Decentralized Merging Control in Traffic Networks: A Control Barrier Function Approach *

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ABSTRACT

In this paper, we aim to optimize the process of Connected and Automated Vehicles (CAVs) merging at a traffic intersection while guaranteeing the state, control and safety constraints. We decompose the task of automatic merging for all the CAVs in a control zone around a merging point into same-lane safety constraints and different-lane safe merging, and implement these requirements using control barrier functions (CBFs). We consider two main objectives. First, to minimize travel time, we make the CAVs reach the road maximum speed with exponentially stabilizing control Lyapunov functions (CLF). Second, we penalize energy consumption as a cost in an optimization problem. We then decompose the merging problem into decentralized subproblems formulated as a sequence of quadratic programs (QP), which are solved in real time. Our simulations and comparisons show that the method proposed here outperforms ad hoc controllers used in traffic system simulators and provides comparable results to the optimal control solution of the merging problem in earlier work.

CCS CONCEPTS

• **Networks** → **Network algorithms**; • **Applied computing** → *Physical sciences and engineering*; • **Computing methodologies** → Modeling and simulation;

KEYWORDS

Decentralized Control, Traffic Merging, Control Barrier Function, Control Lyapunov Function, Optimal Control.

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1 INTRODUCTION

Traffic management at merging points (usually, highway on-ramps) is one of the most challenging problems within a transportation system in terms of safety, congestion, and energy consumption, in addition to being a source of stress for many drivers [19, 22, 25]. Advancements in next generation transportation system technologies and the emergence of CAVs (also known as self-driving cars or autonomous vehicles) have the potential to drastically improve a transportation network's performance by better assisting drivers in making decisions, ultimately reducing energy consumption, air pollution, congestion and accidents. One of the very early efforts exploiting the benefit of CAVs was proposed in [10], where an optimal linear feedback regulator was introduced for the merging problem to control a single string of vehicles. An overview of automated intelligent vehicle-highway systems was provided in [23].

There has been significant research in assisted traffic merging offering guidance to drivers so as to avoid congestion and collisions. A Classification and Regression Tree (CART) method was used in [26] to model merging behavior and assist decisions in terms of the time-to-collision between vehicles. The Long Short-Term Memory (LSTM) network was used in [7] to predict possible long-term congestion. In [30], a Radial Basis Function-Artificial Neural Network (RBF-ANN) is used to forcast the traffic volume in a merging area. However, such assisted merging methods do not take advantage of autonomous driving so as to possibly automate the merging process in a cooperative manner.

A number of centralized or decentralized merging control mechansims have been proposed [6, 12–15, 17, 18, 22]. In the case of decentralized control, all computation is performed on board each vehicle and shared only with a small number of other vehicles which are affected by it. Optimal control problem formulations are used in some of these approaches, while Model Predictive Control (MPC) techniques are employed in others, primarily to account for additional constraints and to compensate for disturbances by re-evaluating optimal actions. The objectives specified for optimal control problems may target the minimization of acceleration as in [17] or the maximization of passenger comfort (measured as the acceleration derivative or jerk) as in [14, 16]. MPC approaches were used in [6, 13], as well as in [14] where inequality constraints were added to the originally considered optimal control problem.

The optimal control approaches to the merging problem usually assume that no constraints are active in order to get simple analytical solutions for the controller or they become complicated when the constraints are included in the derivation of optimal trajectories [11, 14, 28, 29]. The number of possible constrained cases is determined by the number of constraints, and the constraints may even recursively become active, which makes this problem very expensive computationally when there are many constraints. In the existing work, the objective functions are also restricted to simple forms, such as the 2-norm square of the control input. Moreover, the dynamics are restricted to linear when analytical solutions are desired.

In this paper, we combine all the constraints in a quadratic program (QP) or linear program (LP) by using control barrier functions (CBF) [4, 5, 21] and solve it in real time. Therefore, the enumeration of specific cases is avoided, which makes our method easy to implement even under many constraints. We can accommodate complex objective functions and nonlinear dynamics since we solve the optimization problem in discrete time. Most recent work in the merging problem is restricted to minimizing acceleration and jerk [14, 16] or just to achieving safe merging [6, 13]. In addition to minimizing energy consumption, we also consider minimization of travel time, while satisfying all the state, control, safety, and safe merging constraints. We regard travel time as another objective using control Lyapunov functions (CLF) [2, 3, 20]. As in [28], we also solve the merging problem in a decentralized way in this paper. Finally, our solution provides an increased degree of robustness since the merging problem is solved in real time and can therefore deal with perturbations or noise.

The rest of the paper is structured as follows. In Section 2 we include some preliminaries. In Section 3, we introduce the merging process model and formulate the merging control problem, including all the safety requirements that must be satisfied at all times. In Sections 4 and 5, we present the control strategy and discuss the feasibility of the merging problem, respectively. We illustrate the proposed method with simulations in Section 6 and conclude with final remarks in Section 7.

2 PRELIMINARIES

Definition 2.1. Class $\mathcal{K}$ function [9]: A continuous function $\alpha : [0, a) \to [0, \infty), a > 0$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

In this paper, we consider affine control systems of the form

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u}, \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times q}$ are locally Lipschitz, $\boldsymbol{x} \in \mathbb{R}^n$ denotes the state vector and $\boldsymbol{u} \in U \subset \mathbb{R}^q$ (U denotes the control constraint set). Solutions $\boldsymbol{x}$ of (1) are forward complete.

Definition 2.2. A set C is forward invariant for system (1) if its solutions starting at all $\boldsymbol{x}(t_0) \in C$ satisfy $\boldsymbol{x}(t) \in C$ for $\forall t \geq t_0$.

Definition 2.3. Control barrier function (CBF) [4]: Let $C := \{\boldsymbol{x} \in \mathbb{R}^n : h(\boldsymbol{x}) \geq 0\}$, where $h : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable. A function $B : C \to \mathbb{R}$ is a control barrier function (CBF) for system (1) if there exist class $\mathcal{K}$ functions β_1, β_2 and $\gamma > 0$ such that

$$\frac{1}{\beta_1(h(\boldsymbol{x}))} \leq B(\boldsymbol{x}) \leq \frac{1}{\beta_2(h(\boldsymbol{x}))}, \tag{2}$$

$$\inf_{\boldsymbol{u}\in U} [L_f B(\boldsymbol{x}) + L_g B(\boldsymbol{x})\boldsymbol{u} - \frac{\gamma}{B(\boldsymbol{x})}] \leq 0, \tag{3}$$

for all $\boldsymbol{x} \in Int(C)$, where L_f, L_g denote the Lie derivatives [1] along f and g, respectively, and $Int(C)$ is the interior of C.

Given a CBF B, any Lipschitz continuous controller $\boldsymbol{u} \in K_{cbf}(\boldsymbol{x})$, with

$$K_{cbf}(\boldsymbol{x}) := \{\boldsymbol{u} \in U : L_f B(\boldsymbol{x}) + L_g B(\boldsymbol{x})\boldsymbol{u} - \frac{\gamma}{B(\boldsymbol{x})} \leq 0\},$$

renders set C forward invariant [4] for affine control system (1).

Definition 2.4. Control Lyapunov function (CLF) [2]: A continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}$ is a globally and exponentially stabilizing control Lyapunov function (CLF) if there exist constants $c_1 > 0, c_2 > 0, c_3 > 0$ such that

$$c_1||\boldsymbol{x}||^2 \leq V(\boldsymbol{x}) \leq c_2||\boldsymbol{x}||^2 \tag{4}$$

$$\inf_{u\in U} [L_f V(\boldsymbol{x}) + L_g V(\boldsymbol{x})\boldsymbol{u} + c_3 V(\boldsymbol{x})] \leq 0. \tag{5}$$

for $\forall \boldsymbol{x} \in \mathbb{R}^n$.

Given an exponentially stabilizing CLF V, any Lipschitz continuous controller $\boldsymbol{u} \in K_{clf}(\boldsymbol{x})$, with

$$K_{clf}(\boldsymbol{x}) := \{\boldsymbol{u} \in U : L_f V(\boldsymbol{x}) + L_g V(\boldsymbol{x})\boldsymbol{u} + c_3 V(\boldsymbol{x}) \leq 0\},$$

exponentially stabilizes system (1) to its zero dynamics (defined by the dynamics of the internal part if we transform the system to standard form and set the output to zero [9]).

3 PROBLEM FORMULATION AND APPROACH

The merging problem arises when traffic must be joined from two different roads, usually associated with a main lane and a merging lane as shown in Fig. 1. We consider the case where all traffic consists of CAVs randomly arriving from two lanes joined at the Merging Point (MP) M, where collisions may occur. The segments from the origins O and O' to the merging point M are assumed to have the same length L, and are referred to as the Control Zone (CZ). We assume that CAVs do not overtake each other in the CZ. A coordinator is associated with the MP whose function is to maintain a First-In-First-Out (FIFO) queue of CAVs based on their arrival time at the CZ and enable real-time communication with the CAVs that are in the CZ as well as the last one leaving the CZ (see Fig. 1). The FIFO assumption imposed so that CAVs cross the MP in their order of arrival is made for simplicity and often to ensure fairness, but can be relaxed through dynamic resequencing schemes, e.g., as described in [31].

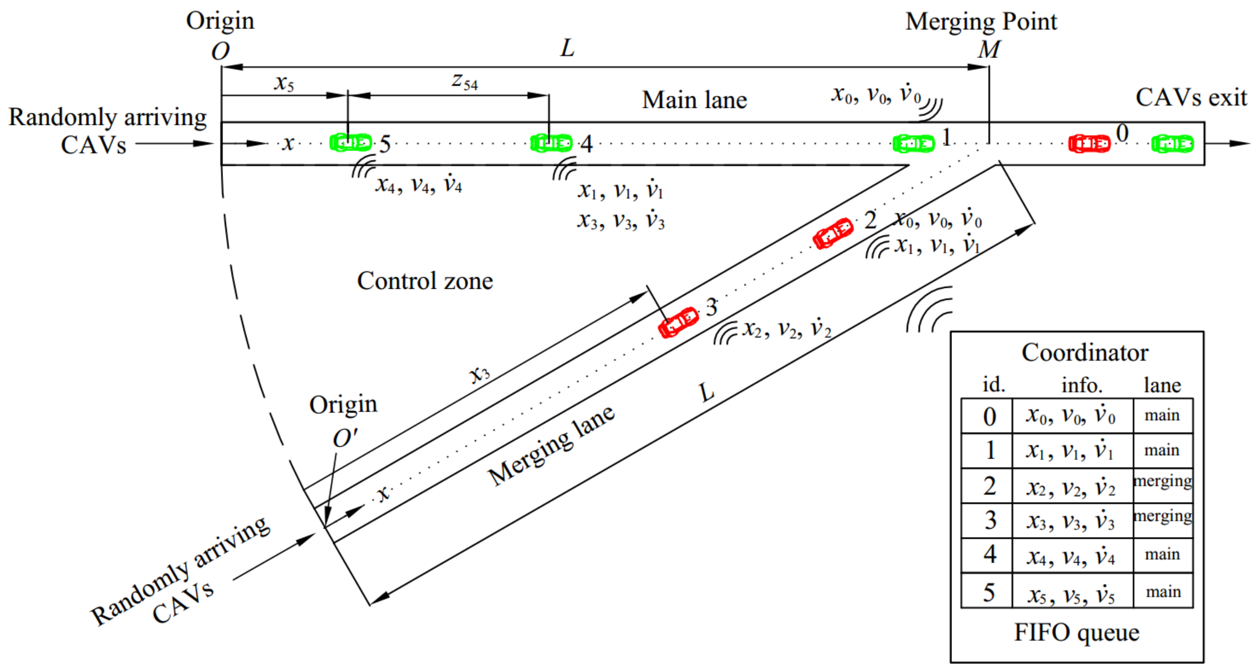
3.1 Vehicle Dynamics

Let $S(t)$ be the set of CAV indices in the CZ at time t, including the vehicle that has just left the CZ (whose index is 0 as shown in Fig. 1). Let $N(t)$ be the cardinality of $S(t)$. Therefore, the next arriving CAV will be assigned index $N(t)$. All the vehicle indices decrease by one when a vehicle leaves the CZ and the vehicle whose index is -1 is dropped.

The vehicle dynamics for each CAV $i \in S(t)$ along the lane to which it belongs take the form

$$m_i \frac{dv_i(t)}{dt} = u_i(t) - F_r(v_i(t)), \tag{6}$$

where $u_i(t)$ is the control input of CAV i, m_i denotes its mass, and $v_i(t)$ is its the velocity. $F_r(v_i(t))$ denotes the resistance force, which



id.	info.	lane
0	$x_0, v_0, \dot{v}_0$	main
1	$x_1, v_1, \dot{v}_1$	main
2	$x_2, v_2, \dot{v}_2$	merging
3	$x_3, v_3, \dot{v}_3$	merging
4	$x_4, v_4, \dot{v}_4$	main
5	$x_5, v_5, \dot{v}_5$	main

Figure 1: The traffic merging problem: CAVs randomly arrive at O and O' and merge at M, where collisions may occur.

is normally expressed [9] as:

$$F_r(v_i(t)) = \alpha_0 sgn(v_i(t)) + \alpha_1 v_i(t) + \alpha_2 v_i^2(t), \quad (7)$$

where $\alpha_0 > 0, \alpha_1 > 0$ and $\alpha_2 > 0$ are scalars determined empirically, and sgn is the signum function. The first term in $F_r(v_i(t))$ denotes the Coulomb friction force, the second term denotes the viscous friction force and the last term denotes the aerodynamic drag.

With $\boldsymbol{x}_i(t) := (x_i(t), v_i(t))$, we can rewrite the vehicle dynamics in standard form:

$$\underbrace{\begin{bmatrix} \dot{x}_i(t) \\ \dot{v}_i(t) \end{bmatrix}}_{\dot{\boldsymbol{x}}_i(t)} = \underbrace{\begin{bmatrix} v_i(t) \\ -\frac{1}{m_i} F_r(v_i(t)) \end{bmatrix}}_{f(\boldsymbol{x}_i(t))} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m_i} \end{bmatrix}}_{g(\boldsymbol{x}_i(t))} u_i(t), \quad (8)$$

where $x_i(t)$ denotes the distance to the origin O (O') along the main (merging) lane if the vehicle i is located in the main (merging) lane.

3.2 Objectives and Constraints

The following objectives and constraints are exactly the same as the ones from [28] to enable the comparison presented later in the paper.

Objective 1 (Minimizing travel time): Let t_i^0 and t_i^m denote the times that CAV $i \in S(t)$ arrive at the origin O or O' and the merging point M, respectively. We wish to minimize the travel time $t_i^m - t_i^0$ for CAV i.

Objective 2 (Minimizing energy consumption): We also want to minimize the energy consumption for each vehicle $i \in S(t)$ expressed as

$$J_i(t_i^m, u_i(t)) = \int_{t_i^0}^{t_i^m} C(u_i(t))dt, \quad (9)$$

where $C(\cdot)$ is a class $\mathcal{K}$ function of its argument.

Constraint 1 (Safety): Let i_p denote the index of the CAV that immediately precedes i in the same lane in the CZ (if one is present). We require that the distance $z_{i,i_p}(t) := x_{i_p}(t) - x_i(t)$ be constrained by the speed $v_i(t)$ of vehicle $i \in S(t)$ so that

$$z_{i,i_p}(t) \geq \varphi v_i(t) + l, \quad \forall t \in [t_i^0, t_i^m], \quad (10)$$

where φ denotes the reaction time (the general rule $\varphi = 1.8$ is used as in [24]). If we define z_{i,i_p} to be the distance from the center of CAV i to the center of CAV i_p, then l is a constant determined by the length of these two CAVs (generally dependent on i and i_p but taken to be a constant over all CAVs for simplicity) .

Constraint 2 (Safe merging): There should be enough safe space at the merging point M, i.e., the distance between the vehicle at M and the preceding one should satisfy:

$$z_{1,0}(t_1^m) \geq \varphi v_1(t_1^m) + l, \quad (11)$$

Constraint 3 (Vehicle limitations): Finally, there are constraints on the speed and control input for each $i \in S(t)$, i.e., $v_i(t) \in [v_{min}, v_{max}]$ and $u_i(t) \in [-c_d m_i g, c_a m_i g]$ for all $t \in [t_i^0, t_i^m]$, where $v_{max} \geq 0$ and $v_{min} \geq 0$ denote the maximum and minimum speeds allowed in the road, $c_d > 0$ and $c_a > 0$ are deceleration and acceleration coefficients, respectively, and g is the gravity constant.

3.3 Problem Formulation

Problem 1. *Our goal is to determine control laws to achieve Objectives 1, 2 subject to Constraints 1, 2, 3, for each vehicle $i \in S(t)$ governed by dynamics (8).*

Constraints 1, 2, 3 are safety critical constraints (or hard constraints) that should be always satisfied. To enforce them, we will use CBFs. Unlike [28] that forms a convex combination explicitly for Objectives 1, 2, we indirectly achieve Objective 1 through a CLF so as to exponentially stabilize the speed $v_i(t)$ to v_{max}. We also capture Objective 2 as the optimization objective function, which is combined implicitly as shown in the next section.

4 PROBLEM REFORMULATION

In this section, we formulate Problem 1 as a nonlinear constrained optimization problem.

Recall that i_p is the index of the CAV that immediately (in the same lane) precedes CAV $i \in S(t)$. We need to distinguish between the following two cases:

- (i) $i_p = i-1$, *i.e.,* i_p is the CAV immediately preceding i in the FIFO queue (such as CAVs 3 and 5 in Fig. 1), and
- (ii) $i_p < i-1$ (such as CAVs 2 and 4 in Fig.1), which implies CAV $i-1$ is in the different lane as i.

We can solve Problem 1 for all $i \in S(t)$ in a decentralized way, in the sense that CAV i can solve Problem 1 using only its own local information (position, velocity and acceleration) along with that of its "neighbor" CAVs $i-1$ and i_p (in case (ii) only). Observe that if $i_p = i-1$, then (11) is a redundant constraint. Otherwise, we need to consider (10) and (11) independently.

4.1 Safety & Vehicle Limitations (Constraints 1 & 3)

We can use CBFs to map the safety constraints and limitations from the state $\boldsymbol{x}_i(t)$ to control input $u_i(t)$. Consider the function $B_{i,q}(\boldsymbol{x}_i(t)) := \frac{1}{h_{i,q}(\boldsymbol{x}_i(t))}$, where $q \in \{1,2,3\}$, $h_{i,1}(\boldsymbol{x}_i(t)) = v_{max} - v_i(t)$, $h_{i,2}(\boldsymbol{x}_i(t)) = v_i(t) - v_{min}$, $h_{i,3}(\boldsymbol{x}_i(t)) = z_{i,i_p}(t) - \varphi v_i(t) - l$. Therefore, in Definition 2.3, we choose $\beta_1(h_{i,q}) = \beta_2(h_{i,q}) = h_{i,q}$, $\gamma = 1$. Then, each $B_{i,q}(\boldsymbol{x}_i(t))$ is a CBF. Any control input $u_i(t)$ should satisfy

$$\underbrace{\frac{-F_r(v_i(t))}{m_i(v_{max}-v_i(t))^2}}_{L_fB_{i,1}(\boldsymbol{x}_i(t))}+\underbrace{\frac{1}{m_i(v_{max}-v_i(t))^2}}_{L_gB_{i,1}(\boldsymbol{x}_i(t))}u_i(t)\le\underbrace{v_{max}-v_i(t)}_{\frac{1}{B_{i,1}(\boldsymbol{x}_i(t))}} \tag{12}$$

$$\underbrace{\frac{F_r(v_i(t))}{m_i(v_i(t)-v_{min})^2}}_{L_fB_{i,2}(\boldsymbol{x}_i(t))}+\underbrace{\frac{-1}{m_i(v_i(t)-v_{min})^2}}_{L_gB_{i,2}(\boldsymbol{x}_i(t))}u_i(t)\le\underbrace{v_i(t)-v_{min}}_{\frac{1}{B_{i,2}(\boldsymbol{x}_i(t))}} \tag{13}$$

$$\underbrace{-\frac{m_i(v_{i_p}(t)-v_i(t))+\varphi F_r(v_i(t))}{m_i(z_{i,i_p}(t)-\varphi v_i(t)-l)^2}}_{L_fB_{i,3}(\boldsymbol{x}_i(t))} + \underbrace{\frac{\varphi}{m_i(z_{i,i_p}(t)-\varphi v_i(t)-l)^2}}_{L_gB_{i,3}(\boldsymbol{x}_i(t))}u_i(t)\le\underbrace{z_{i,i_p}(t)-\varphi v_i(t)-l}_{\frac{1}{B_{i,3}(\boldsymbol{x}_i(t))}} \tag{14}$$

$\forall t \in [t_i^0, t_i^m]$. Note that $u_i(t) \in [-c_d m_i g, c_a m_i g]$ is already a constraint on the control input, hence, we do not need to use the CBF for it.

4.2 Safe Merging (Constraint 2)

We want to avoid collision when CAVs from different lanes arrive at the merging point M. Note that the safe merging constraint (11) is only imposed at t_1^m and does not vary continuously in time. For example, vehicles 4 and 3 in Fig. 1 are not constrained before they arrive at the merging point M, but have to satisfy (11) at the merging point M. In order to use a CBF approach, we need a version of (11) that is continuous in time when $i-1 > i_p$.

Vehicles i and $i-1$ both arrive randomly at O or O', and the minimum distance along the lane $z_{i,i-1}(t_i^0)$ between vehicle i and $i-1$ is 0, i.e., these two CAVs are allowed to arrive at the origin O or O' at the same time. The coordinator FIFO queue preserves the order in which i and $i-1$ arrive at the merging point M according to the order in which they arrive at the origin O or O'. When vehicles i and $i-1$ arrive at the merging point M, they will merge into the same lane. Therefore, we want the distance between vehicle i and $i-1$ to be greater than or equal to $\varphi v_i(t_i^m)$, which is in the form of (11). However, we have some freedom in choosing the reaction time φ from Eqn, (11) for vehicle i $(i-1 > i_p)$ $\forall t \in (t_i^0, t_i^m)$. In the following, we provide a definition for the allowed variation of φ as an approximation to the optimal reaction time variation in [28]:

Definition 4.1. The variation of reaction time φ for vehicle i $(i-1 > i_p)$ is a class $\mathcal{K}$ function $\Phi : \mathbb{R} \to \mathbb{R}$ that satisfies the initial condition $\Phi(x_i(t_i^0)) = -\frac{l}{v_i(t_i^0)}$ and final condition $\Phi(x_i(t_i^m)) = \varphi$.

For example, in Fig. 1, $x_i(t_i^0) = 0$ and $x_i(t_i^m) = L$, *e.g.,* $\Phi(x_i(t)) = \frac{\varphi x_i(t)}{L}$. Examples of variation functions Φ are shown in Fig.2. The lower bound of the distance from Eqn. (11) becomes greater as vehicle i approaches the merging point M such that there is enough space for the vehicle in the merging lane to merge into the main lane. Therefore, a continuous version of the constraint from Eqn. (11) on i for $i-1 > i_p$ in the control zone is:

$$z_{i,i-1}(t) \ge \Phi(x_i(t))v_i(t) + l, \forall t \in [t_i^0, t_i^m]. \tag{15}$$

To enforce safe merging, we employ a control barrier function that is similar to the ones used for safety: $B_{i,4}(\boldsymbol{x}_i(t)) = \frac{1}{h_{i,4}(\boldsymbol{x}_i(t))}$, where $h_{i,4}(\boldsymbol{x}_i(t)) = z_{i,i-1}(t) - \Phi(x_i(t))v_i(t) - l$. Any control input $u_i(t)$ should satisfy

$$\underbrace{-\frac{(v_{i-1}(t)-v_i(t))+\frac{\Phi(x_i(t))}{m_i}F_r(v_i(t))-\dot{\Phi}(x_i(t))v_i(t)}{(z_{i,i-1}(t)-\Phi(x_i(t))v_i(t)-l)^2}}_{L_fB_{i,4}(\boldsymbol{x}_i(t))} + \underbrace{\frac{\Phi(x_i(t))}{m_i(z_{i,i-1}(t)-\Phi(x_i(t))v_i(t)-l)^2}}_{L_gB_{i,4}(\boldsymbol{x}_i(t))}u_i(t)\le\underbrace{z_{i,i-1}(t)-\Phi(x_i(t))v_i(t)-l}_{\frac{1}{B_{i,4}(\boldsymbol{x}_i(t))}} \tag{16}$$

$\forall t \in [t_i^0, t_i^m]$.

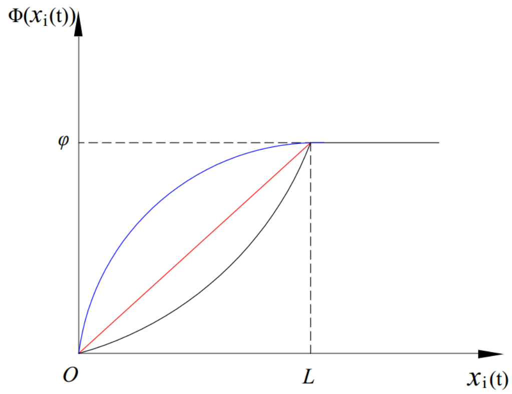


Figure 2: The CAVs reaction time variation between origin O and merging point M for the vehicle i ($i-1 > i_p$). Blue, red and black lines denote three different choices.

4.3 Minimizing Travel Time (Objective 1)

The lower bound to the minimum travel time is achieved when a vehicle runs at v_{max} in the control zone. Therefore, the lower bound is $\frac{L}{v_{max}}$. If the traffic is heavy, the speed of vehicle $i \in S(t)$ is constrained by the physically preceding vehicle i_p. However, we can try to drive the speed to v_{max} as fast as possible such that the travel time is close to the lower bound under heavy traffic, which can be done using a CLF.

We define an output $y_i(t) := v_i(t) - v_{max}$, and choose a CLF $V(y_i(t)) = y_i^2(t)$ with $c_1 = c_2 = 1$ and $c_3 = \epsilon > 0$ in Definition 2.4. Any control input $u_i(t)$ should satisfy

$$\underbrace{-\frac{2(v_i(t) - v_{max})}{m_i}F_r(v_i(t))}_{L_f V(y_i(t))} + \underbrace{\epsilon(v_i(t) - v_{max})^2}_{\epsilon V(y_i(t))} + \underbrace{\frac{2(v_i(t) - v_{max})}{m_i}}_{L_g V(y_i(t))} u_i(t) \leq \delta_i(t) \tag{17}$$

$\forall t \in [t_i^0, t_i^m]$. Here $\delta_i(t)$ denotes a relaxation variable that makes (17) become a soft constraint. We show how to minimize $\delta_i^2(t)$ in Sec. 4.4.

4.4 Decentralized Optimization Problem

To achieve Objective 2, we consider minimizing the square of the CAV's acceleration since energy consumption is proportional to it [8]. Therefore, Problem 1 for CAV $i \in S(t)$ can be formalized as:

$$\boldsymbol{u}_i^*(t) = \underset{\boldsymbol{u}_i(t)}{\arg\min} \left(\frac{u_i(t) - F_r(v_i(t))}{m_i}\right)^2 + p_i \delta_i^2(t) \tag{18}$$

subject to (8), (12)-(14), (17) if $i - 1 = i_p$ and (8), (12)-(14), (16), (17) if $i - 1 > i_p$, $u_i(t) \in [-c_d m_i g, c_a m_i g], \forall i \in S(t)$, where $p_i > 0$ is a penalty coefficient and $\boldsymbol{u}_i(t) := (u_i(t), \delta_i(t))$.

This decentralized problem, which is defined for $\forall t \in [t_i^0, t_i^m], \forall i \in S(t)$, is hard to solve because both the objective and the constraints are nonlinear. In the next section we provide an approximate solution to this problem that is based on partitioning the time interval $[t_i^0, t_i^m]$ and defining the solution as a piecewise constant controller.

5 SOLUTION

We assume that all the constraints are strict at t_i^0 (if not, we can include a feasibility enforcement zone [32]):

Assumption 1: The speed constraint (in Constraint 3) and the safety constraint (10) are not active at t_i^0, $\forall i \in S(t)$. The arrival times for i and $i-1$ at O or O' in case (ii) are not the same, i.e., $z_{i,i-1}(t_i^0) > 0$, if $i - 1 > i_p$.

We begin by partitioning the continuous time interval set $[t_i^0, t_i^m]$ into equal time intervals $\{[t_i^0 + k\Delta t, t_i^0 + (k+1)\Delta t)\}, k = 0, 1, 2, \ldots$. In each interval $[t_i^0 + k\Delta t, t_i^0 + (k+1)\Delta t)$, we assume the control is constant and find a solution to the optimization problem in Eqn. (18). Specifically, at $t = t_i^0 + k\Delta t$ ($k = 0, 1, 2, \ldots$), we solve (18) and update (8) for $\forall t \in (t_i^0 + k\Delta t, t_i^0 + (k+1)\Delta t)$. The workflow is shown in Fig. 3.

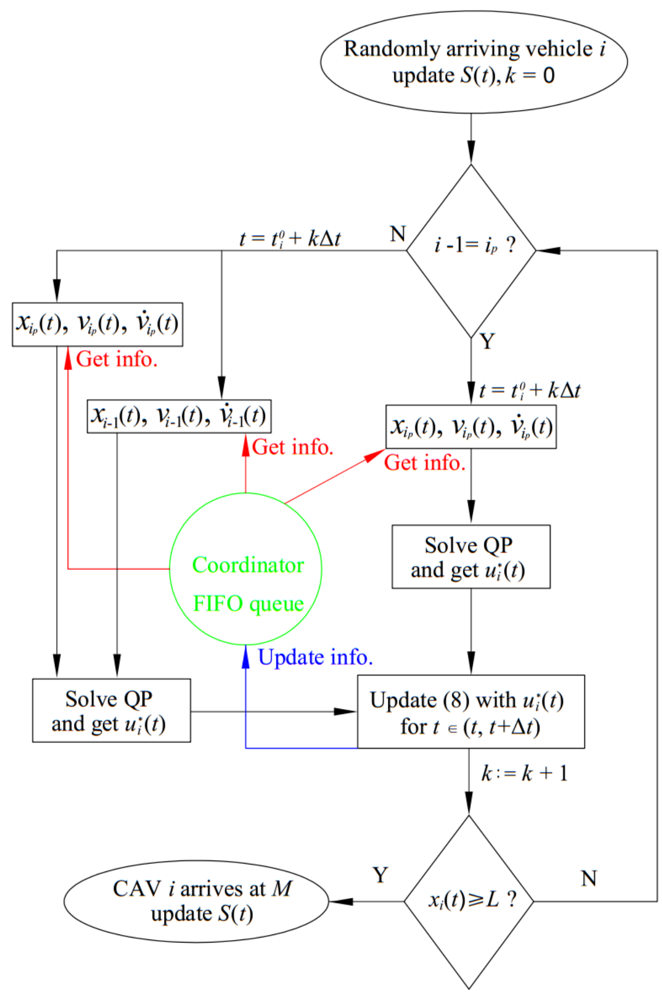


Figure 3: Decentralized problem workflow for CAV i.

For the QP in (18), the constraint set is a polyhedron with $u_i(t)$ and $\delta_i(t)$ as decision variables for each $i \in S(t)$. We need to make sure that this polyhydron is non-empty so that a feasible solution of (18) always exists. Therefore, we make the following assumption:

Assumption 2: c_a, c_d, v_{min} and v_{max} are such that (13), (12) and $u_i(t) \in [-c_d m_i g, c_a m_i g]$ form a full-dimensional (basis) polyhedron P in the $u_i(t)$ - $\delta_i(t)$ space, $\forall i \in S(t), \forall t \in [t_i^0, t_i^m]$.

Because each i has limited control input (Constraint 3), we define a receding horizon in the form of a minimum braking time (distance) horizon to avoid the violation of the control constraint. Under Assumption 2, we only have to deal with the conflicts between (14), (16) and the basis polyhydron P. First, consider (16) that may conflict with $u_i(t) \in [-c_d m_i g, c_a m_i g]$. Vehicle $i \in S(t)$ should stop at least at a safe distance ($z_{i,i-1}(t+T) \geq \Phi(x_i(t+T))v_i(t+T) + l = \Phi(x_i(t+T))v_{i-1}(t+T) + l$) when its speed $v_i(t+T)$ approaches the speed $v_{i-1}(t+T)$ of vehicle $i-1$ (where T denotes the minimum braking time under the maximum braking $u_i(t) = -c_d m_i g$)[4]. Suppose we choose $\Phi(x_i(t)) = \frac{\varphi x_i(t)}{L}$ in this case (Let $l = 0$ for simplicity).

It is safe to drop the resistance force term in (6) since $F_r(v_i(t))$ can only make the braking distance shorter. Assuming $v_{i-1}(t)$ does not change during $[t, t+T]$ when calculating the minimum braking time T, we have $z_{i,i-1}(t+T) \geq \Phi(x_i(t+T))v_{i-1}(t) + l$. The braking distance should be taken into consideration only if $v_i(t) \geq v_{i-1}(t)$. Therefore, $z_{i,i-1}(t+T) \geq \Phi(x_i(t+T))v_i(t) + l \Rightarrow z_{i,i-1}(t+T) \geq \Phi(x_i(t+T))v_{i-1}(t) + l$.

The minimum braking time is

$$v_i(t+T) - v_i(t) = v_{i-1}(t) - v_i(t) = -Tc_d g. \tag{19}$$

Then, we have

$$T = \frac{v_{i-1}(t) - v_i(t)}{-c_d g}. \tag{20}$$

The distance between vehicle i and $i-1$ after time T is

$$\begin{aligned} z_{i,i-1}(t+T) &= z_{i,i-1}(t) + \int_0^T v_{i-1}(t) - v_i(t+\tau)d\tau \\ &= z_{i,i-1}(t) + \int_0^T v_{i-1}(t) - v_i(t) + \tau c_d g d\tau \\ &= z_{i,i-1}(t) - \frac{1}{2}\frac{(v_{i-1}(t) - v_i(t))^2}{c_d g}. \end{aligned} \tag{21}$$

The distance travelled by vehicle i after T is:

$$x_i(t+T) - x_i(t) = v_i(t)T - \frac{1}{2}c_d g T^2 = \frac{1}{2}\frac{v_i^2(t) - v_{i-1}^2(t)}{c_d g}. \tag{22}$$

After maximum braking, the distance should still be

$$z_{i,i-1}(t+T) \geq \frac{\varphi(x_i(t) + \frac{1}{2}\frac{v_i^2(t)-v_{i-1}^2(t)}{c_d g})v_i(t)}{L} + l. \tag{23}$$

Therefore,

$$z_{i,i-1}(t) \geq \frac{\varphi(x_i(t) + \frac{1}{2}\frac{v_i^2(t)-v_{i-1}^2(t)}{c_d g})v_i(t)}{L} + \frac{1}{2}\frac{(v_{i-1}(t) - v_i(t))^2}{c_d g} + l. \tag{24}$$

Then we can use a CBF $B_{i,5}(\boldsymbol{x}_i(t)) = \frac{1}{h_{i,5}(\boldsymbol{x}_i(t))}$ with $h_{i,5}(\boldsymbol{x}_i(t))$ defined as

$$\begin{aligned} h_{i,5}(\boldsymbol{x}_i(t)) &= z_{i,i-1}(t) - \frac{1}{2}\frac{(v_{i-1}(t) - v_i(t))^2}{c_d g} \\ &\quad - \frac{\varphi(x_i(t) + \frac{1}{2}\frac{v_i^2(t)-v_{i-1}^2(t)}{c_d g})v_i(t)}{L} - l. \end{aligned} \tag{25}$$

Then, the control input $u_i(t)$ should satisfy

$$L_f B_{i,5}(\boldsymbol{x}_i(t)) + L_g B_{i,5}(\boldsymbol{x}_i(t))u_i(t) \leq \frac{1}{B_{i,5}(\boldsymbol{x}_i(t))}. \tag{26}$$

Similarly, we can also use a CBF $B_{i,6}(\boldsymbol{x}_i(t)) = \frac{1}{h_{i,6}(\boldsymbol{x}_i(t))}$ with $h_{i,6}(\boldsymbol{x}_i(t))$ defined as

$$h_{i,6}(\boldsymbol{x}_i(t)) = z_{i,i_p}(t) - \frac{1}{2}\frac{(v_{i_p}(t) - v_i(t))^2}{c_d g} - \varphi v_i(t) - l \tag{27}$$

for (14) when considering the conflict with $u_i(t) \in [-c_d m_i g, c_a m_i g], \forall i \in S(t)$. The control input $u_i(t)$ should satisfy

$$L_f B_{i,6}(\boldsymbol{x}_i(t)) + L_g B_{i,6}(\boldsymbol{x}_i(t))u_i(t) \leq \frac{1}{B_{i,6}(\boldsymbol{x}_i(t))}. \tag{28}$$

Then at time $t = t_i^0 + k\Delta t$ ($k = 0, 1, 2, \ldots$), we can form a new feasible QP:

$$\underset{t=t_i^0+k\Delta t}{\textbf{QP:}} \quad \boldsymbol{u}_i^*(t) = \underset{\boldsymbol{u}_i(t)}{\arg\min} \frac{1}{2}\boldsymbol{u}_i(t)^T H \boldsymbol{u}_i(t) + F^T \boldsymbol{u}_i(t) \tag{29}$$

$$\boldsymbol{u}_i(t) = \begin{bmatrix} u_i(t) \\ \delta_i(t) \end{bmatrix}, H = \begin{bmatrix} \frac{2}{m_i^2} & 0 \\ 0 & 2p_i \end{bmatrix}, F = \begin{bmatrix} \frac{-2F_r(v_i(t))}{m_i^2} \\ 0 \end{bmatrix}$$

subject to

$$A_{\text{clf}}\boldsymbol{u}_i(t) \leq b_{\text{clf}}, \text{ if } i \in S(t)$$
$$A_{\text{cbf\_lim}}\boldsymbol{u}_i(t) \leq b_{\text{cbf\_lim}}, \text{ if } i \in S(t)$$
$$A_{\text{cbf\_safety}}\boldsymbol{u}_i(t) \leq b_{\text{cbf\_safety}}, \text{ if } i \in S(t)$$
$$A_{\text{cbf\_merge}}\boldsymbol{u}_i(t) \leq b_{\text{cbf\_merge}}, \text{ if } i-1 > i_p$$

where the constraint parameters are

$$\begin{aligned} A_{\text{clf}} &= [L_g V(y_i(t)), \quad -1], \\ b_{\text{clf}} &= -L_f V(y_i(t)) - \epsilon V(y_i(t)). \end{aligned} \tag{30}$$

$$\begin{aligned} A_{\text{cbf\_lim}} &= \begin{bmatrix} L_g B_{i,1}(\boldsymbol{x}_i(t)), & 0 \\ L_g B_{i,2}(\boldsymbol{x}_i(t)), & 0 \\ 1, & 0 \\ -1, & 0 \end{bmatrix}, \\ b_{\text{cbf\_lim}} &= \begin{bmatrix} -L_f B_{i,1}(\boldsymbol{x}_i(t)) + \frac{1}{B_{i,1}(\boldsymbol{x}_i(t))} \\ -L_f B_{i,2}(\boldsymbol{x}_i(t)) + \frac{1}{B_{i,2}(\boldsymbol{x}_i(t))} \\ c_a m_i g \\ c_d m_i g \end{bmatrix}. \end{aligned} \tag{31}$$

$$\begin{aligned} A_{\text{cbf\_safety}} &= \begin{bmatrix} L_g B_{i,3}(\boldsymbol{x}_i(t)), & 0 \\ L_g B_{i,6}(\boldsymbol{x}_i(t)), & 0 \end{bmatrix}, \\ b_{\text{cbf\_safety}} &= \begin{bmatrix} -L_f B_{i,3}(\boldsymbol{x}_i(t)) + \frac{1}{B_{i,3}(\boldsymbol{x}_i(t))} \\ -L_f B_{i,6}(\boldsymbol{x}_i(t)) + \frac{1}{B_{i,6}(\boldsymbol{x}_i(t))} \end{bmatrix}. \end{aligned} \tag{32}$$

$$A_{\text{cbf\_merge}} = \begin{bmatrix} L_g B_{i,4}(\boldsymbol{x}_i(t)), & 0 \\ L_g B_{i,5}(\boldsymbol{x}_i(t)), & 0 \end{bmatrix},$$
$$b_{\text{cbf\_merge}} = \begin{bmatrix} -L_f B_{i,4}(\boldsymbol{x}_i(t)) + \frac{1}{B_{i,4}(\boldsymbol{x}_i(t))} \\ -L_f B_{i,5}(\boldsymbol{x}_i(t)) + \frac{1}{B_{i,5}(\boldsymbol{x}_i(t))} \end{bmatrix}. \quad (33)$$

Remark 1: We have conducted an approximate calculation when calculating the minimum braking time T by assuming $v_{i-1}(t)$ does not change during $[t, t+T]$. This can still work well as long as Δt is chosen small enough to handle the error due to the approximation.

The following theorem ensures the existence of a choice for the simulation time Δt to ensure the feasiblity of (29) due to the discretization of (18).

Theorem 5.1. *Under* Assumptions 1 and 2*,* $\exists \delta > 0, 0 < \Delta t < \delta$*, such that* $h_{i,q}(\boldsymbol{x}_i(t)) > 0, \forall q \in \{1,2,3,4,5,6\}, \forall i \in S(t), \forall t \in [t_i^0, t_i^m]$.

Proof. *Assumption 1* ensured that all functions $h_{i,q}(\boldsymbol{x}_i(t)), q \in \{1,2,3,4,5,6\}$ are greater than 0 at $t_i^0, \forall i \in S(t)$, and *Assumption 2* together with (26), (28) make the decentralized problem feasible. We solve (29) at each $t = t_i^0 + k\Delta t$ ($k = 0, 1, 2, \dots$), and then we update (8) for $\forall t \in (t_i^0 + k\Delta t, t_i^0 + (k+1)\Delta t)$ with the $u_i^*(t_i^0 + k\Delta t)$ solved from (29). System (8) will be "uncontrolled" for $\forall t \in (t_i^0 + k\Delta t, t_i^0 + (k+1)\Delta t)$ because we have a constant control input $u_i^*(t_i^0 + k\Delta t)$.

From [4], we know that CBFs are allowed to grow and decrease. Suppose we have a initial $\Delta t > 0$, If the $u_i^*(t_i^0 + k\Delta t)$ we get from (29) happens to make $L_f B_{i,q}(\boldsymbol{x}_i(t)) + L_g B_{i,q}(\boldsymbol{x}_i(t)) u_i^*(t_i^0 + k\Delta t) \le 0$ for $\forall t \in [t_i^0 + k\Delta t, t_i^0 + (k+1)\Delta t)$, i.e., $\dot{B}_{i,q}(\boldsymbol{x}_i(t)) \le 0$, since $B_{i,q}(\boldsymbol{x}_i(t)) = \frac{1}{h_{i,q}(\boldsymbol{x}_i(t))}$, then $h_{i,q}(\boldsymbol{x}_i(t)) \ge h_{i,q}(\boldsymbol{x}_i(t_i^0 + k\Delta t)) > 0$ for $\forall t \in [t_i^0 + k\Delta t, t_i^0 + (k+1)\Delta t)$. $B_{i,q}(\boldsymbol{x}_i(t))$ may grow during some time intervals and decrease during others. If $h_{i,q}(\boldsymbol{x}_i(t))$ never goes to zero during the "uncontrolled" time interval, $h_{i,q}(\boldsymbol{x}_i(t)) > 0, \forall t \in [t_i^0, t_i^m]$. Otherwise, by the continuity of (8), there will be a time $\delta_{i,q,k} > 0$ that the function $h_{i,q}(\boldsymbol{x}_i(t))$ will reach 0 during each $[t_i^0 + k\Delta t, t_i^0 + (k+1)\Delta t)$. Let $\delta = \min_{i \in S(t)} \min_{q=1}^{6} \min_{k=0}^{t_i^0 + k\Delta t \le t_i^m} \delta_{i,q,k}$. If we choose a new $0 < \Delta t < \delta$, then $h_{i,q}(\boldsymbol{x}_i(t)) > 0$ for $\forall i \in S(t), \forall q \in \{1,2,3,4,5,6\}, \forall t \in [t_i^0, t_i^m]$. □

Remark 2: For $i = 0$, we can just set $z_{i,i_p} = \infty$ if there are no restrictions on the first vehicle in the FIFO queue. Theorem 5.1 cannot ensure that the zeno behavior is prevented, and we will do future study about the zeno behavior in future work. In general, the CBFs will always make $h_{i,q}(\boldsymbol{x}_i(t)) > 0$ if $h_{i,q}(\boldsymbol{x}_i(t))$ is initially positive. If we choose Δt small enough such that the CBFs can regulate the control input $u_i(t)$ according to the state changes of itself and other vehicles, the set C of each CBF will be rendered forward invariant. We may even choose a different Δt for each step k to make the computation more efficient.

Complexity: The time complexity of QP (active-set method) is polynomial in the dimension of decision variables on average. In general, the complexity is $O(n^3)$, where n denotes the dimension of the decision variable space. In the merging problem, if the number of time intervals is N_i for CAV i, then the complexity for CAV i is $O(N_i n^3)$ in the control zone. The complexity for the overall merging problem is $O(S(t)n^3)$ at time t. In our case, $n = 2$. The time consumption should be fast enough for real time application, and the time consumption in MATLAB to get optimal control for (29) at each step is less than $0.01s$ (Intel(R) Core(TM) i7-8700 CPU @ 3.2GHz 3.2GHz). We may consider the state errors that are due to the computation time as noise to be studied in future work.

6 IMPLEMENTATION AND CASE STUDIES

We implemented the method described above in MATLAB. We used quadprog for solving QPs and ode45 to integrate dynamics. In our implementation, all the CAVs solve (29) and update (8) independently, and they are connected to a coordinator.

We considered the scenario shown in Fig. 1 with cars arriving randomly (the arriving is sampled from a uniform distribution over (0,1) and determined by the arrival rate) at the origins O and O', respectively. We assume that all vehicles enter the control zone with the same speed 20m/s, *i.e.,* the initial conditions are $(x_i(t_i^0), v_i(t_i^0)) = (0, 20)$, for all $i \in S(t)$. The parameters for (29) and (8) are given in Table 1.

In order to emphasize the main features of the proposed method, we provide four comparisons. First, we compare the results of our method with those produced by the *Intersection Method* [11]. Second, we change the cost from (29) to a linear cost and compare our QP results with the ones obtained from the corresponding LP. Third, we compare our CBF approach with the optimal control approach from [28]. Fourth, we compare our results with those obtained using the off-the-shelf tool for simulation of traffic networks Vissim, which is based on the model from [27].

In order to provide meaningful comparisons, we use the fuel consumption metamodel proposed in [8], which is a function of speed $v_i(t)$ and acceleration $a_i(t) := \frac{u_i(t) - Fr(v_i(t))}{m_i}$. The model is defined as

$$\dot{f}_v(t) = \dot{f}_{cruise}(t) + \dot{f}_{accel}(t), \quad (34)$$

where

$$\dot{f}_{cruise}(t) = \omega_0 + \omega_1 v_i(t) + \omega_2 v_i^2(t) + \omega_3 v_i^3(t),$$
$$\dot{f}_{accel}(t) = (r_0 + r_1 v_i(t) + r_2 v_i^2(t)) a_i(t),$$

and $\omega_0, \omega_1, \omega_2, \omega_3, r_0, r_1$ and r_2 are positive coefficients (we used the values reported in [8]). The unit of fuel consumption is in milliliter (mL). It is assumed that during braking from a high velocity when $a_i(t) < 0$, no fuel is consumed.

Comparison with the Intersection Method [11]: The approach proposed in [11] has a constant safety constraint for all CAVs. We set this constant as the value of the safety constraint under maximum speed in this paper because CAVs tend to reach v_{max} when they arrive at M in [11]. The simulations are conducted simultaneously with the same input and all parameters are the same.

We calculate the average fuel consumption and travel time for all vehicles that pass over the merging point M under four different traffic conditions (light traffic, arrival rate = 0.01), normal traffic (arrival rate =0.05), heavy traffic (arrival rate =0.10) and very heavy traffic (arrival rate =0.15)). The results are shown in Figs. 4 and 5.

The average travel time of our method is close to [11] when traffic is very light, but performs a little better in both metrics, and the improvement becomes greater as the traffic load increases. In short, this means our method can handle more heavy traffic and results in lower fuel consumption.

Table 1: Simulation parameters

Parameter	Value	Units
φ	1.8	s
$\Phi(x_i(t))$	$\varphi x_i(t)/L$	s
L	400	m
l	0	m
m_i	1650	kg
g	9.81	m/s^2
α_0	0.1	N
α_1	5	Ns/m
α_2	0.25	Ns^2/m
v_{max}	30	m/s
v_{min}	0	m/s
Δt	0.1	s
ϵ	10	unitless
c_a	0.4	unitless
c_d	0.6	unitless
p_i	1	unitless

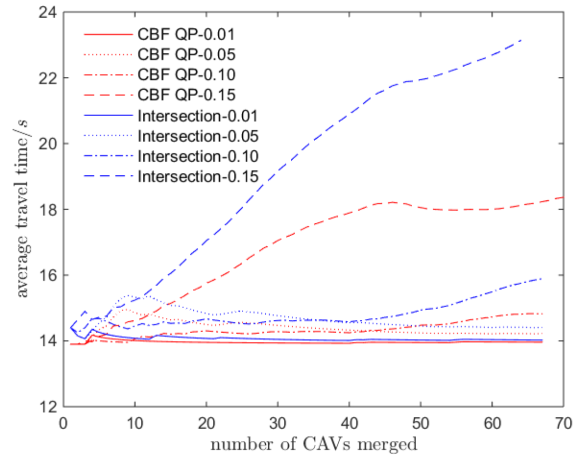


Figure 5: Travel time comparison with the Intersection Method [11]

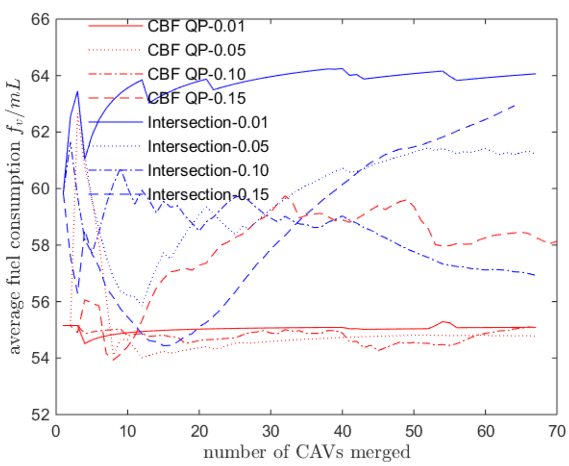


Figure 4: Fuel consumption comparison with the Intersection Method [11]

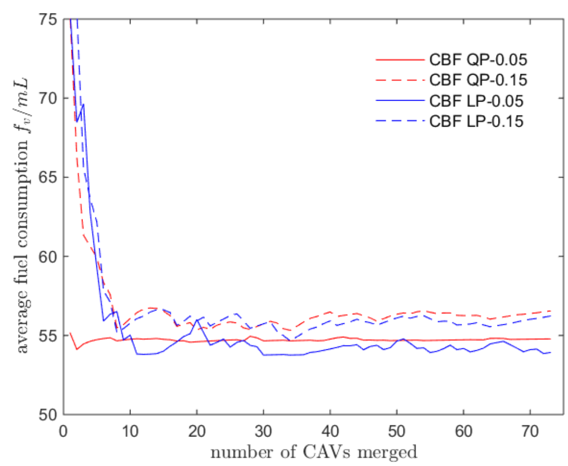


Figure 6: QP and LP fuel consumption comparison

Comparison between quadratic and linear costs: The objective function (18) may not be the best form for the fuel model (34). Therefore, we can also define a feasible LP by just replacing (18) with

$$\boldsymbol{u}_i^*(t) = \underset{\boldsymbol{u}_i(t)}{\arg\min} \, |u_i(t) - F_r(v_i(t))| + p_i \delta_i(t) \tag{35}$$

subject to the same constraints as in the original QP ($\delta_i(t) \geq 0$).

We compared the simulation results between the QP and the LP methods under two traffic conditions, as shown in Figs. 6 and 7. The LP tends to have a little more travel time than the QP, but it consumes less fuel, which is consistent with the fact that LP objective form (35) is closer to the fuel model (34). The LP is solved using *linprog* (active-set) in MATLAB. The runtime ratio between the QP and the LP methods is 1.3.

Comparison with [28] and with Vissim: Finally, we used the Vissim microscopic multi-model traffic flow simulation tool as a baseline to compare our CBF-based method with the optimal control approach from [28]. The car following model in Vissim is based on [27] and it simulates human psycho-physiological driving behavior. In this framework, the vehicles in the merging lane give priority to the main lane vehicles when there is conflict at the merging point.

The simulation results are shown in Table 2 (OC denotes the optimal control method from [28], CBF-QP stands for the approach from this paper, and $\alpha \in [0, 1]$ denotes the weight factor on travel time with respect to energy consumption). We need to consider the trade-off between time and energy consumption. The CBF method results in much lower travel time than Vissim. The fuel consumption in CBF is greater than Vissim. However, this can be due to the different forms of objective functions ((18) compared to (34)). If

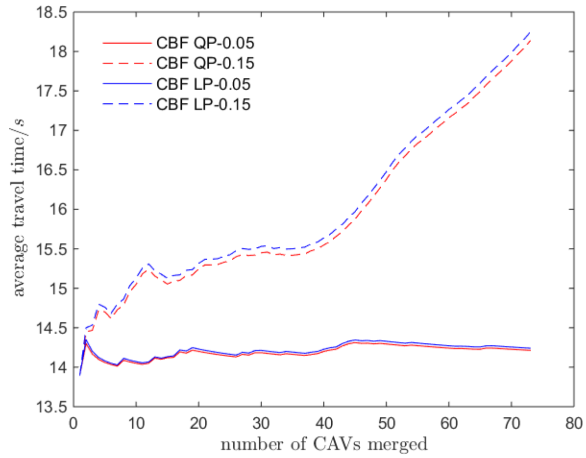


Figure 7: QP and LP travel time comparison

Table 2: Comparison among the method from this paper (CBF-QP), the optimal control approach from [28] (OC), and Vissim ($u_i(t)$ denotes the acceleration for all the methods).

Items	CBF-QP	OC		Vissim
Weight		α=0.26	α=0.41	
Ave. time(s)	**14.6978**	17.1989	15.3132	25.0813
Main time(s)	**14.7000**	17.2109	15.3261	17.9935
Merg. time(s)	**14.6956**	17.1867	15.3000	32.3267
Ave. $\frac{1}{2}u_i^2(t)$	26.2678	**4.9517**	10.7603	20.0918
Main $\frac{1}{2}u_i^2(t)$	26.9178	**4.9027**	10.6644	9.4066
Merg. $\frac{1}{2}u_i^2(t)$	25.6034	**5.0018**	10.8583	31.0144
Ave. fuel(mL)	57.9532	47.6372	67.2234	**36.9954**
Main fuel(mL)	57.7028	47.6971	67.0743	**42.6925**
Merg. fuel(mL)	58.2092	47.5759	67.3757	**31.1717**

we look at both travel time and fuel consumption (i.e., the tradeoff between these two metrics), the CBF method is better since it consumes half of the travel time as Vissim (α can be viewed as 1 since we significantly care travel time and drive $v_i(t)$ to v_{max} as fast as possible). The CBF method tends to favor travel time compared with the OC method, but the fuel consumption may be worse depending on the weight factor α in the OC method.

The results for the method proposed in this paper are only preliminary as we can see that the CBF method is too aggresive in travel time, and thus, has bigger average $\frac{1}{2}u_i^2(t)$ compared with OC. We will study further how to relax the travel time and make the CBF method approach the OC method in objectives 1-2 under any α values as in [28]. This allows us to further investigate the tradeoff between travel time and energy consumption.

7 CONCLUSIONS & FUTURE WORK

In this paper, we brought together control barrier functions (CLF), control Lyapunov functions (CLF), and optimal control to provide an efficient solution to the merging problem in traffic networks. Our approach provides a decentralized implementation that requires minimal communication with a coordinator in a control zone surrounding the merging point. The complexity of our algorithm is linear in the number of CAVs in the control zone. The calculation can be performed on each vehicle individually, which makes this method expandable to large traffic networks. We compared our method with a recently developed optimal control approach and with an off-the-shelf tool for simulation of traffic networks. The results show that our approach provides significant improvements in terms of fuel consumption and travel time.

In future work, we plan to consider different choices of the variation in reaction time, and study their effect on the results of the optimization problem. We will also consider more sophisticated policies to manage the queue (not necessarily FIFO) in order to accommodate different traffic rates and possibly different lane priorities. Finally, we will extend these results to roads with multiple lanes, where we will consider lane switching strategies to avoid traffic congestion.

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