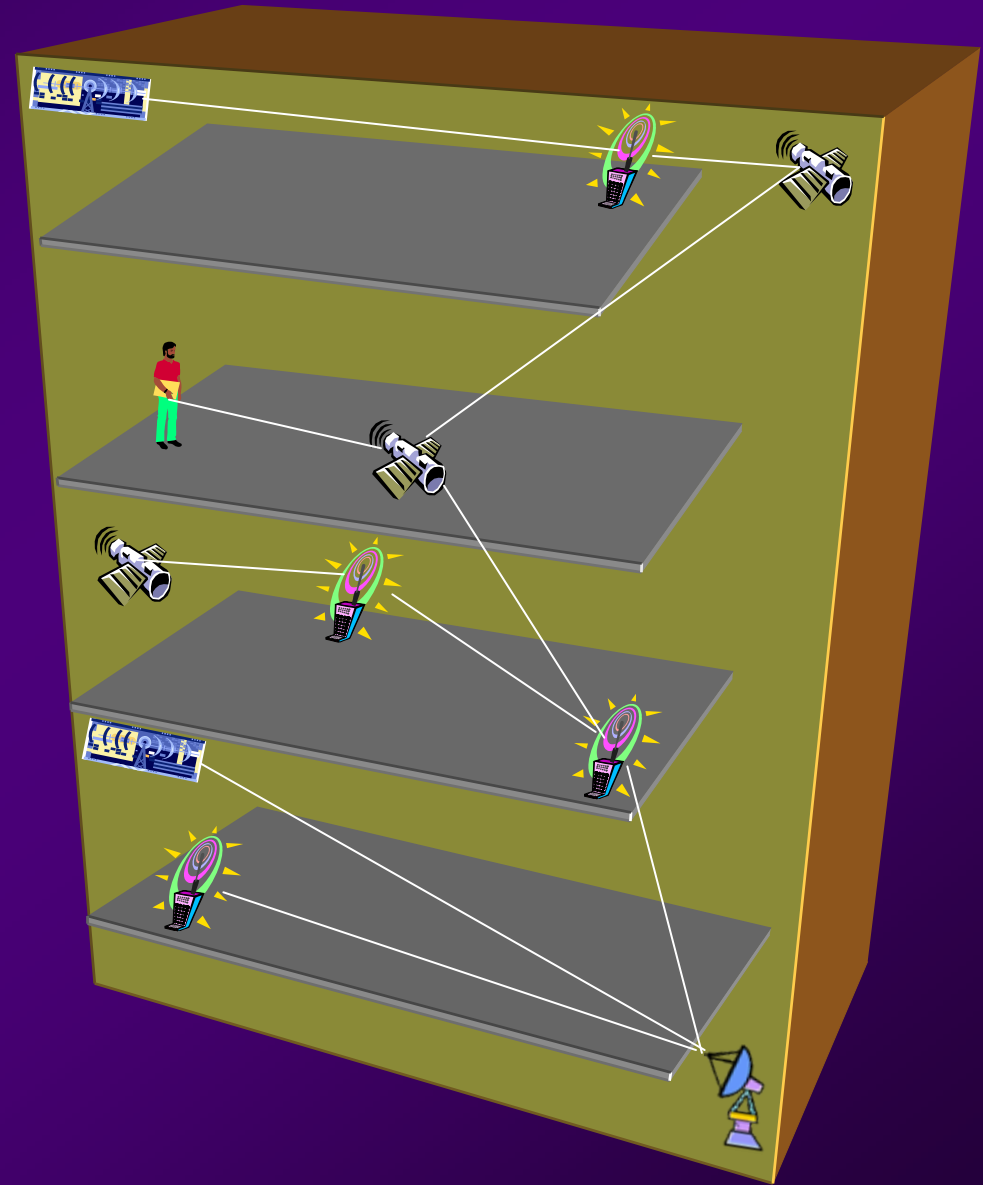


- Meguerdichian et al, INFOCOM, 2001,
- Cortes et al, IEEE Trans. on Robotics and Auto., 2004
- Li and Cassandras, CDC, 2005

WHAT'S A SENSOR NETWORK (SNET)?

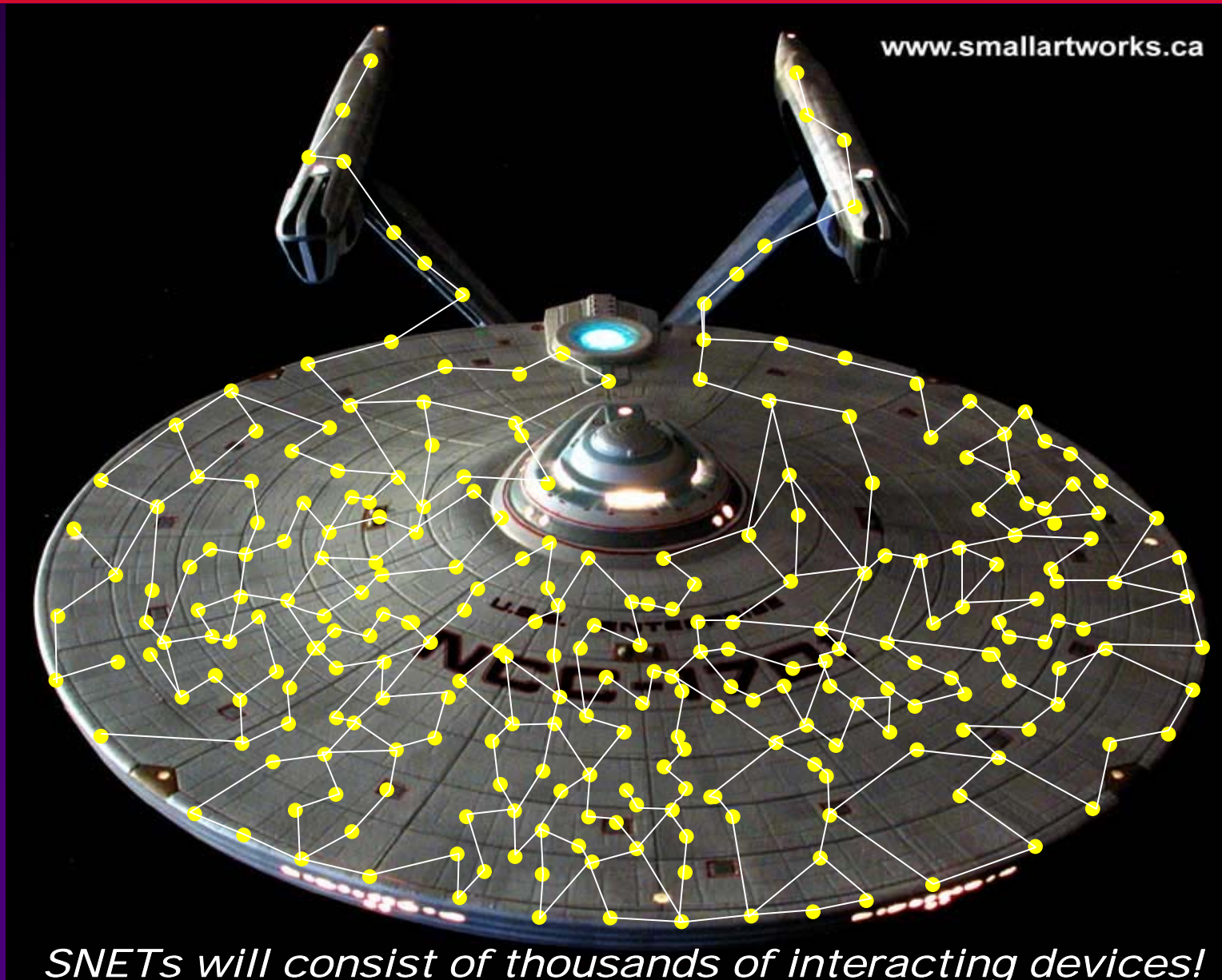
A NETWORK consisting of devices (sensors) that:

- ... communicate wirelessly
- ... are battery-powered
- ... may have different characteristics
- ... have limited processing capabilities
- ... have limited life
- ... often operate in noisy/adversarial environments
- ... monitor/control physical processes



WHAT'S A SENSOR NETWORK (SNET)?

CONTINUED



SNETs will consist of thousands of interacting devices!

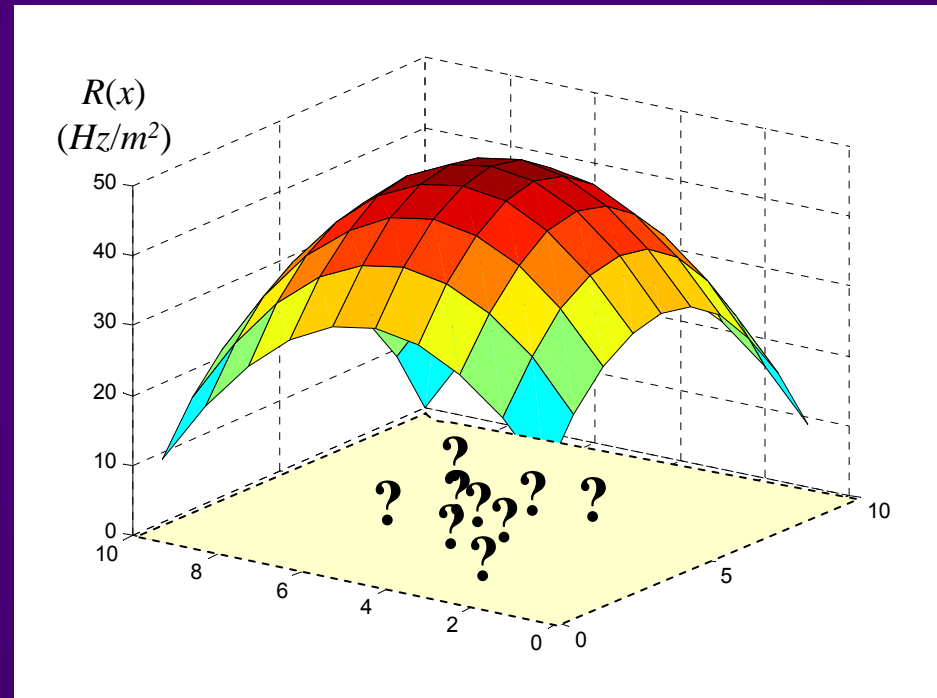
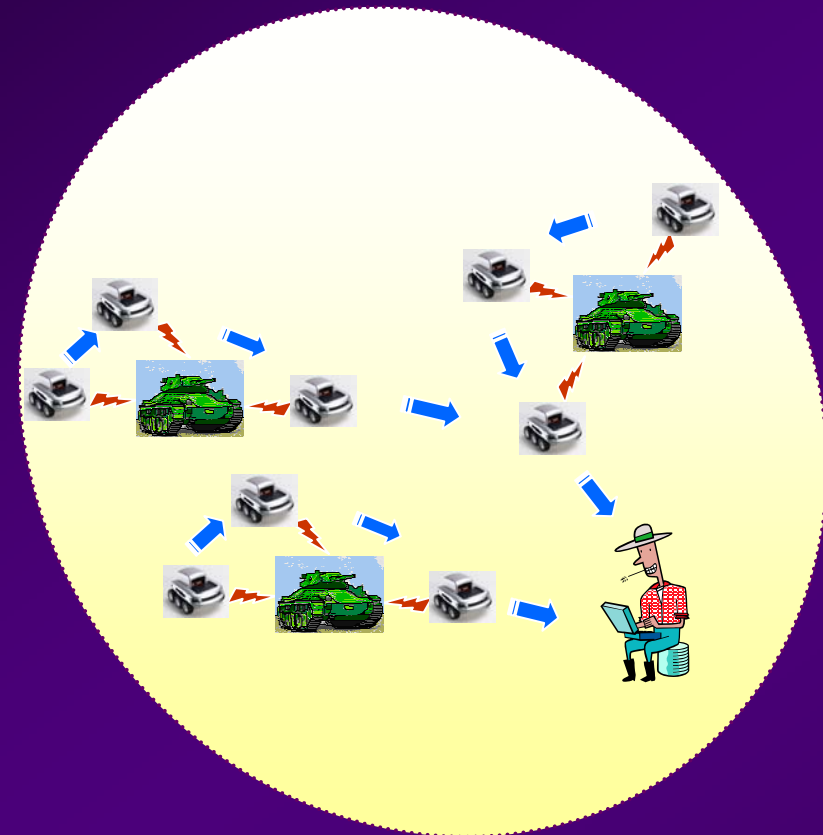
WHY ARE **SNET**s EXCITING?

- They interact with the *physical* world
- They promise *fascinating applications*:
 - **Smart Buildings** (locate persons/objects, find closest resource, adjust environment, detect emergency conditions)
 - **Health monitoring**
 - **Security** and **military** applications
 - **Environmental monitoring**
 - **Inventory** monitoring/replenishment (smart shelves)
 - **Equipment condition monitoring** and **active maintenance** (smart appliances)
- They realize a convergence of the "**3 Cs**":
Communications + **Computing** + **Control**

COVERAGE CONTROL MISSION

GOAL: Deploy mobile nodes to maximize data source detection probability

- unknown data sources
- data sources may be mobile



Perceived data source density
over mission space

PROBLEM FORMULATION

- N mobile sensors, each located at $s_i \in \mathbf{R}^2$
- Data source at x emits signal with energy E
- Signal observed by sensor node i (at s_i)

- Sensing model:

$$p_i(x) \equiv p(\text{Detected by } i \mid A(x), s_i)$$

($A(x)$ = data source emits at x)

- Sensing attenuation:

$p_i(x)$ is a decreasing function of $d_i(x) \equiv \|x - s_i\|$
(distance between x and s_i)

- Joint detection prob. assuming sensor independence:

$$P(x) = 1 - \prod_{i=1}^N [1 - p_i(x)]$$

- OBJECTIVE:

*Determine locations s_i ($i=1, \dots, N$)
to maximize total detection probability:*

$$\max_{s_i \in \Omega} \int_{\Omega} R(x) \left\{ 1 - \prod_{i=1}^N [1 - p_i(x)] \right\} dx$$

Perceived data source density

DISTRIBUTED COOPERATIVE SCHEME

- Denote

$$F(s_1, \dots, s_N) = \int_{\Omega} R(x) \left\{ 1 - \prod_{i=1}^N [1 - p_i(x)] \right\} dx$$

- Maximize $F(s_1, \dots, s_N)$ by forcing nodes to move using gradient information:

$$\frac{\partial F}{\partial s_k} = \int_{\Omega} R(x) \prod_{i=1, i \neq k}^N [1 - p_i(x)] \frac{\partial p_k(x)}{\partial d_k(x)} \frac{s_k - x}{d_k(x)} dx$$

$$\frac{\partial F}{\partial s_k} = \int_{\Omega} R(x) \prod_{i=1, i \neq k}^N [1 - p_i(x)] \frac{\partial p_k(x)}{\partial d_k(x)} \frac{s_k - x}{d_k(x)} dx$$

This has to be evaluated numerically.

Not doable for a mobile sensor with limited computation capacity.

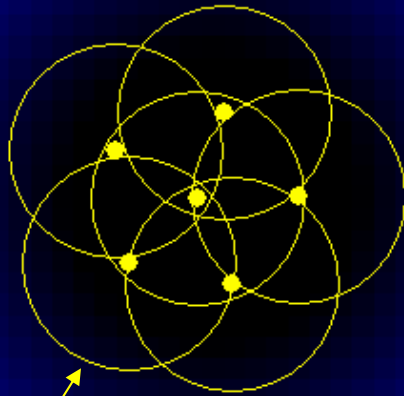
- Approximate $p_i(x)$ by truncating sensing attenuation
- Discretize $p_i(x)$ using a grid

COVERAGE CONTROL MISSION DEMO

SOFTWARE DEMO OF COVERAGE CONTROL ALGORITHM:

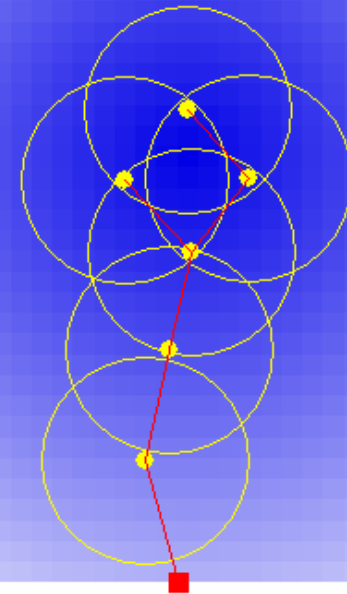
<http://frontera.bu.edu/Applets/CoverageContr/index.html>

No communication cost



Sensing Range

With communication cost



SOME FINAL THOUGHTS...

- Small, cheap cooperating devices cannot handle complexity
⇒ we need **DISTRIBUTED** control !
 - Cooperating agents operate asynchronously
⇒ we need **ASYNCHRONOUS** control/optimization schemes
 - Wireless communication is alarmingly vulnerable to security threats
 - Different views/aspects of “Cooperative Control” abound...
-

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ΕΥΧΑΡΙΣΤΟ