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Structural Properties of Optimal Uplink Transmission Scheduling in Energy-Efficient Wireless Networks with Real-Time Constraints

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***Abstract*— It has been shown that the energy efficiency of wireless networks can be greatly improved by utilizing transmission control techniques, which dynamically adjust the transmission speed subject to real-time operating constraints. In this paper, we focus on the uplink transmission scheduling problem for minimizing the total transmission cost in the setting that multiple nodes share the spectrum and transmit to the same destination. Our formulation is more general than that in [1], where the FlowRight algorithm is proposed, in the sense that we allow each individual task have its own deadline. We identify several structural properties of the optimal control to the two-node uplink scheduling problem, including: i) the optimal rates only change at known event times; ii) at each rate changing point, there exists an explicit relationship between the event type and the rate changing direction; and iii) at each rate changing point, the directions of rate change are reverse. These properties are helpful to establish an efficient decomposition approach towards solving the general transmission scheduling problem.**

***Index Terms*— wireless networks, energy-efficiency, real-time, optimization**

I. INTRODUCTION

Energy efficiency is extremely important in order to extend the lifetime of wireless networks. It is well known that there exists an explicit relationship between transmission power and channel capacity [2]; transmission power can be adjusted by changing the transmission rate, provided that appropriate coding schemes are used. This provides an option to conserve the transmission energy of a wireless node by slowing down the transmission rate. Increased latency is a direct side effect caused by the low transmission rate and it can affect other Quality-of-Service (QoS) metrics as well. For example, excessive delay may cause buffer overflow, which increases the packet dropping rate. The existence of this trade-off between energy and latency motivates *Dynamic Transmission Control* techniques for designing energy-efficient wireless systems.

To the best of our knowledge, the earliest work that captures the trade-off between energy and latency in transmission scheduling is [3], in which Collins and Cruz formulated a Markov decision problem for minimizing transmission cost subject to some power constraints. By assuming a linear dependency between transmission cost and time, their model did not consider the potential of more energy saving by varying the transmission rate. Berry [4] considered a Markov decision process in the context of wireless fading channels to minimize the weighted sum of average transmission power and a buffer cost, which corresponds to either average delay or probability of buffer overflow. Using dynamic programming and assuming the transmission cost to be a convex function of time, Berry discovered some structural properties of the optimal adaptive control policy, which relies on information on the arrival state, the queue state, and the channel state. In [5] and [6], Ata developed optimal dynamic power control policies subject to a QoS constraint for Markovian queues in wireless static channels and fading channels respectively. In his work, the optimization problem was formulated to minimize the long-term average transmission power, given a constraint of buffer overflow probability in equilibrium; dynamic programming and Lagrangian relaxation approaches were used in deriving the optimal policies, which can be expressed as functions of the packet queue length and the channel state.

Another line of research does not rely on the Markovian assumption; instead, it assumes that each packet is associated with an arrival time (generally random) and a deadline that must be met. A relatively new problem which is referred to as the "downlink scheduling" problem, has been studied initially in [7] with follow-up work in [8] where a "homogeneous" case is considered assuming all packets have the same deadline and number of bits. By identifying some properties of this convex optimization problem, Gamal et al. proposed the "MoveRight" algorithm in [8] to solve it iteratively. However, the rate of convergence of the MoveRight algorithm is only obtainable for a special case of the problem when all packets have identical energy functions; in general the MoveRight algorithm may converge slowly. Yu et al. [9] formulated a problem to minimize the overall energy dissipation of the sensor nodes in the aggregation tree subject to a latency constraint. They solved the problem using an extended MoveRight algorithm. However, the extended MoveRight algorithm inevitably inherits the limitations of the MoveRight algorithm. In [10], we have considered a more general *Downlink Transmission Scheduling* (DTS) problem which is formulated assuming that each packet has a different deadline and number of bits. By analyzing the structure of the

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optimal sample path, we proposed the Generalized Critical Task Decomposition Algorithm (GCTDA) which solves the problem efficiently using a two-fold decomposition approach and is numerically shown to be typically an order of magnitude faster than the MoveRight algorithm in [8].

The above results consider the transmission control problem of a single transmitter (the destinations can be multiple though). The problem becomes more challenging when multiple wireless nodes share the spectrum and transmit simultaneously to a common receiver. This is referred to as the "uplink" scheduling problem [1], where Uysal-Bikikoglu et al. proposed the "FlowRight" algorithm that converges to the optimal solution when each task has a common deadline. In this paper, we consider the uplink scheduling problem in the more general case that each task has its individual deadline. By using sample path analysis techniques, we are able to identify a number of interesting structural properties of the optimal uplink schedule. Our results in this paper are helpful in designing efficient sample path decomposition approaches for explicitly solving the general uplink scheduling problem.

The organization of this paper is as follows: we formulate the uplink scheduling problem in Section II; Section III presents several structural properties of the optimal control; conclusions and future research are discussed in Section IV.

II. Problem Formulation

The uplink transmission scheduling problem can be described as follows. Several wireless nodes sharing the RF spectrum transmit packets to a common destination. We model each transmitter as a single-server queueing system operating on a first-come-first-served basis, whose dynamics are given by the well-known max-plus equation

$$x_i = \max(x_{i-1}, a_i) + s_i$$

where a_i is the arrival time of task $i = 1, 2, \ldots$, x_i is the time when task i completes service, and s_i is its (generally random) service time.

The service time s_i is controlled by the transmission rate, which is determined by transmission power and coding scheme. Since each packet can be considered as a communication task, we will use the term "task", rather than "packet" in what follows. Using the same setting as in the DTS problem in [10], each task in the uplink problem is also associated with three parameters: arrival time, deadline, and the number of bits. By controlling the transmission rate at each node, the objective of the uplink problem is to minimize the total energy consumption of all nodes while guaranteeing hard deadline satisfaction for each individual task.

Before formulating the optimization problem, we first derive the cost function in multiaccess fading channels. Note that this cost function was first established in [1]. We summarize the procedure used in [1] for the sake of self-sufficiency.

As shown in [11], the capacity region for a multiaccess Additive White Gaussian Noise (AWGN) channel of N nodes under Gaussian noise σ^2 is given by:

$$r(Q) \leq \frac{1}{2}\log(1 + \frac{\sum_{i\in S} g_i P_i}{\sigma^2}), \text{ for every } Q \subset \{1, \ldots, N\} \tag{1}$$

where $r(Q)$ is the maximum sum rate of nodes in a set Q, with other nodes' information known to the receiver; $g = (g_1, \ldots, g_N)$ and $P = (P_1, \ldots, P_N)$ are the channel fading vector and power vector respectively. For notational and analytical simplicity, we assume the number of transmitting nodes is two and the channels are static for both nodes, i.e., g is time-invariant.

It has been discussed in [1] that if the codewords of optimal coding are long enough, the boundary of the above capacity region is achievable. We can set $\sigma^2 = 1$, normalize g_1 to 1, and rewrite (1) into power-rate relationships for two nodes as follows:

$$\begin{aligned} P_1 &\geq f(r_1) \\ aP_2 &\geq f(r_2) \\ P_1 + aP_2 &\geq f(r_1 + r_2) \end{aligned} \tag{2}$$

where $f(r) \triangleq 2^{2r} - 1$, a is a constant. Recall that our goal is to minimize the total power consumption $P_{total} = P_1 + P_2$, given r_1 and r_2, and subject to the constraints in (2). Note that we control the transmission power under fixed transmission rates here. Later on, we will control the transmission rates to minimize the transmission energy.

The optimal solution to this linear program can only be one of the two corner points $(f(r_1), [f(r_1+r_2) - f(r_1)]/a)$ and $(f(r_2)/a, f(r_1+r_2) - f(r_2))$, depending on the value of a. Without loss of generality, we assume $a \geq 1$. It can be easily verified that in this case the optimal solution is $(f(r_1), [f(r_1+r_2) - f(r_1)]/a)$ with $P_{total} = f(r_1+r_2) + (1 - 1/a)f(r_1)$. Therefore, the total RF transmission power can be defined as a function of both r_1 and r_2

$$\zeta(r_1, r_2) = f(r_1 + r_2) + cf(r_1)$$

where $0 \leq c \leq 1$. Note that $\zeta(r_1, r_2)$ is convex, twice continuously differentiable, and monotonically increasing in both r_1 and r_2. When $c = 0$, the two uplinks are symmetric in the sense that they contribute the same amount to the total energy consumption. This is unlikely to happen in real applications, due to the nature of wireless communications and the difference between transmission distances. It has been shown in [1] that in the symmetric case, a transmission schedule utilizing time-division between two nodes is optimal.

We will now focus on the case when the two uplinks are not symmetric, i.e., $c \in (0, 1]$. First of all, let us show that $\zeta(r_1, r_2)$ is strictly convex. We write down the explicit form of $\zeta(r_1, r_2)$:

$$\begin{aligned} \zeta(r_1, r_2) &= f(r_1 + r_2) + cf(r_1) \\ &= 2^{2(r_1+r_2)} + c2^{2r_1} - (c+1) \end{aligned}$$

After some simple algebra (we omit the details), we establish that the two eigenvalues of $\nabla^2\zeta(r_1, r_2)$ are positive. This shows that $\nabla^2\zeta(r_1, r_2)$ is positive definite and $\zeta(r_1, r_2)$ is strictly convex. Although we have used the explicit form of $\zeta(r_1, r_2)$ to show its strict convexity, our analysis will not rely on the form of $\zeta(r_1, r_2)$, as long as it is strictly convex, twice continuously differentiable, and monotonically increasing in both r_1 and r_2. Also note that when there are more than two nodes, the power/rate region becomes a more complicated polyhedron with more corner points. However, the function ζ will still possess the properties above. Therefore, our results can be extended to the case when more than two nodes are transmitting as well.

Let us now consider the two-node case and formulate optimization problem **P1**:

$$\min_{\substack{r_1(t),\, x_{1,i},\, r_2(t),\, x_{2,j} \\ i=1,\dots,N,\ j=1,\dots,M}} \int_{\min(a_{1,1},a_{2,1})}^{\max(d_{1,N},d_{2,M})} \zeta(r_1(t), r_2(t))dt$$

$$\text{s.t.} \quad \int_{\max(a_{1,i},x_{1,i-1})}^{x_{1,i}} r_1(t)dt = \nu_{1,i}, \ i = 1, \dots, N,$$

$$x_{1,0} = 0, \ x_{1,i} \le d_{1,i}$$

$$\int_{\max(a_{2,j},x_{2,j-1})}^{x_{2,j}} r_2(t)dt = \nu_{2,j}, \ j = 1, \dots, M,$$

$$x_{2,0} = 0, \ x_{2,j} \le d_{2,j}$$

where $r_1(t)$ is the transmission rate at nodes 1, $\nu_{1,i}$ is the number of bits of task i at node 1, $a_{1,i}$ is the arrival time of task i at node 1, $d_{1,i}$ is the deadline of task i at node 1, $x_{1,i}$ is the departure time of task i at node 1, and $r_2(t), \nu_{2,j}, a_{2,j}, d_{2,j}, x_{2,j}$ are the corresponding ones for node 2 respectively. In this paper, we consider the *off-line* control case in which the task information at both nodes is known a priori. Note that in **P1** above we control the departure times of each task and the transmission rates at both nodes.

Problem **P1** is a complicated nonlinear optimization problem with nondifferentiable constraints. A simpler form of the problem was formulated in [1], where the tasks at each node have the same deadline. In [1], Uysal-Bikikoglu et al. solve the problem using the FlowRight algorithm, which is based on the MoveRight algorithm in [8]. The FlowRight algorithm is an iterative algorithm that converges to the optimal solution, rather than providing the exact form solution. Note that the FlowRight algorithm cannot even guarantee convergence when solving **P1** above, since, with the presence of task-dependent deadlines, the monotonicity property of the FlowRight algorithm no longer exists and the information now may flow to the left.

III. PROPERTIES OF OPTIMAL UPLINK SCHEDULING

Although **P1** is hard to solve, we will proceed by exploring structural properties that may lead to an efficient approach utilizing the decomposition idea developed in [10]. The first question we address is when the optimal rate changes, i.e., should the optimal rate of a node dynamically vary all the time or only change at some specific times? The question has been partially answered for tasks with common deadlines in [1], where Uysal-Bikikoglu et al. showed that the optimal transmission rate is static between adjacent task arrivals. In what follows, we will show that a similar result can be obtained in our setting where each task has its own deadline.

Let A_1, D_1, A_2, D_2 be the sets of arrival times and deadlines of tasks at node 1 and node 2 respectively. By ordering all the arrival times and deadlines of tasks at both nodes, we obtain a sequence of event times $\boldsymbol{\tau} = \{\tau_1, \dots, \tau_S\}$, where S is the cardinality of $A_1 \bigcup D_1 \bigcup A_2 \bigcup D_2$.

We begin with an important auxiliary lemma.

Lemma 3.1: Let B_1, B_2, τ_1, $\tau_2 \in \mathbb{R}^+$, $\tau_1 < \tau_2$, $r_1(t) : \mathbb{R}^+ \to \mathbb{R}^+$, $r_2(t) : \mathbb{R}^+ \to \mathbb{R}^+$. Consider the following optimization problem:

$$\begin{array}{ll} \min\limits_{r_1(t),\, r_2(t)} & \int_{\tau_1}^{\tau_2} \zeta(r_1(t), r_2(t))dt \\ \text{s.t.} & \int_{\tau_1}^{\tau_2} r_1(t)dt = B_1 \\ & \int_{\tau_1}^{\tau_2} r_2(t)dt = B_2 \end{array}$$

Then, the unique solution to this problem is:

$$r_1^*(t) = \frac{B_1}{\tau_2 - \tau_1} \text{ and } r_2^*(t) = \frac{B_2}{\tau_2 - \tau_1}$$

Proof: Due to space limitation, we omit the proof which can be found in [12]. ■

This lemma asserts that during any time interval, if the number of bits that need to be transmitted at each node is fixed, then static rates are the unique optimal solution. Therefore, dynamically adjusting the transmission rates will not pay off. Based on this result, we are able to establish the following with $r_1^*(t)$ and $r_2^*(t)$ denoting the solution of problem **P1**.

Theorem 3.1: Both $r_1^*(t)$ and $r_2^*(t)$ are constant during two adjacent event times in the sequence $\boldsymbol{\tau} = \{\tau_1, \dots, \tau_S\}$.

Proof: We use a contradiction argument to prove it. Suppose that both $r_1^*(t)$ and $r_2^*(t)$ vary during a time interval $[\tau_{k-1}, \tau_k)$. Then, the total number of bits sent by node 1 and node 2 during this time interval are given below:

$$B_1 = \int_{\tau_{k-1}}^{\tau_k} r_1^*(t)dt \text{ and } B_2 = \int_{\tau_{k-1}}^{\tau_k} r_2^*(t)dt.$$

Invoking Lemma 3.1, to minimize the total energy during $[\tau_{k-1}, \tau_k)$ of transmitting B_1 and B_2 bits at node 1 and node 2 respectively, i.e., $\int_{\tau_{k-1}}^{\tau_k} \zeta(r_1(t), r_2(t))dt$, static rates provide the unique optimal solution. This contradicts the assumption that $r_1^*(t)$ and $r_2^*(t)$ vary during $[\tau_{k-1}, \tau_k)$. ■

Theorem 3.1 and its proof is very similar to Lemma 2 in [1]. The difference is that our result here applies to the more general case when each task has its own deadline. Similar to Lemma 2 in [1], Theorem 3.1 shows that the optimal transmission rates at both nodes only change at points in event time sequence $\boldsymbol{\tau}$. Therefore, the cost in **P1** can be reduced to the following expression:

$$\sum_{k=2}^{S} \zeta(r_{1,k-1}, r_{2,k-1})(\tau_k - \tau_{k-1}),$$

where $r_{1,k-1}$ and $r_{2,k-1}$ are the transmission rates during time interval $[\tau_{k-1}, \tau_k)$ at node 1 and node 2 respectively. Note that the optimal transmission rates certainly may stay unchanged at task arrivals and deadlines.

Having shown that the optimal transmission rates are static during certain time intervals, we now further explore the special properties of the optimal solution to **P1**.

Lemma 3.2: Suppose the optimal transmission rates change at an event time τ_k in $\boldsymbol{\tau}$, $1 < k < S$. If $\tau_k \in A_i \cup D_i$, $i = 1$ or 2, then the optimal transmission rate must change at node i.

Proof: See [12]. ■

Lemma 3.2 implies that at any time τ_k of rate change at either or both nodes, the optimal transmission rate of the node that τ_k belongs to must change. The next result discusses how the rate changes at this node.

Theorem 3.2: Suppose the optimal transmission rates change at time τ_k in $\boldsymbol{\tau}$, $1 < k < S$. If $\tau_k \in A_i \cup D_i$, $i = 1$ or 2, then the following holds:

i) τ_k is an arrival time at node i if and only if node i's optimal transmission rate increases at τ_k.

ii) τ_k is a deadline at node i if and only if node i's optimal transmission rate decreases at τ_k.

Proof: Without loss of generality, let us assume $i = 1$. We will first show the sufficient conditions for both parts. Then, we will show the necessary conditions using the sufficiency.

Sufficiency proof for part i): By assumption, $r^*_{1,k-1} < r^*_{1,k}$. Since τ_k is either a task arrival or a deadline of node 1, we only need to show that it cannot be a task deadline. We use a contradiction argument to prove it. Suppose τ_k is a task deadline at node 1. Consider a solution $r'_1(t)$ and $r'_2(t)$ of node 1 and node 2 respectively:

$$r'_1(t) = \begin{cases} r'_{1,k}, & t \in [\tau_{k-1}, \tau_{k+1}), \\ r^*_1(t), & o.w. \end{cases}, \tag{3}$$

where

$$r'_{1,k} = \frac{r^*_{1,k-1}(\tau_k - \tau_{k-1}) + r^*_{1,k}(\tau_{k+1} - \tau_k)}{\tau_{k+1} - \tau_{k-1}} \tag{4}$$

$$r'_2(t) = \begin{cases} r'_{2,k}, & t \in [\tau_{k-1}, \tau_{k+1}), \\ r^*_2(t), & o.w. \end{cases}, \tag{5}$$

where

$$r'_{2,k} = \frac{r^*_{2,k-1}(\tau_k - \tau_{k-1}) + r^*_{2,k}(\tau_{k+1} - \tau_k)}{\tau_{k+1} - \tau_{k-1}}. \tag{6}$$

Note that $r'_1(t)$ and $r'_2(t)$ are identical to $r^*_1(t)$ and $r^*_2(t)$ respectively, except that they use a constant speed (averaged over $r^*_{1,k-1}$, $r^*_{1,k}$ at node 1 and $r^*_{2,k-1}$, $r^*_{2,k}$ at node 2) to transmit in time interval $[\tau_{k-1}, \tau_{k+1})$. The optimal control $r^*_{1,k-1}$ and $r^*_{1,k}$ certainly are feasible in the sense that they guarantee deadline τ_k. Because $r'_{1,k}$ is a convex combination of $r^*_{1,k-1}$ and $r^*_{1,k}$, $r^*_{1,k-1} < r'_{1,k} < r^*_{1,k}$. Since $r'_{1,k} > r^*_{1,k-1}$ implies a faster task completion time, $r'_1(t)$ can guarantee deadline τ_k as well. Since τ_k is a task deadline at node 1, $r'_2(t)$ is clearly feasible for node 2. Therefore, $r'_1(t)$ and $r'_2(t)$ are a feasible solution of **P1**. Because $r'_1(t)$ ($r'_2(t)$, respectively) and $r^*_1(t)$ ($r^*_2(t)$, respectively) transmit the same amount of bits during time interval $[\tau_{k-1}, \tau_{k+1})$ (as seen from (4) and (6)), we can invoke Lemma 3.1 to establish that $r'_1(t)$ and $r'_2(t)$, which are static during $[\tau_{k-1}, \tau_{k+1})$, incur a lower cost than the latter in the same time interval. Since they are identical except in $[\tau_{k-1}, \tau_{k+1})$, $r'_1(t)$ and $r'_2(t)$ outperform $r^*_1(t)$ and $r^*_2(t)$. This contradicts the optimality of the latter. Therefore, τ_k can only be a task arrival time at node 1.

Sufficiency proof for part ii): By assumption, $r^*_{1,k-1} > r^*_{1,k}$. Since τ_k is either a task arrival or a deadline of node 1, we only need to show that it cannot be a task arrival. We use a contradiction argument to prove it. Suppose τ_k is a task arrival at node 1. Again, consider solution $r'_1(t)$ and $r'_2(t)$ of node 1 and node 2 in (3) and (5) respectively. Note that $r'_1(t)$ and $r'_2(t)$ are identical to $r^*_1(t)$ and $r^*_2(t)$ respectively, except that they use a constant speed (averaged over $r^*_{1,k-1}$, $r^*_{1,k}$ at node 1 and $r^*_{2,k-1}$, $r^*_{2,k}$ at node 2) to transmit in time interval $[\tau_{k-1}, \tau_{k+1})$. The optimal control $r^*_{1,k-1}$ and $r^*_{1,k}$ certainly are feasible in the sense that causality is not violated, i.e., tasks will not be transmitted before their arrival times. Because $r'_{1,k}$ is a convex combination of $r^*_{1,k-1}$ and $r^*_{1,k}$, $r^*_{1,k-1} > r'_{1,k} > r^*_{1,k}$. Since $r'_{1,k} < r^*_{1,k-1}$ implies a slower task completion time, $r'_1(t)$ does not violate causality as well. Since τ_k is a task arrival at node 1, $r'_2(t)$ is clearly feasible for node 2. Therefore, $r'_1(t)$ and $r'_2(t)$ are a feasible solution of **P1**. Because $r'_1(t)$ ($r'_2(t)$, respectively) and $r^*_1(t)$ ($r^*_2(t)$, respectively) transmit the same amount of bits during time interval $[\tau_{k-1}, \tau_{k+1})$ (as seen from (4) and (6)), we can invoke Lemma 3.1 to establish that $r'_1(t)$ and $r'_2(t)$, which are static during $[\tau_{k-1}, \tau_{k+1})$, incur a lower cost than the latter in the same time interval. Since they are identical except in $[\tau_{k-1}, \tau_{k+1})$, $r'_1(t)$ and $r'_2(t)$ outperform $r^*_1(t)$ and $r^*_2(t)$. This contradicts the optimality of the latter. Therefore, τ_k can only be a task deadline at node 1.

After proving the sufficiency, we will proceed to show the necessity, which is a direct result from sufficiency. The sufficiency of part *i)* shows that if node 1's optimal transmission rate increases at τ_k, then τ_k is an arrival time at node 1. Using a simple contrapositive argument, we can establish that if τ_k is not an arrival time at node 1, i.e., τ_k is a deadline at node 1, then node 1's optimal transmission rate decreases or remains unchanged at τ_k. Using Lemma 3.2, node 1's optimal transmission rate must change at τ_k. Therefore, we can obtain the necessity of part *ii)*: if τ_k is a deadline at node 1, then node 1's optimal transmission rate decreases at τ_k. Similarly, using the sufficiency of part *ii)* and a contrapositive argument, we can establish that if τ_k is an arrival at node 1, then node 1's optimal transmission rate increases or remain unchanged at τ_k. Again, invoking Lemma 3.2, the latter case cannot happen. This gives the necessity of part *i)* and completes the proof. ■

So far, Theorem 3.1 shows that the optimal transmission rates at both nodes can only change at task arrivals or deadlines. Lemma 3.2 indicates that at any time τ_k of rate change, the optimal transmission rate of the node that τ_k belongs to does not remain unchanged. Theorem 3.2 further explores the structure of the optimal sample path of **P1**. It basically establishes that at a rate changing point τ_k, there exists a relationship between the directions of optimal rate changes (i.e., increasing or decreasing) and the types of τ_k (i.e., an arrival or a deadline) at the node that τ_k belongs to.

Specifically, the necessity of Theorem 3.2 shows that if τ_k is an arrival, then the optimal transmission rate at the node that τ_k belongs to will increase; if τ_k is a deadline, then the optimal transmission rate at the node that τ_k belongs to will decrease. Note that at this point we do not know the rate changing event times τ_k yet; however, Theorem 3.2 applies to all τ_k.

After obtaining Theorem 3.2, a natural question arises: what happens to the optimal transmission rate at the other node?

Theorem 3.3: Suppose *i)* the optimal transmission rates change at a time τ_k in $\boldsymbol{\tau}$, $1 < k < S$, and *ii)* neither $r_1^*(t)$ nor $r_2^*(t)$ is constant zero throughout $[\tau_{k-1}, \tau_{k+1})$. Then, they must change toward opposite directions, i.e., if one increases, the other decreases, and vice versa.

Proof: See [12]. ■

Theorem 3.3 shows that if one node's optimal transmission rate changes at τ_k and the other node is not fully idling during $[\tau_{k-1}, \tau_{k+1})$, then the directions of rate change are opposite. This result is intuitive since the theorem suggests that a node should transmit using a high (low, respectively) rate when the other node's transmission rate is slow (fast, respectively).

The next result is helpful to determine the optimal transmission rates once the "critical" time instances at which the rates change, are known.

Lemma 3.3: Suppose the optimal transmission rate of node $l = 1$ or 2 changes at time $\tau_k \in A_i \cup D_i$, $i = 1$ or 2. Then, the following holds:

i) If τ_k is an arrival time of task i, $i = \arg\min_j\{a_{l,j} : a_{l,j} = \tau_k\}$, then $x_{1,i-1}^* \leq \tau_k$.

ii) If τ_k is a deadline of task i, $i = \arg\max_j\{d_{l,j} : d_{l,j} = \tau_k\}$, then $x_{1,i}^* = \tau_k$.

Proof: Without loss of generality, we assume τ_k is an event time at node 1.

i) $\tau_k = a_{1,i}$ and task i is the one with the smallest index among all tasks that arrive at τ_k (we assume it is possible that multiple tasks arrive at the same time). We need to show $x_{1,i-1}^* \leq a_{1,i}$. We use a contradiction argument to prove the result. Suppose that the optimal departure time of task $i-1$ is later than $a_{1,i}$, i.e., $x_{1,i-1}^* > a_{1,i}$. Let B_1 and B_2 be the number of bits transmitted by node 1 and node 2 during time interval $[\tau_{k-1}, x_{1,i-1}^*)$ using the optimal schedule respectively. Recall that, by assumption, the optimal rate changes at $\tau_k \in [\tau_{k-1}, x_{1,i-1}^*)$.

Now consider feasible rates

$$r_1'(t) = \begin{cases} \frac{B_1}{x_{1,i-1}^* - \tau_{k-1}}, & t \in [\tau_{k-1}, x_{1,i-1}^*), \\ r_1^*(t), & o.w. \end{cases}$$
$$r_2'(t) = \begin{cases} \frac{B_2}{x_{1,i-1}^* - \tau_{k-1}}, & t \in [\tau_{k-1}, x_{1,i-1}^*), \\ r_2^*(t), & o.w. \end{cases},$$

which are identical to $r_1^*(t)$ and $r_2^*(t)$ respectively, except in time interval $[\tau_{k-1}, x_{1,i-1}^*)$. Note that $r_1'(t)$ and $r_2'(t)$ are feasible since there is no deadline in time interval $(\tau_{k-1}, x_{1,i-1}^*)$, tasks transmitted during $(\tau_{k-1}, x_{1,i-1}^*)$ have arrived by τ_{k-1} so that causality is not violated, and they use static rates that transmit the same amount of data in this time interval. Invoking Lemma 3.1, $r_1'(t)$, $r_2'(t)$ incur a lower cost than $r_1^*(t)$, $r_2^*(t)$ in $[\tau_{k-1}, x_{1,i-1}^*)$. Since the former is identical to the latter elsewhere, this contradicts the optimality of the latter and in turn, the assumption of $x_{1,i-1}^* > a_{1,i}$.

ii) $\tau_k = d_{1,i}$ and task i is the one with the largest index among all tasks whose deadlines are τ_k (we assume different tasks may have a common deadline). We need to show $x_{1,i}^* = d_{1,i}$. Note that due to the real-time constraint, $x_{1,i}^* \leq d_{1,i}$. We use a contradiction argument to prove the result. Suppose that the optimal departure time of task i is earlier than $d_{1,i}$, i.e., $x_{1,i}^* < d_{1,i}$. Consider time interval $[x_{1,i}^*, \tau_{k+1})$, during which node 1 and node 2 transmit B_1 and B_2 bits respectively using the optimal schedule. Recall that by assumption, the optimal rate changes at $\tau_k \in [x_{1,i}^*, \tau_{k+1})$. Now consider feasible rates

$$r_1'(t) = \begin{cases} \frac{B_1}{\tau_{k+1} - x_{1,i}^*}, & t \in [x_{1,i}^*, \tau_{k+1}), \\ r_1^*(t), & o.w. \end{cases}$$
$$r_2'(t) = \begin{cases} \frac{B_2}{\tau_{k+1} - x_{1,i}^*}, & t \in [x_{1,i}^*, \tau_{k+1}), \\ r_2^*(t), & o.w. \end{cases},$$

which are identical to $r_1^*(t)$ and $r_2^*(t)$ respectively, except in time interval $[x_{1,i}^*, \tau_{k+1})$. Note that $r_1'(t)$, $r_2'(t)$ are feasible since *i)* there is no task arrival or deadline in $[x_{1,i}^*, \tau_{k+1})$ which belongs to a task transmitted during the same time interval, and *ii)* they utilize static rates that send the same amount of bits as the ones transmitted by $r_1^*(t)$ and $r_2^*(t)$ respectively. Invoking Lemma 3.1, static rates $r_1'(t)$, $r_2'(t)$ incur a lower cost than $r_1^*(t)$, $r_2^*(t)$ in $[x_{1,i}^*, \tau_{k+1})$. Since the former is identical to the latter elsewhere, this contradicts the optimality of the latter and in turn, the assumption of $x_{1,i}^* < d_{1,i}$. ■

Although Lemma 3.3 is helpful, the optimal schedule is not yet fully determined after the "critical" time instances of rate change are known. The difficulty is that when a node's optimal rate changes at the arrival time of task i, we still need to find out the time when task $i-1$ departs. According to Lemma 3.3, task $i-1$ departs at a time no later than the arrival time of task i. Because the optimal rate at node 1 decreases to zero after task $i-1$ departs, using Theorem 3.1, task $i-1$ can only depart at a time in $\boldsymbol{\tau}$. One way of finding this time is to try all the possible arrivals and deadlines before the arrival time of task i and sift the one with the minimum cost. However, smarter ways may exist due to the special properties of the problem.

Lemma 3.4: Consider tasks $i-1$ and i at node $l = 1$ or 2. If $x_{l,i-1}^* < a_{l,i} \leq d_{l,i-1}$, then task $i-1$ must depart upon a task arrival at the other node.

Proof: Without loss of generality, we assume $l = 1$. The lemma states that if *i)* the optimal departure time of task $i-1$ at node 1 is earlier than the arrival time of task i and *ii)* the deadline of task $i-1$ is not earlier than the arrival time of task i, then the optimal departure time of task $i-1$ must be a task arrival time at node 2.

Since task $i-1$ departs at $x^*_{1,i-1}$ and node 1 idles between $x^*_{1,i-1}$ and $a_{1,i}$, $x^*_{1,i-1}$ is a time of rate change. According to Theorem 3.1, $x^*_{1,i-1} \in \boldsymbol{\tau}$. In order to show $x^*_{1,i-1}$ is an arrival time of node 2, we only need to prove it cannot be other types of event times. We consider two cases:

Case 1: $x^*_{1,i-1}$ is either the arrival time or the deadline of task j at node 1.

We use a contradiction argument to show this case is impossible. Suppose it is true. Invoking Theorem 3.2, because node 1's optimal schedule decreases at $x^*_{1,i-1}$,

$$x^*_{1,i-1} = d_{1,j}. \tag{7}$$

Using part $ii)$ of Lemma 3.3,

$$x^*_{1,j} = d_{1,j}. \tag{8}$$

We will use contradiction arguments to show that (7) and (8) cannot hold. We discuss three cases:

Case 1.1: $j > i-1$. If it is true, (7) and (8) show that tasks $\{i, \ldots, j\}$ need to be sent in no time using the optimal schedule. This contradicts the feasibility of the optimal rates.

Case 1.2: $j = i-1$. (8) contradicts the assumption that $x^*_{1,i-1} < a_{1,i} \leq d_{1,i-1}$.

Case 1.3: $j < i-1$. If it is true, (7) and (8) show that tasks $\{j+1, \ldots, i-1\}$ need to be sent in no time using the optimal schedule. This contradicts the feasibility of the optimal rates.

Case 2: $x^*_{1,i-1}$ is a task deadline at node 2.

We use a contradiction argument to prove it. Suppose it is true. Invoking Theorem 3.2, node 2's optimal rate decreases at $x^*_{1,i-1}$. Using Theorem 3.3, node 1's optimal rate should increase at $x^*_{1,i-1}$. This contradicts the assumption that node 1's rate decreases at $x^*_{1,i-1}$ (because by assumption, $x^*_{1,i-1} < a_{1,i}$ so that node 1 idles between $[x^*_{1,i-1}, a_{1,i})$).

Therefore, task $i-1$ must depart upon a task arrival at node 2. ■

The above results identify a number of structural properties of the optimal sample path of **P1**. We first establish in Theorem 3.1 that the optimal transmission rates at both nodes can only change at points in an ordered time sequence containing all arrival times and deadlines. The next result, Lemma 3.2, shows that at any time τ_k of rate change, the optimal transmission rate of the node that τ_k belongs to must change. Then, Theorem 3.2 establishes that there is a relationship between the directions of rate change and the times of rate change at the node that τ_k belongs to. Theorem 3.3 shows that when optimal transmission rates change and both nodes do not idle, the rates change toward opposite directions. Finally, Lemmas 3.3 and 3.4 help us to determine the optimal transmission rate when the rate changing points are known.

After obtaining the above structural results, solving **P1** boils down to identifying those event times $\tau_k \in \boldsymbol{\tau}$ at which the optimal transmission rates actually change. Since the structure of the optimal sample path of **P1** is very similar to that of the DTS problem in [10], we expect an efficient algorithm, which is similar to the GCTDA algorithm, to be developed.

IV. Conclusions and Future Work

In this paper, we study the uplink transmission scheduling problem, which is aiming at minimizing the total transmission energy in AWGN multiaccess channels subject to a real-time constraint for each task. The problem is hard to solve for wireless devices since it is a complex nonlinear optimization problem. By analyzing the optimal sample path of this uplink scheduling problem for the two-node case, we discover several structural properties of the optimal control. These results are very helpful in developing an efficient algorithm that decomposes the optimal sample path and solves the problem. This algorithm, although still unknown at this point, will depend on full task information at both nodes. Our future work also includes developing on-line controllers that require minimal information sharing between nodes.

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