

A Decentralized Optimal Control Framework for Connected Automated Vehicles at Urban Intersections with Dynamic Resequencing

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Abstract—Earlier work has established a decentralized framework to optimally control Connected Automated Vehicles (CAVs) crossing an urban intersection without using explicit traffic signaling while following a strict First-In-First-Out (FIFO) queueing structure. The proposed solution minimizes energy consumption subject to a FIFO-based throughput maximization requirement. In this paper, we extend the solution to account for asymmetric intersections by relaxing the FIFO constraint and including a dynamic resequencing process so as to maximize traffic throughput. To investigate the tradeoff between throughput maximization and energy minimization objectives, we exploit several alternative problem formulations. In addition, the computational complexity of the resequencing process is analyzed and proved to be bounded, which makes the online implementation computationally feasible. The effectiveness of the dynamic resequencing process in terms of throughput maximization is illustrated through simulation.

I. INTRODUCTION

To date, traffic light control is the prevailing method for controlling the traffic flow in an urban area. Recent technological developments (e.g., [1]) have exploited data-driven control and optimization approaches and enabled the adaptive control of traffic light cycles, which reduces the travel delay. However, in addition to the obvious infrastructure cost, safety issues, e.g., rear-end collisions, often arise under traffic light control. These issues have motivated research efforts to explore new approaches capable of enabling a smoother traffic flow while also improving safety.

Connected and Automated Vehicles (CAVs) have the potential to drastically improve a transportation network's performance by assisting drivers in making better decisions, ultimately reducing energy consumption, air pollution, congestion and accidents. One of the very early efforts exploiting the benefit of CAVs was proposed in [2], where an optimal linear feedback regulator is introduced for the merging problem to control a single string of vehicles. More recently, several research efforts have been reported in the literature for CAV coordination at intersections. Dresner and Stone [3] proposed a reservation-based scheme for centralized automated vehicle intersection management. Since then, numerous research efforts have explored safe and efficient control strategies, e.g., [4]–[6]. Some approaches have focused on coordinating vehicles so as to reduce travel

delay and increase intersection throughput, e.g., [7]–[9] and some have studied intersections as polling systems [10] so as to determine a sequence of times assigned to vehicles on each road. Reducing energy consumption is one of the desired objectives which has been considered in recent literature [11]–[14]. A detailed discussion of the overall research in this area can be found in [15].

Our earlier work [16] has established a decentralized optimal control framework for coordinating on line a continuous flow of CAVs crossing an urban intersection without using explicit traffic signaling. For each CAV, an energy minimization optimal control problem is formulated where the time to cross the intersection is first determined through a throughput maximization problem. We also established conditions under which feasible solutions to the optimal control problem exist.

The crossing sequence for the CAVs based on which the throughput maximization problem in [17] is formulated adopts a strict First-In-First-Out (FIFO) queueing structure. This can be effective when the intersection is physically symmetrical and the vehicle arrival rates at all intersection entries do not differ much. However, when the intersection is asymmetrical, the FIFO queueing structure may lead to poor scheduling and possible congestion. Even with a fully symmetrical intersection, a strict FIFO crossing sequence is conservative in the sense that it prevents the intersection from further exploiting the benefits of CAVs and achieving traffic throughput maximization. Hence, it is necessary to design a coordination approach for CAVs to maximize the traffic throughput. Zohdy et al. [18] presented an approach based on Cooperative Adaptive Cruise Control (CACC) for minimizing intersection delay and hence maximizing the throughput. Lee and Park [19] considered minimizing the overlap between vehicle positions. In this paper, we extend the optimal control solution in [17] by relaxing the FIFO constraint and including a dynamic resequencing process so as to maximize traffic throughput.

The paper is structured as follows. In Section II, we review the model in [16] and its generalization in [17]. In Section III, we extend the solution in [17] by relaxing the FIFO constraint and including a dynamic resequencing process so as to maximize traffic throughput. We consider several alternative problem formulations in order to investigate the tradeoff between throughput maximization and energy minimization objectives. In Section IV, we analyze the computational complexity of the resequencing process and show it to be bounded and, on average, limited to the number of lanes in the intersection. Conclusions and future

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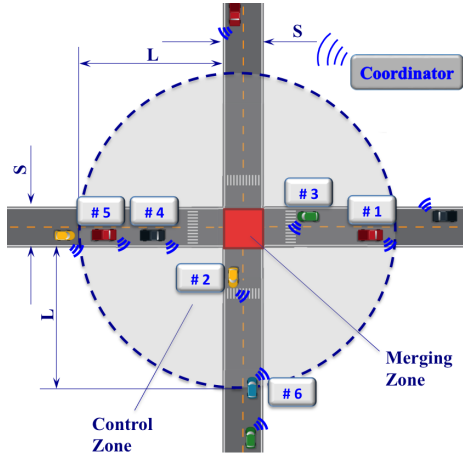


Fig. 1: Connected Automated Vehicles crossing an urban intersection.

work are given in Section V.

II. THE MODEL

The model introduced in [16] and [17] is briefly reviewed. We consider an intersection (Fig. 1) where the region at the center of each intersection, called *Merging Zone* (MZ), is the area of potential lateral CAV collision and assumed to be a square of side S . The intersection has a *Control Zone* (CZ) and a coordinator that can communicate with the CAVs traveling within it. The road segment from the CZ entry to the CZ exit (i.e., the MZ entry) is referred as a CZ segment. The length of CZ segment is $L > S$, and it is assumed to be the same for all entry points to a given CZ.

Let $N(t) \in \mathbb{N}$ be the cumulative number of CAVs which have entered the CZ by time t and formed a queue that designates the crossing sequence in which these CAVs will enter the MZ. There is a number of ways to form the queue. In [16] and [17], a strict First-In-First-Out (FIFO) crossing sequence is assumed, that is, when a CAV reaches the CZ, the coordinator assigns it an integer value $i = N(t) + 1$. If two or more CAVs enter a CZ at the same time, then the corresponding coordinator selects randomly the first one to be assigned the value $N(t) + 1$.

For simplicity, we assume that each CAV is governed by a second order dynamics

$$\dot{p}_i = v_i(t), \quad p_i(t_i^0) = 0; \quad \dot{v}_i = u_i(t), \quad v_i(t_i^0) \text{ given} \quad (1)$$

where $p_i(t) \in \mathcal{P}_i$, $v_i(t) \in \mathcal{V}_i$, and $u_i(t) \in \mathcal{U}_i$ denote the position, i.e., travel distance since the entry of the CZ, speed and acceleration/deceleration (control input) of each CAV i . The sets \mathcal{P}_i , \mathcal{V}_i and \mathcal{U}_i are complete and totally bounded sets of \mathbb{R} . These dynamics are in force over an interval $[t_i^0, t_i^f]$, where t_i^0 and t_i^f are the times that the vehicle i enters the CZ and exits the MZ respectively.

To ensure that the control input and vehicle speed are within a given admissible range, the following constraints are imposed:

$$\begin{aligned} u_{i,min} \leq u_i(t) \leq u_{i,max}, \quad \text{and} \\ 0 \leq v_{min} \leq v_i(t) \leq v_{max}, \quad \forall t \in [t_i^0, t_i^f]. \end{aligned} \quad (2)$$

Definition 1: Depending on its physical location inside the CZ, CAV $i - 1 \in \mathcal{N}(t)$ belongs to only one of the following four subsets of $\mathcal{N}(t)$ with respect to CAV i : 1) $\mathcal{R}_i(t)$ contains all CAVs traveling on the same road as i and towards the same direction but on different lanes, 2) $\mathcal{L}_i(t)$ contains all CAVs traveling on the same road and lane as vehicle i (e.g., $\mathcal{L}_5(t)$ contains CAV #4 in Fig. 1), 3) $\mathcal{C}_i(t)$ contains all CAVs traveling on different roads from i and having destinations that can cause collision at the MZ (e.g., $\mathcal{C}_6(t)$ contains CAV #5 in Fig. 1), and 4) $\mathcal{O}_i(t)$ contains all CAVs traveling on the same road as i and opposite destinations that cannot, however, cause collision at the MZ (e.g., $\mathcal{O}_4(t)$ contains CAV #3 in Fig. 1).

To ensure the absence of any rear-end collision throughout the CZ, we impose the *rear-end safety constraint*:

$$s_i(t) = p_k(t) - p_i(t) \geq \delta, \quad \forall t \in [t_i^0, t_i^f], \quad k \in \mathcal{L}_i(t) \quad (3)$$

where k is the CAV physically ahead of i on the same lane, $s_i(t)$ is the inter-vehicle distance between i and k , and δ is the *minimal safety following distance* allowable.

A lateral collision involving CAV i may occur only if some CAV $j \neq i$ belongs to $\mathcal{C}_i(t)$. This leads to the following definition:

Definition 2: For each CAV $i \in \mathcal{N}(t)$, we define the set Γ_i that includes all time instants when a lateral collision involving CAV i is possible: $\Gamma_i \triangleq \{t \mid t \in [t_i^m, t_i^f]\}$, where t_i^m is the time that CAV i enters the MZ. Consequently, to avoid a lateral collision for any two vehicles $i, j \in \mathcal{N}(t)$ on different roads, the following constraint should hold

$$\Gamma_i \cap \Gamma_j = \emptyset, \quad \forall t \in [t_i^m, t_i^f], \quad j \in \mathcal{C}_i(t). \quad (4)$$

As part of safety considerations, we impose the following assumptions: For CAV i , none of the constraints (2)-(3) is active at t_i^0 . The speed of the CAVs inside the MZ is constant, i.e., $v_i(t) = v_i^m, \quad \forall t \in [t_i^m, t_i^f]$. This implies that $t_i^f = t_i^m + \frac{S}{v_i^m}$. Each CAV i has proximity sensors and can measure local information without errors or delays.

The objective of each CAV is to derive an optimal acceleration/deceleration profile, in terms of minimizing energy consumption, inside the CZ while avoiding congestion between the two intersections. Since the coordinator is not involved in any decision making process on the vehicle control, we can formulate $N(t)$ decentralized tractable problems that can be solved online, that is,

$$\begin{aligned} \min_{u_i \in \mathcal{U}_i} \frac{1}{2} \int_{t_i^0}^{t_i^m} K_i \cdot u_i^2 dt \\ \text{subject to : (1), (2), } t_i^m, p_i(t_i^0) = 0, \\ p_i(t_i^m) = L, \text{ and given } t_i^0, v_i(t_i^0), \end{aligned} \quad (5)$$

where K_i is a factor to capture CAV diversity ($K_i = 1$ for simplicity). Note that the terminal speed v_i^m is undefined and obtained from the energy minimization problem.

The terminal times for CAVs entering the MZ, i.e., t_i^m , can be obtained as the solution to a throughput maximization problem [17] based on a FIFO crossing sequence subject to

rear-end and lateral collision avoidance constraints inside the MZ.

An analytical solution of problem (5) may be obtained through a Hamiltonian analysis [17]. Assuming that all constraints are satisfied upon entering the CZ and that they remain inactive throughout $[t_i^0, t_i^m]$, the optimal control input (acceleration/deceleration) over $t \in [t_i^0, t_i^m]$ is given by

$$u_i^*(t) = a_i t + b_i \quad (6)$$

where a_i and b_i are constants of integration, which can be determined given initial and terminal conditions. The optimal speed and position are obtained by using (6) in the CAV dynamics (1).

Note that the analytical solution (6) is valid while none of the constraints becomes active for $t \in [t_i^0, t_i^m]$. Otherwise, the optimal solution should be modified considering the active constraints as discussed in [17]. Also note that this formulation (5) does not include the safety constraint (3). The conditions under which the rear-end collision avoidance constraint does not become active inside the CZ are provided in [20], where it is also shown how they can be enforced through an appropriately designed *Feasibility Enforcement Zone* that precedes the CZ.

III. DYNAMIC RESEQUENCING OF CONNECTED AUTOMATED VEHICLES

The crossing sequence for the CAVs based on which the throughput maximization problem in [17] is formulated adopts a strict FIFO queueing structure. This can be effective when the CZ is physically symmetrical and the vehicle arrival rates at all CZ entries do not differ much. However, when the CZ is asymmetrical (see Fig. 2), the FIFO queueing structure may lead to poor scheduling and possible congestion. For example, in Fig. 2(a) where the CZ is asymmetrical in terms of the vehicle arrival rates, CAV #4 entering the intersection from a CZ segment with a lower arrival rate should wait under FIFO for the first three CAVs crossing the MZ, which leads to unnecessary travel delay and extra energy consumption. In Fig. 2(b) where the CZ is asymmetrical in terms of the physical lengths of CZ segments, CAV #4 enters the intersection from a shorter CZ segment and it is closer to the MZ entry, while #4 has to decelerate in order to let the CAV #1, #2 and #3 cross the MZ first. This again will increase travel delay. Even with a fully symmetrical CZ, a strict FIFO queueing structure is conservative in the sense that it prevents the CZ from achieving higher traffic throughput. For example, a CAV with higher initial speed may tend to cross the MZ before another CAV which arrives at the CZ earlier but with lower initial speed.

A. Feasible Crossing Sequence

A natural approach dealing with the sequencing issue is to dynamically resequence the CAVs when a new one enters the CZ. The resequencing policy can be position-based, i.e., the CAV closer to the MZ entry is prioritized to cross it. Alternatively, the crossing sequence can be determined based on the estimated travel time to the MZ. However, these

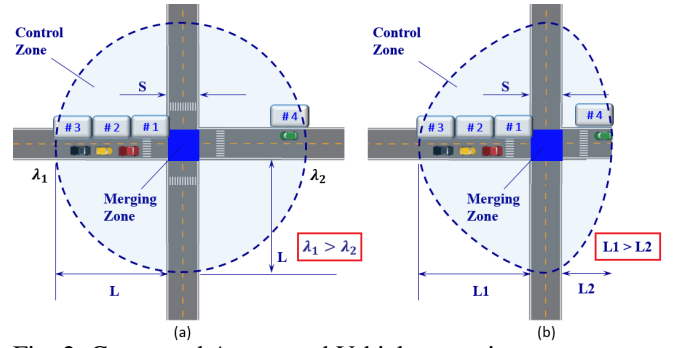


Fig. 2: Connected Automated Vehicles crossing an asymmetrical urban intersection.

methods may not be fair since CAVs entering from the shorter CZ segment are always prioritized over those entering from the longer CZ segment, which leads to congestion on the longer CZ segment. A better approach is to evaluate all feasible crossing sequences whenever a new CAV enters the CZ and select the one that maximizes traffic throughput.

Thus, our objective is to assign each arriving CAV an appropriate order to maximize traffic throughput while maintaining the relative order of the remaining CAVs. The problem reduces to finding all feasible crossing sequences, computing the corresponding terminal times recursively, and determining the one providing maximal throughput. Denoting a crossing sequence as s and the set containing all the feasible crossing sequences as \mathcal{S}_i when CAV i enters the CZ, we basically traverse all the possible options and insert feasible crossing sequences into \mathcal{S}_i . The computational complexity of this process will be discussed in Section IV.

B. Throughput Maximization Problem Formulation

For each feasible crossing sequence s in \mathcal{S}_i , we can determine the terminal time for each CAV iteratively and obtain a terminal time sequence $\mathbf{t}_{(2:i)} = [t_2^m, \dots, t_i^m]$. As in [16] and [17], we aim at minimizing the gaps between the terminal times of two adjacent CAVs i and $i-1$ in the sequence. Given the recursive structure of the terminal times, this objective is equivalent to minimizing $t_i^m - t_1^m$. Thus, our objective is

$$\min_{s \in \mathcal{S}_i} \sum_{j=2}^i (t_j^m - t_{j-1}^m) = \min_{s \in \mathcal{S}_i} (t_i^m - t_1^m) \quad (7)$$

subject to: (1), (2), (4),

$$s_i(t) = p_k(t) - p_i(t) \geq \delta, \quad \forall t \in [t_i^m, t_i^f], \quad k \in \mathcal{L}_i(t).$$

Observe that t_1^m is not included in the terminal time sequence since its selection is subject to a degree of freedom reflecting the tradeoff between energy minimization and throughput maximization. In our earlier work [16] and [17], CAV #1 is assumed to cruise at its initial speed so that $t_1^m = t_1^0 + \frac{L}{v_1^0}$ and its terminal speed is $v_1^m = v_1^0$. However, with resequencing, several alternatives are possible as discussed in the sequel.

As shown in [17], the terminal time of CAV i is dependent not only on the terminal time of CAV $i-1$ and/or k , but also

on the terminal speed of CAV $i - 1$ and/or k . Note that the terminal speed is unspecified and obtained from the energy minimization problem (5). However, there is a number of ways to specify the terminal speed.

C. Alternative Energy Minimization Problem Formulations

The effectiveness of the resequencing process may be affected by the way we formulate the energy minimization problem (5). Next, we are going to explore several alternative problem formulations.

1) Modifying the terminal time of CAV #1 by including the term $\rho \cdot (t_1^m - t_1^0)$ below to penalize longer travel times:

$$\min_{u_1} \frac{1}{2} \int_{t_1^0}^{t_1^m} u_1^2 dt + \rho(t_1^m - t_1^0) \quad (8)$$

subject to: (1), (2), $p_1(t_1^m) = L$, given $t_1^0, v_1(t_1^0), p_1(t_1^0)$.

The coefficient ρ allows trading off the throughput maximization and energy minimization objectives. Note that the terminal time t_1^m is now unspecified. Alternatively, we can force CAV #1 to reach the MZ as quickly as possible by setting $t_1^m = t_1^c$, where t_1^c is the lower bound of t_1^m .

2) Modifying the terminal speed of CAV i by including a quadratic deviation of v_i^m from the maximum speed v_{max} to penalize lower terminal speeds, that is,

$$\min_{u_i} \frac{1}{2} \int_{t_i^0}^{t_i^m} u_i^2 dt + \frac{\sigma}{2} (v_i^m - v_{max})^2 \quad (9)$$

subject to: (1), (2), $t_i^m, p_i(t_i^m) = L$, given $t_i^0, v_i(t_i^0), p_i(t_i^0)$.

The coefficient σ allows trading off the throughput maximization and energy minimization objectives. Alternatively, we can directly set $v_i^m = v_{max}$. Note that CAV i may not be able to reach v_{max} . In that case, v_i^m is set to the maximal speed that CAV i can reach given its initial conditions.

D. Case Study for Dynamic Resequencing

The effectiveness of the resequencing process in terms of maximizing the traffic throughput is validated through simulation in MATLAB considering 20 CAVs crossing an urban intersection. The intersection is asymmetric by setting the lengths of the CZ segments to $L_{E2W} = L_{W2E} = 400\text{m}$ and $L_{N2S} = L_{S2N} = 300\text{m}$, respectively. The width of the merging zone is $S = 30\text{m}$. The vehicle arrival process is assumed to be given by a Poisson process with the same rate $\lambda = 0.4$ (veh/s) for each CZ segment. The initial speeds are assumed to be given by a uniform distribution defined over $[8, 12]$ m/s. The maximum speed and maximum acceleration are $v_{max} = 16$ m/s and $u_{max} = 2$ m/s², while the minimum speed and maximum deceleration (minimum acceleration) are set to $v_{min} = 4$ m/s and $u_{min} = -5$ m/s².

We consider 10 different alternative energy minimization problem formulations for comparison ([R] indicates a case with resequencing, and [NR] without resequencing):

- (1) [NR] CAV #1 cruises and reaches MZ at $t_1^m = t_1^0 + \frac{L}{v_1^0}$;
- (2) [NR] CAV #1 is penalized for longer travel time by including the term $\rho(t_1^m - t_1^0)$ in the cost functional, where $\rho = 5$;

- (3) [NR] CAV #1 is forced to reach MZ at $t_1^m = t_1^c$;
- (4) [R] CAV #1 cruises and reaches MZ at $t_1^m = t_1^0 + \frac{L}{v_1^0}$;
- (5) [R] CAV #1 is penalized for longer travel time by including the term $\rho(t_1^m - t_1^0)$ in the cost functional, where $\rho = 5$;
- (6) [R] CAV #1 is forced to reach MZ at $t_1^m = t_1^c$;
- (7) [R] CAVs are penalized from deviating v_{max} at t_i^m by including the term $\frac{\sigma}{2}(v_i(t) - v_{max})^2$ in the cost functional, where $\sigma = 0.1$;
- (8) [R] similar to case (7), except that $\sigma = 1$;
- (9) [R] similar to case (7), except that $\sigma = 10$;
- (10) [R] CAVs are forced to reach v_{max} at t_i^m .

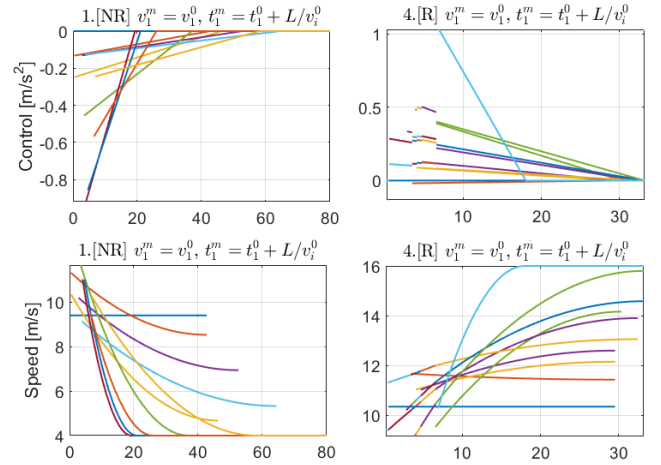


Fig. 3: Optimal control and speed profiles of the first 10 CAVs under problem formulations (1) and (4).

The optimal control and speed trajectories of the first 10 CAVs under problem formulations (1) and (4) are shown in Fig. 3. For other cases, please refer to [21]. Within each trajectory, the change of color indicates an occurrence of a resequencing process. Note that the speed and control constraints (2) are satisfied throughout the trajectories.

To illustrate the resequencing process, part of the speed trajectories for the first 3 CAVs under case 4 are shown in Fig. 4, where CAV #1 is cruising in an energy-optimal way, i.e., $t_1^m = t_1^0 + \frac{L}{v_1^0}$ and the crossing sequence is re-evaluated whenever a CAV enters the CZ. Observe that when CAV #3 arrives at the CZ, it is rescheduled to #1₃ (previously indexed as #3), and CAV #1 and #2 are rescheduled to #2₁ and #3₂. Note that both CAV #2 and #1 are traveling on the longer CZ segments, while CAV #3 is traveling on the shorter CZ segment. Intuitively, since CAV #3 enters the CZ right after CAV #2 ($t_2^0 = 0.43\text{s}$, and $t_3^0 = 0.51\text{s}$), it is natural to let CAV #3 cross the MZ first as it is closer to the MZ. Without the resequencing process (case 1), CAV #3 can only enter the MZ when #2 leaves the MZ, which makes the total gap $t_3^m - t_1^m = 3.52\text{s}$; with the resequencing process (case 4), CAV #3 becomes #1₃, and the total gap reduces to $t_3_2^m - t_1_3^m = 2.9\text{s}$, hence, improving the traffic throughput.

Under case 1, where CAV #1 is assumed to cruise at its initial speed in terms of minimizing energy consumption and no resequencing is considered, CAV#3 results in a low terminal speed $v_3^m = 4.67\text{m/s}$. Under case 4 where the

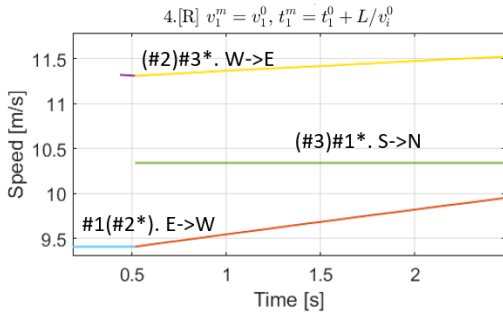


Fig. 4: Illustration of the resequencing process.

resequencing process is included, CAV #3 is rescheduled to #1₃ and assumed to cruise at its initial speed. Therefore, the terminal times for CAV #2₁ and #3₂ are updated based on the recursive terminal time structure and observe that $t_{3_2}^m < t_{3_3}^m$. This forces CAV #3₂ to accelerate, which leads to a higher terminal speed $v_{3_2}^m = 13.1\text{m/s}$ and further minimizes the gap.

E. Performance Metrics

To quantify the effectiveness of the resequencing process, we compare the performance metrics, i.e., energy consumption and throughput under different cases. To measure the throughput, we use $t_{N(t)}^m$, the time by which all $N(t)$ vehicles exit the CZ. To measure the energy consumption, we use the polynomial metamodel in [22] that yields vehicle fuel consumption as a function of speed and acceleration. We consider 100 CAVs crossing one intersection given a vehicle arrival rate of $\lambda = 0.4$ (veh/s). The performance metrics are shown in Fig. 5. Observe that with the resequencing process (starting with case 4), the travel time is improved by approximately 34% compared to the cases without resequencing. This validates the efficiency of the resequencing process in terms of traffic throughput maximization.

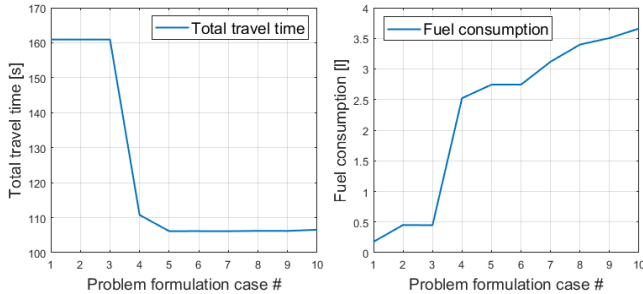


Fig. 5: Travel time (left) and fuel consumption (right) under alternative problem formulations given $\lambda = 0.4$ (veh/s).

In Fig. 5, observe that the resequencing process leads to an increase in energy, counteracting the throughput benefits. This shows the tradeoff between energy minimization and throughput maximization. In general, cases 4 and 5 achieve better performance compared to others.

IV. COMPUTATIONAL COMPLEXITY ANALYSIS

Since the coordinator needs to re-evaluate the crossing sequence every time a new CAV arrives at the CZ, the computational complexity may be significant when the traffic is heavy. A key observation is that CAV i can obviously not

overtake its preceding CAV k , which therefore, guarantees an upper bound in the resequencing computational complexity involved. Since the key to the resequencing process lies in inserting CAV i into different positions of the queue after k , the computational complexity can be represented by the number of feasible crossing sequences.

The worst case corresponds to CAV i entering the CZ when there is no preceding vehicle k traveling on the same lane, while all other CZ road segments operate near capacity. Assuming four CZ segments within an intersection, their lengths are denoted by L_r , $r \in \{1, 2, 3, 4\}$. Vehicle arrivals are assumed to be distributed according to Poisson processes with rates λ_r . Letting the average CAV length be l_v , the capacity for each CZ segment C_r is given by $C_r = \frac{L_r}{l_v + \delta}$, $r \in \{1, 2, 3, 4\}$. Assuming CAV i enters the first CZ segment, i.e., $r = 1$, the computational complexity measured using the number of feasible crossing sequences for CAV i , denoted as N^1 , under the worst case is $N^1 = \frac{L_2 + L_3 + L_4}{l_v + \delta} + 1$. Here, 1 indicates the feasible sequence without resequencing. Taking the vehicle arrivals on other CZ segments into consideration, the worst case of the computational complexity for the whole intersection is $N = \max\{\frac{L_2 + L_3 + L_4}{l_v + \delta}, \frac{L_1 + L_3 + L_4}{l_v + \delta}, \frac{L_1 + L_2 + L_4}{l_v + \delta}, \frac{L_1 + L_2 + L_3}{l_v + \delta}\} + 1$.

This represents the *upper bound* of the computational complexity associated with the resequencing process. The best case occurs when $k = i - 1$, which indicates no necessity to resequence. Hence, the *lower bound* is $N = 1$.

Expected computational complexity $E[N]$: when the traffic flow is stable, we can derive the expected flow rate. Viewed as a queueing system, the intersection is an M/G/1 queue, where the MZ is the server and the vehicles are the customers in the queue. The condition for this M/G/1 queueing system to be stable is $\sum_{r \in \{1, 2, 3, 4\}} \lambda_r < \mu$, where λ_r is the arrival rate on r th road segment, and μ is the service rate of the MZ. Based on the recursive structure of terminal times in [17], vehicles traveling on opposite roads will not generate any collision inside the MZ, hence, they are allowed to cross the MZ at the same time. It follows that we only need $\sum_{r \in \{1, 2, 3, 4\}} \lambda_r < 2\mu$ as a condition for stability.

To compute $E[N]$, we consider the expected interarrival time between CAVs k and i . Assuming CAV i entering the p -th CZ segment, $p \in \{1, \dots, M\}$, the expected interarrival time is $E[\Delta t] = \frac{1}{\lambda_p}$. Over the interarrival time Δt , the expected number of arrivals on the other three CZ segments are given by $E[\Delta t] \cdot (\sum_{r \in \{1, \dots, M\}, r \neq p} \lambda_r)$. Therefore, for vehicles coming from the p -th CZ segment, we have

$$E[N^p] = \frac{1}{\lambda_p} \sum_{r \in \{1, \dots, M\}, r \neq p} \lambda_r + 1,$$

and for the whole intersection, we have

$$E[N] = \frac{\lambda_1 \cdot E[N^1] + \dots + \lambda_M \cdot E[N^M]}{\sum_{r \in \{1, \dots, M\}} \lambda_r} = M \quad (10)$$

This indicates that the expected computational complexity is always determined by the number of lanes associated with the intersection.

The expected computational complexity is validated through simulation in MATLAB considering 100 CAVs crossing an urban intersection, with exactly the same simulation settings as in Sec. III-E with $M = 4$ lanes. The average service time is roughly estimated as 1.25s and the expected service rate is $\mu = 0.8$. Therefore, the stability condition can be determined as $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 < 1.6$. The energy minimization problem is formulated as in case 5 in Section III.D, which penalizes longer travel times for CAV #1.

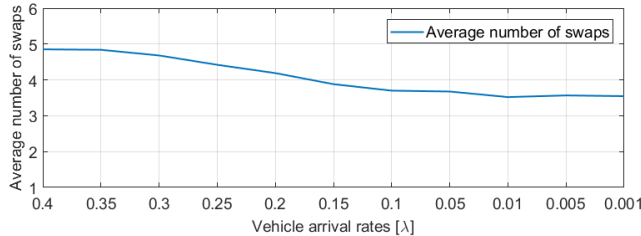


Fig. 6: Expected computational complexity of resequencing process over decreasing traffic intensity.

The simulation results are shown in Fig. 6, where the computational complexity, measured using the number of swaps, is averaged over 10 simulations. We assume $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$, where $\lambda < 0.4$. Over different arrival rates, the computational complexity in performing dynamic resequencing, is approximately 4, as expected. The actual value of $E[N]$, however, may be lower since a resequencing step affects subsequent resequencing steps by altering the vehicle arrival process distribution.

V. CONCLUSIONS AND FUTURE WORK

Earlier work [16], [17] and [20] has established a decentralized optimal control framework for optimally controlling CAVs crossing a signal-free urban intersection while following a strict FIFO queueing order. In this paper, we have extended the solution of this problem to account for asymmetric intersections by relaxing the FIFO constraint and introducing a dynamic resequencing process so as to maximize the traffic throughput. The dynamic resequencing has been shown to be computationally very efficient. It is also shown to reduce the travel time at the cost of additional fuel consumption.

Ongoing research is considering turns (see [23]) and lane changing in the intersection with a diverse set of CAVs and exploring the mixed scenario where both CAVs and human-driven vehicles travel on the roads (see [24]). Future research should also investigate the multi-intersection scenario and how the coupling between multiple intersections would affect throughput maximization and energy minimization problems.

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