BARGAIN HUNTING

IN THE NO-FREE-LUNCH MALL OF COMPLEX SYSTEMS

C. G. Cassandras

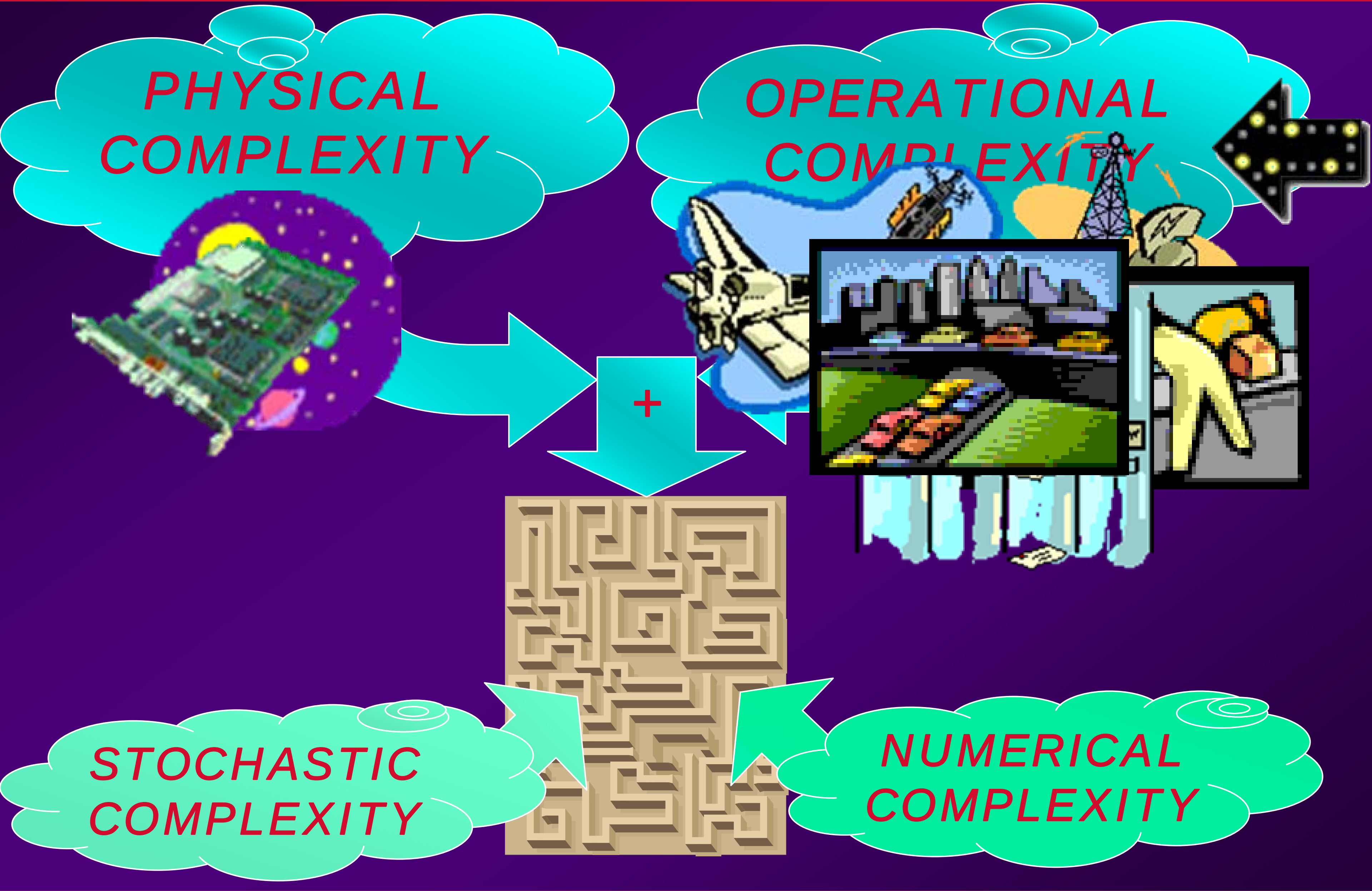
Boston University

cgc@bu.edu, http://vita.bu.edu/cgc

OUTLINE

- COMPLEXITY – FUNDAMENTAL LIMITS
- BARGAIN HUNTING IDEAS – AND PITFALLS…
  - SAMPLE PATH ANALYSIS
  - DECOMPOSITION
  - ABSTRACTION
  - SURROGATE PROBLEMS
  - HIGH PROBABILITY v CERTAINTY
- THOUGHTS ON MANAGING COMPLEXITY

COMPLEXITY



PHYSICAL COMPLEXITY
OPERATIONAL COMPLEXITY
+
STOCHASTIC COMPLEXITY
NUMERICAL COMPLEXITY

THREE FUNDAMENTAL *COMPLEXITY LIMITS*

$1/T^{1/2}$
LIMIT

NP-HARD
LIMIT

C

(e.g

O.
ACE

Tradeoff between
GENERALITY and **EFFICIENCY**
of an algorithm

[*Wolpert and Macready, IEEE TEC, 1997*]

NO-FREE-LUNCH
LIMIT

THREE FUNDAMENTAL *COMPLEXITY LIMITS*

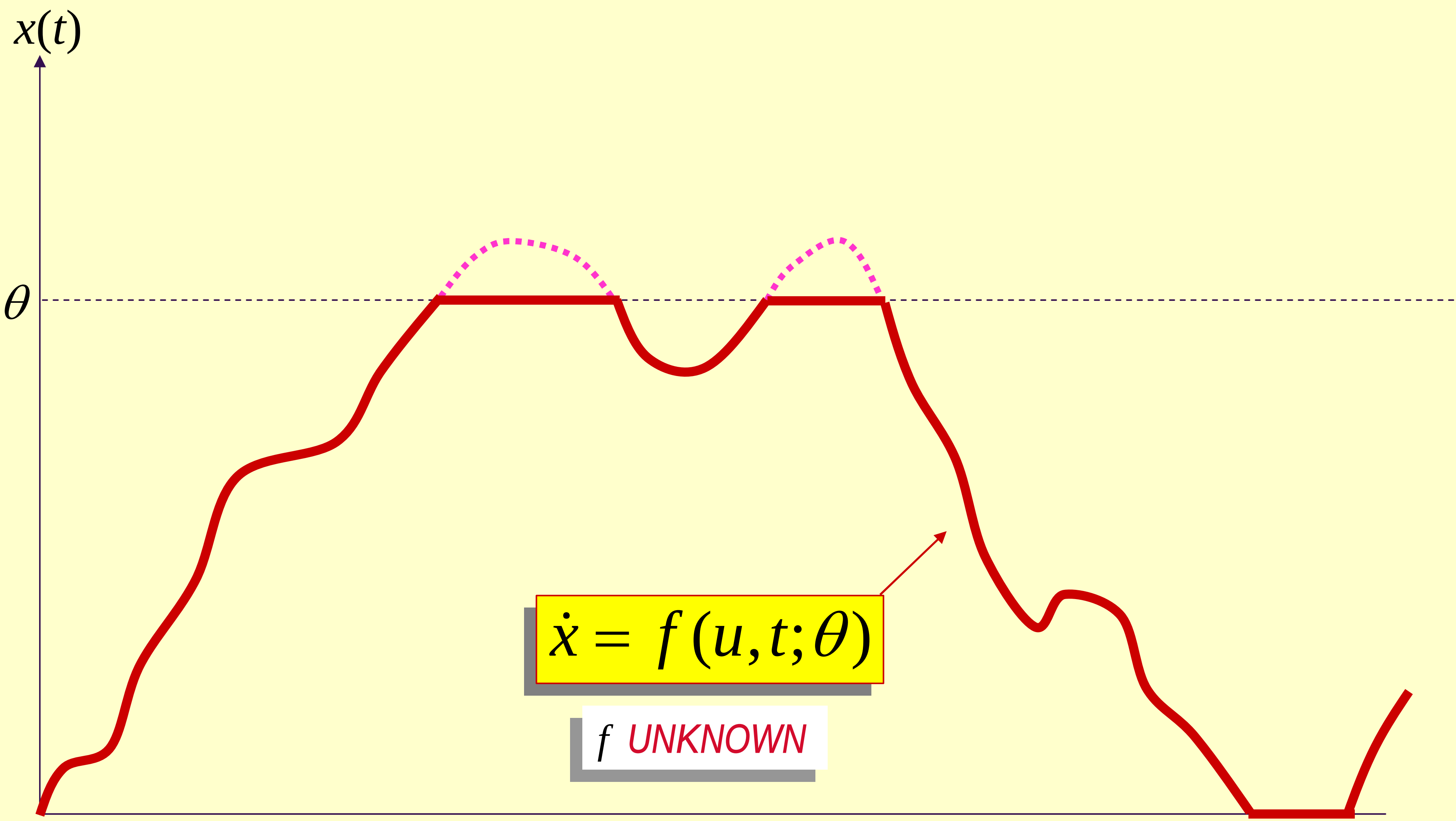
1/$T^{1/2}$
LI

NP-HARD
IMIT

N *H*
LIMIT

A “BARGAIN” EXAMPLE USING SAMPLE PATH ANALYSIS

A COMPLEX SYSTEM IN $[0, \theta]$



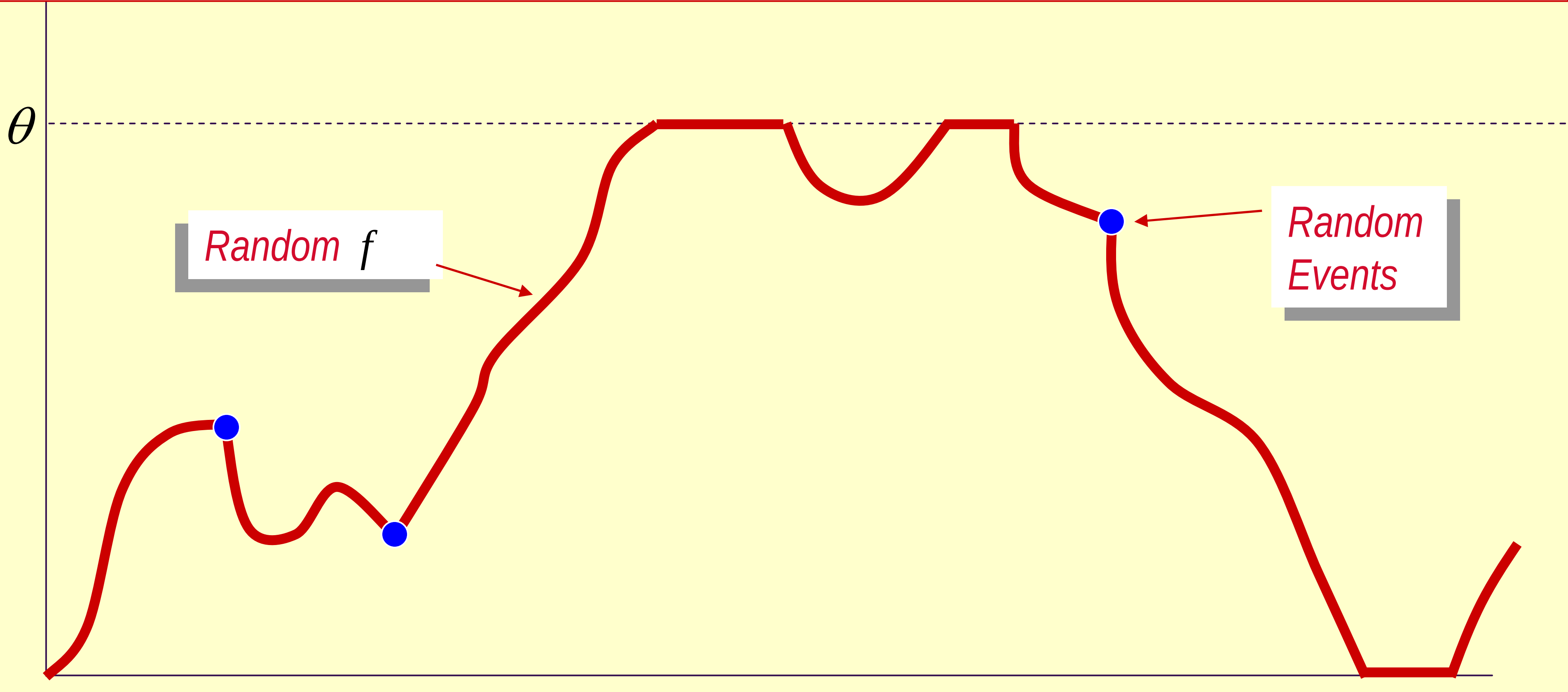
A COMPLEX SYSTEM IN $[0, \theta]$

CONTINUED

PROBLEM: Determine θ to minimize:

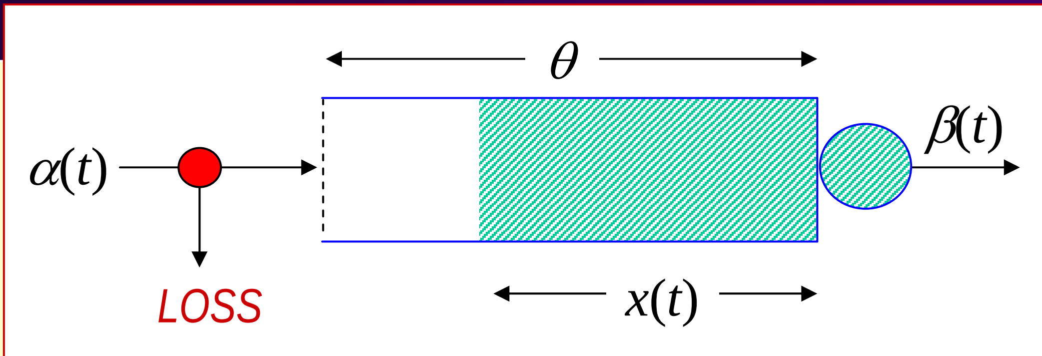
$$J_T(\theta) = \int_0^T L(x(t);\theta)dt$$

subject to $\dot{x} = f(u,t;\theta)$

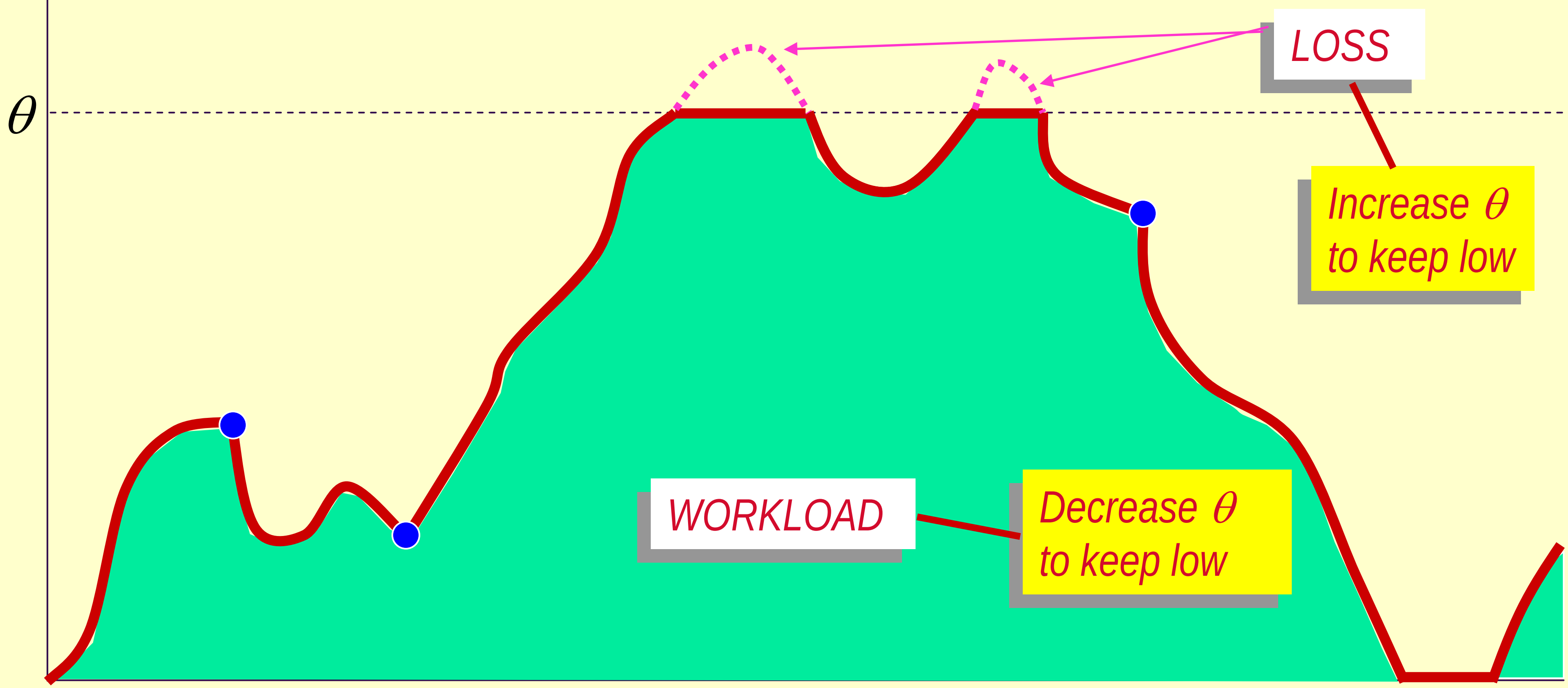


A COMPLEX SYSTEM IN $[0, \theta]$

CONTINUED

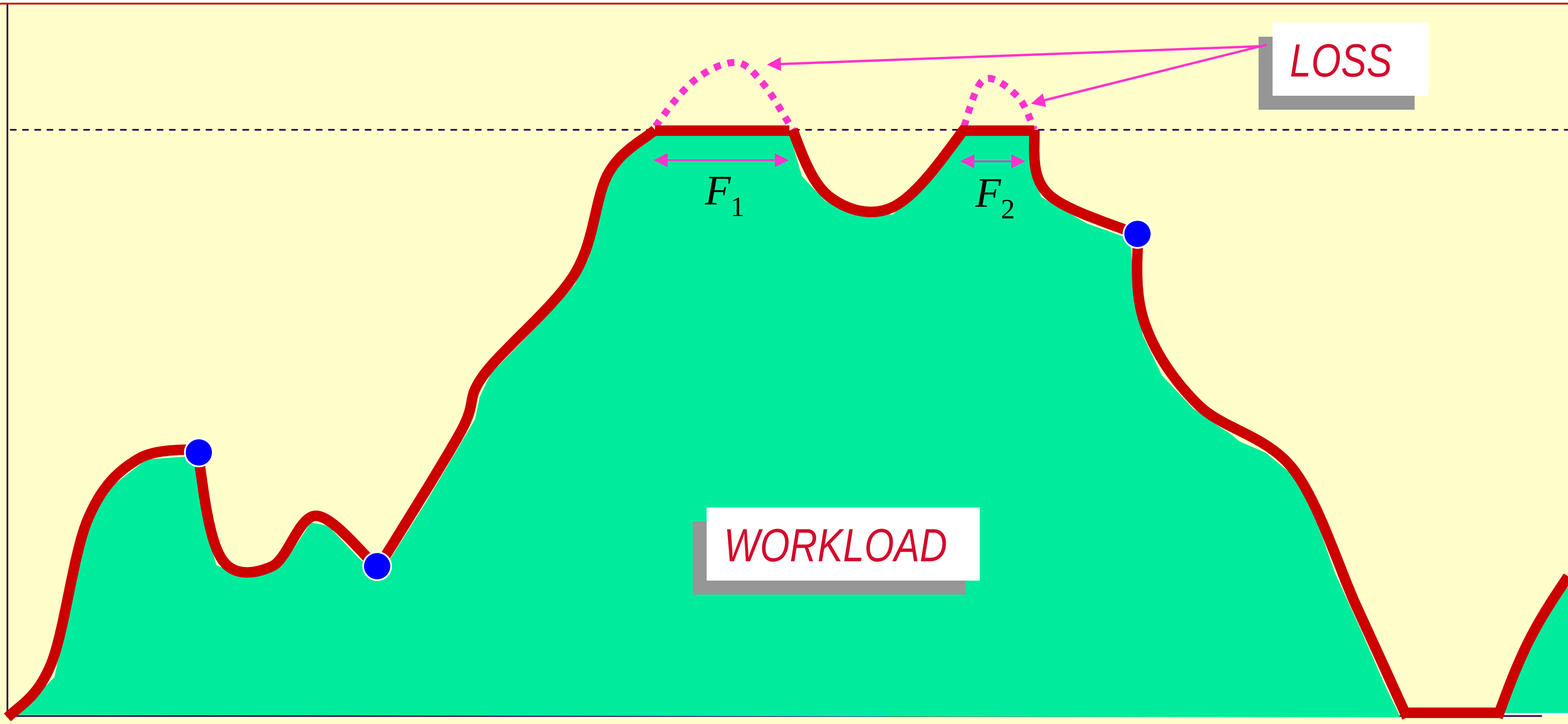


$$\dot{x} = \begin{cases} 0 & x(t) = 0,\ \alpha(t) - \beta(t) \le 0 \\ 0 & x(t) = \theta,\ \alpha(t) - \beta(t) \ge 0 \\ \alpha(t) - \beta(t) & \text{otherwise} \end{cases}$$



PROBLEM: Determine θ to trade off LOSS vs WORKLOAD:

$$E[L_T(\theta)] = E\left[\sum_{i=1}^{N_T} \int_{F_i} dx\right] \qquad E[Q_T(\theta)] = E\left[\int_0^T x\,dt\right]$$



Try and get :

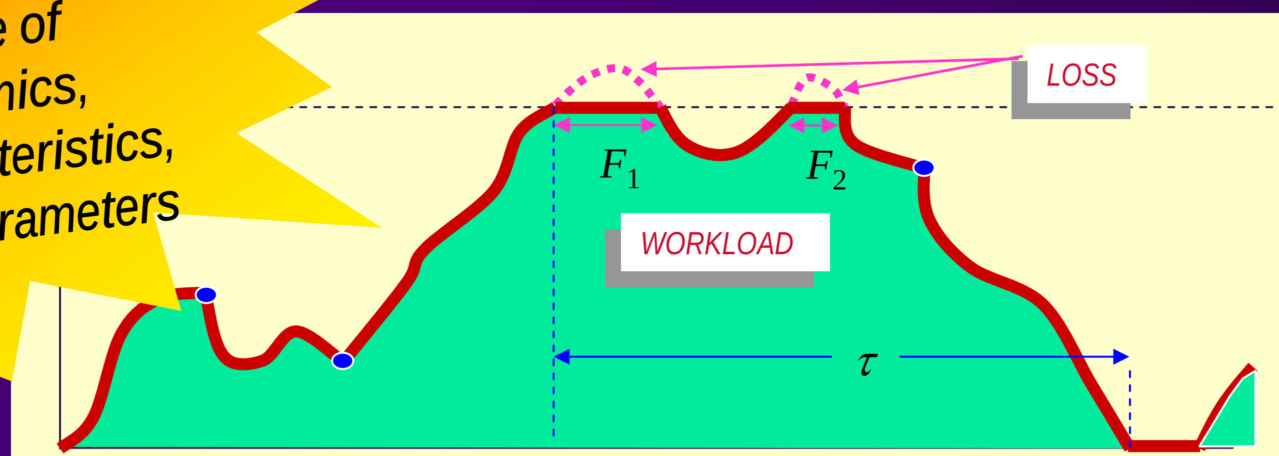
LOSS Sensitivity: $$\frac{dL_T}{d\theta}$$

WORKLOAD Sensitivity: $$\frac{dQ_T}{d\theta}$$

SENSITIVITY ANALYSIS - RESULTS

- $\Phi(\theta)$ = set of periods with at least one overflow interval
- LOSS Sensitivity: $$\frac{dL_T}{d\theta} = -\left|\Phi(\theta)\right|$$ UNBIASED ESTIMATE ← $$E\left[\frac{dL_T}{d\theta}\right] = \frac{d}{dt}E\left[L_T\right]$$
- τ_k = time between first overflow and end of period, $k \in \Phi(\theta)$
- WORKLOAD Sensitivity: $$\frac{dQ_T}{d\theta} = \sum_{k\in\Phi(\theta)} \tau_k$$ UNBIASED ESTIMATE ← $$E\left[\frac{dQ_T}{d\theta}\right] = \frac{d}{dt}E\left[Q_T\right]$$

No knowledge of detailed dynamics, stochastic characteristics, or even model parameters



THE MORAL…

- Often, partial knowledge of system dynamics is adequate to allow useful inferences from observed state trajectory data; in particular: SENSITIVITY INFORMATION

- This "bargain" applies to a large (but not universal) class of problems; otherwise, the NFL limit gets you!

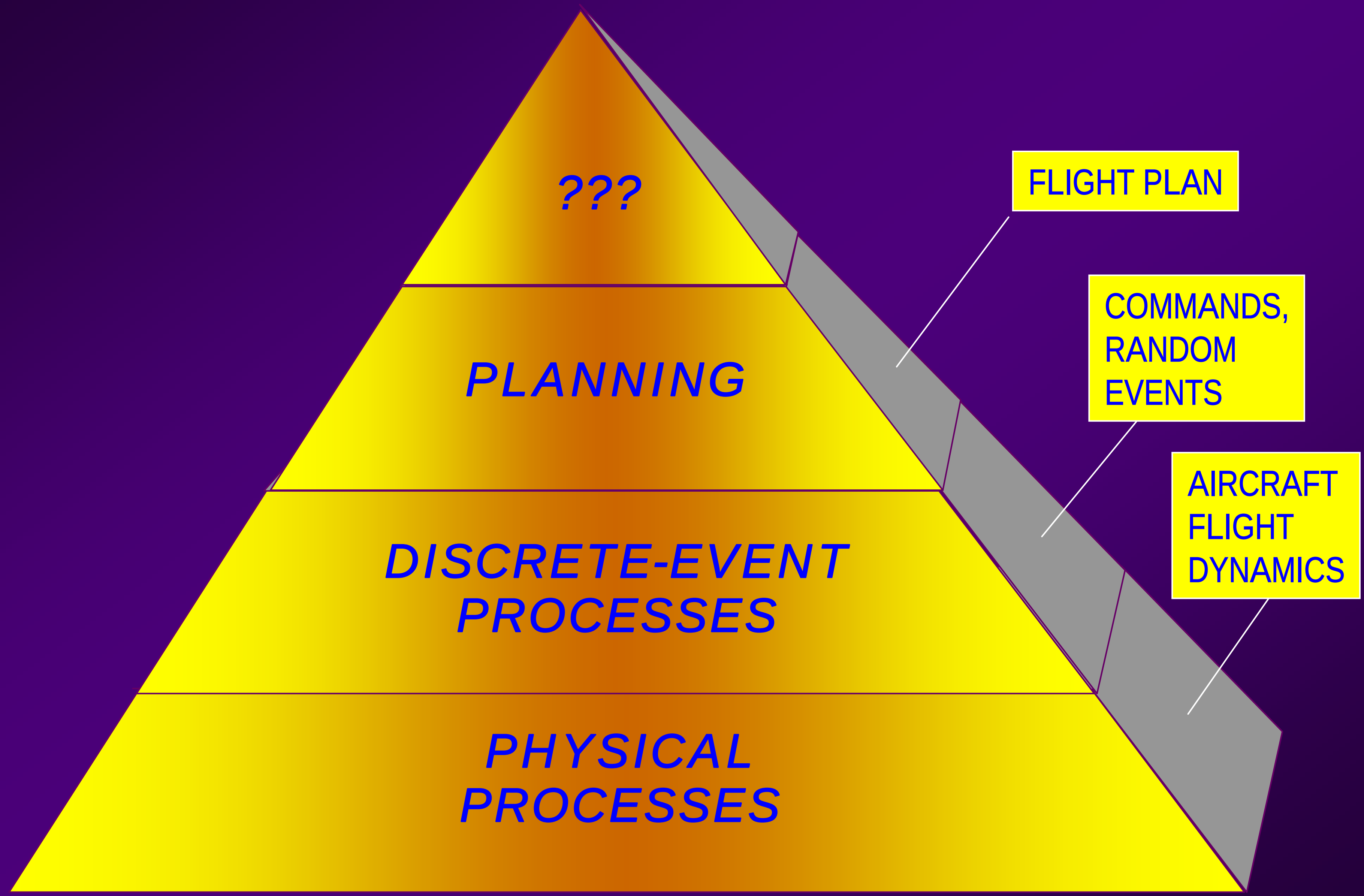
- What about this system with $\dot{x} = f(x, u, t; \theta)$?

  ➡ *Similar results, but more info. needed regarding model parameters*

  Feedback

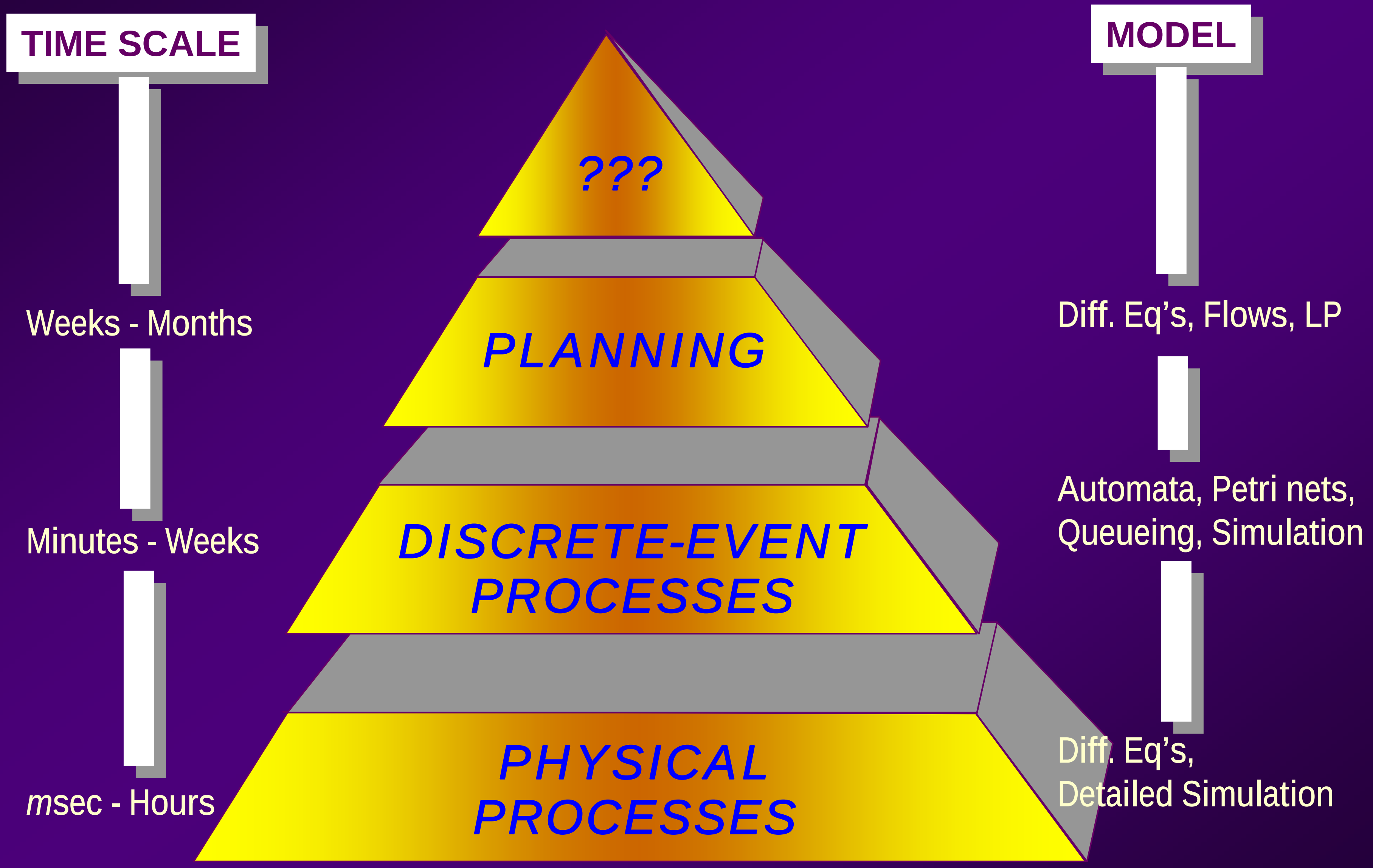
DECOMPOSITION AND ABSTRACTION

HIERARCHICAL DECOMPOSITION

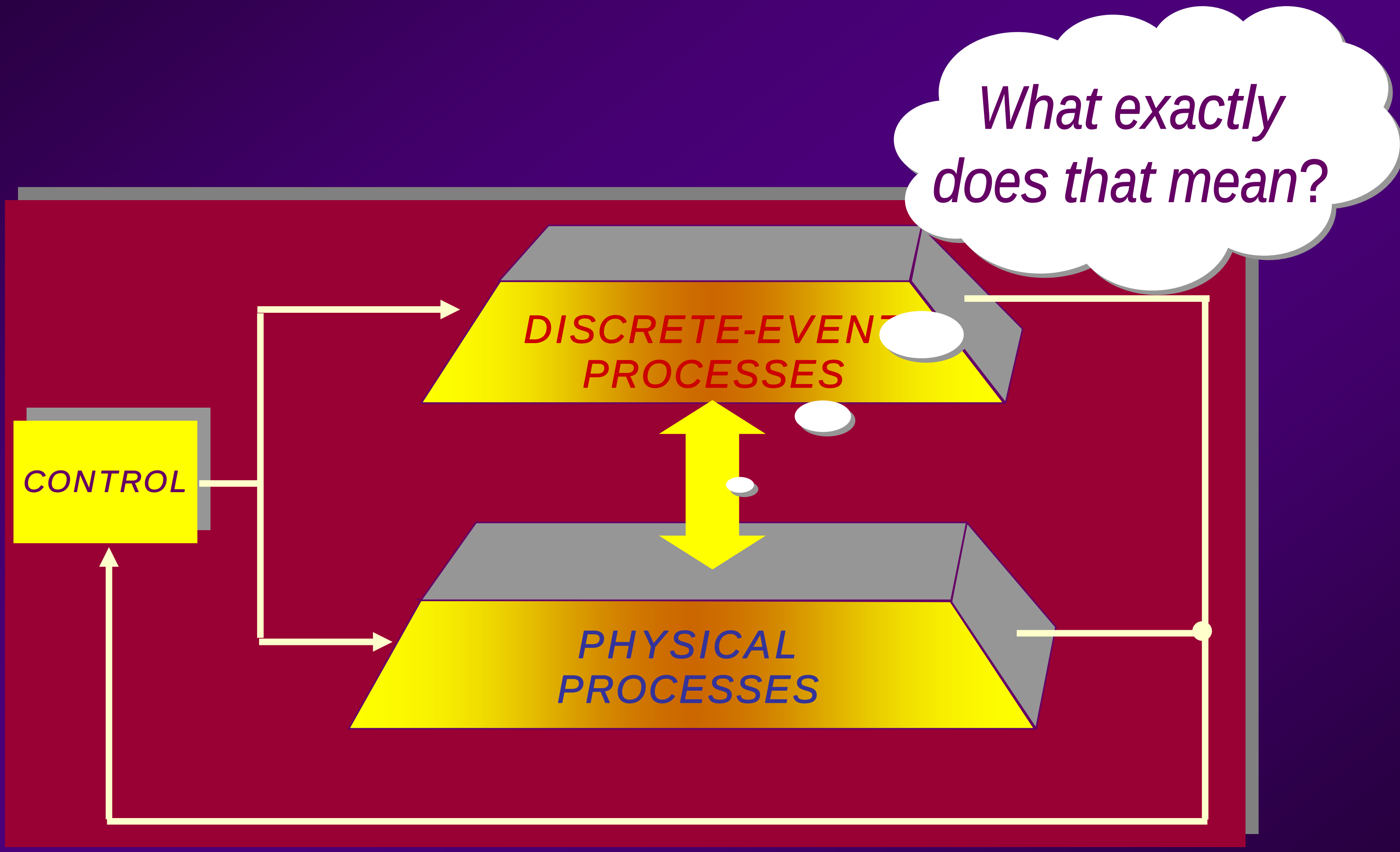


HIEARARCHICAL DECOMPOSITION

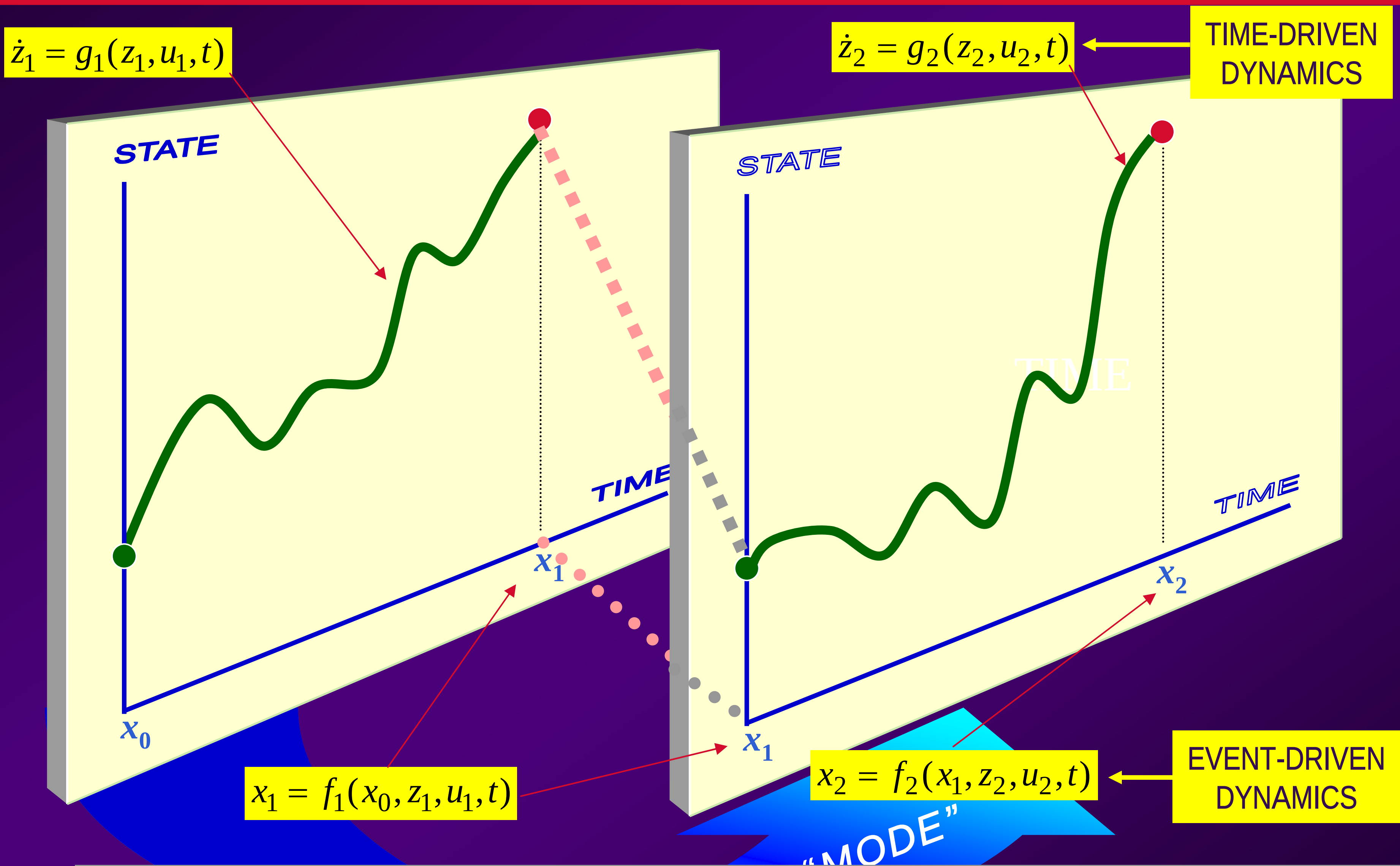
CONTINUED



HYBRID CONTROL SYSTEM

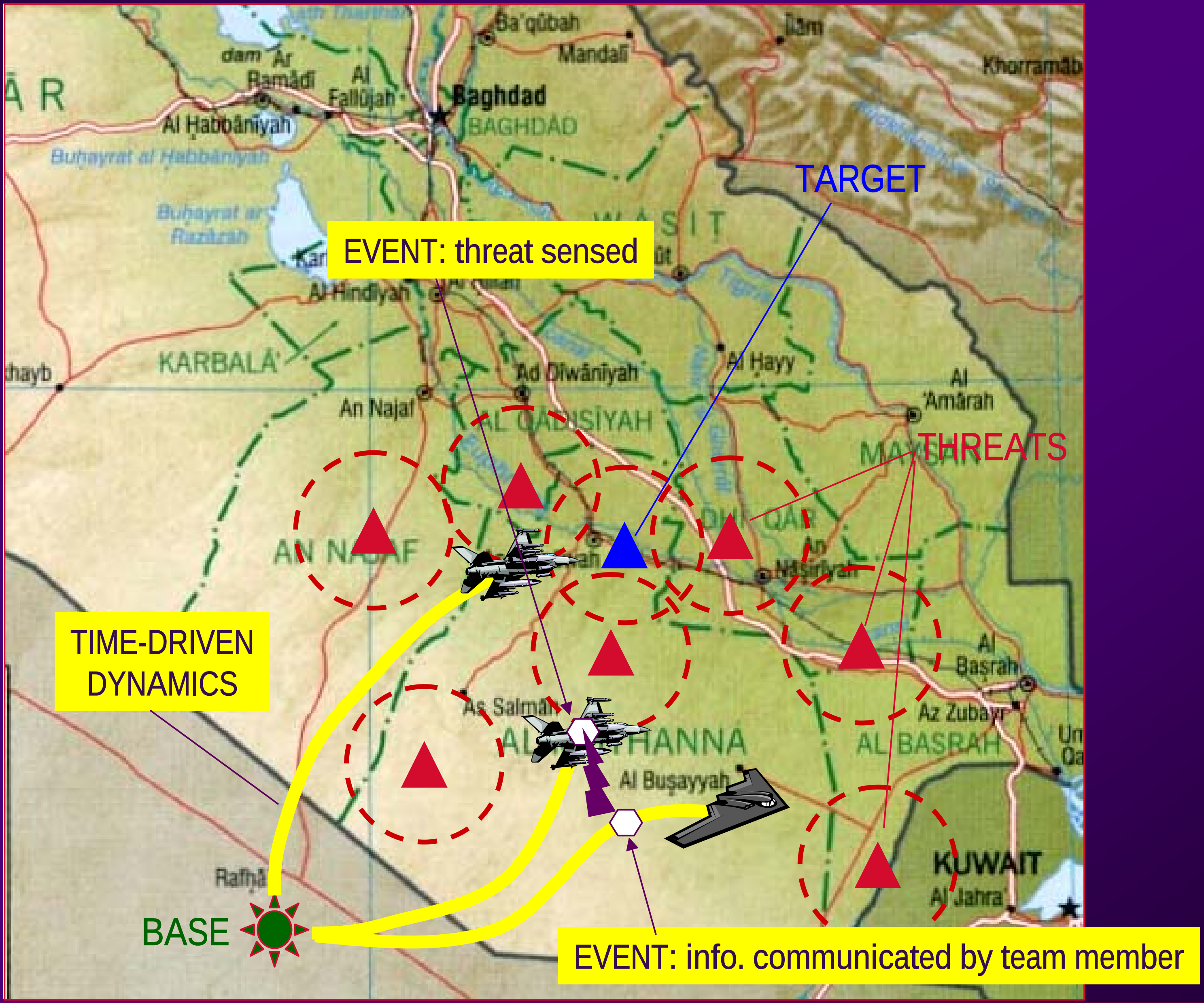


WHAT'S A HYBRID SYSTEM?



More on modeling frameworks, open problems, etc: [*Proc. of IEEE Special Issue (Antsaklis, Ed.), 2000*]

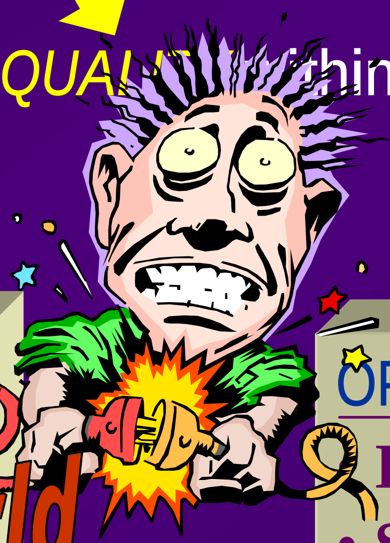
HYBRID SYSTEMS IN *COOPERATIVE CONTROL*



HYBRID SYSTEM IN *MANUFACTURING*

Key questions facing manufacturing system integrators:

- How to integrate *'process control'* with *'operations control'* ?
- How to improve product *QUALITY* within reasonable *TIME* ?



PROCESS CONTROL

- **Physicists**
- **Material Scientists**
- **Chemical Engineers**
- **…**

Time-Driven World

OPERATIONS CONTROL

- **Industrial Engineers, OR**
- **Schedulers**
- **Inventory Control**
- **…**

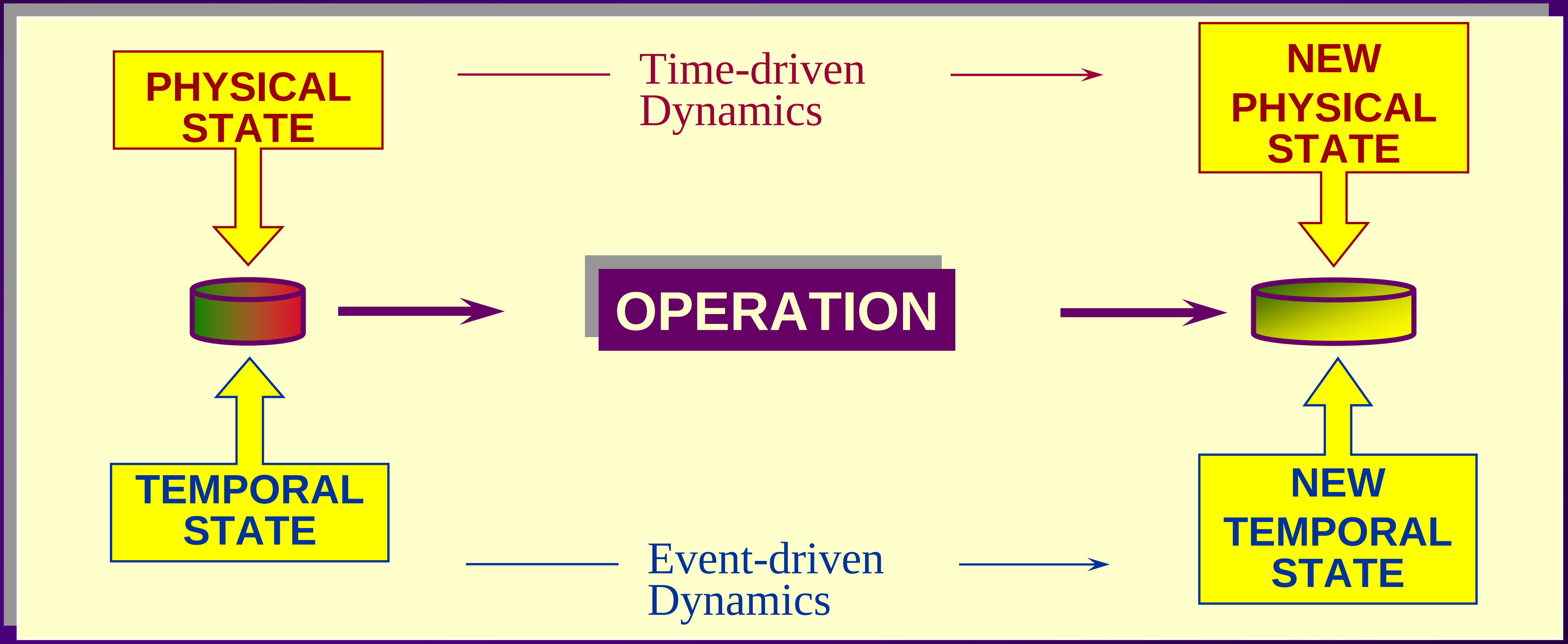
Event-Driven World

HYBRID SYSTEM IN *MANUFACTURING*

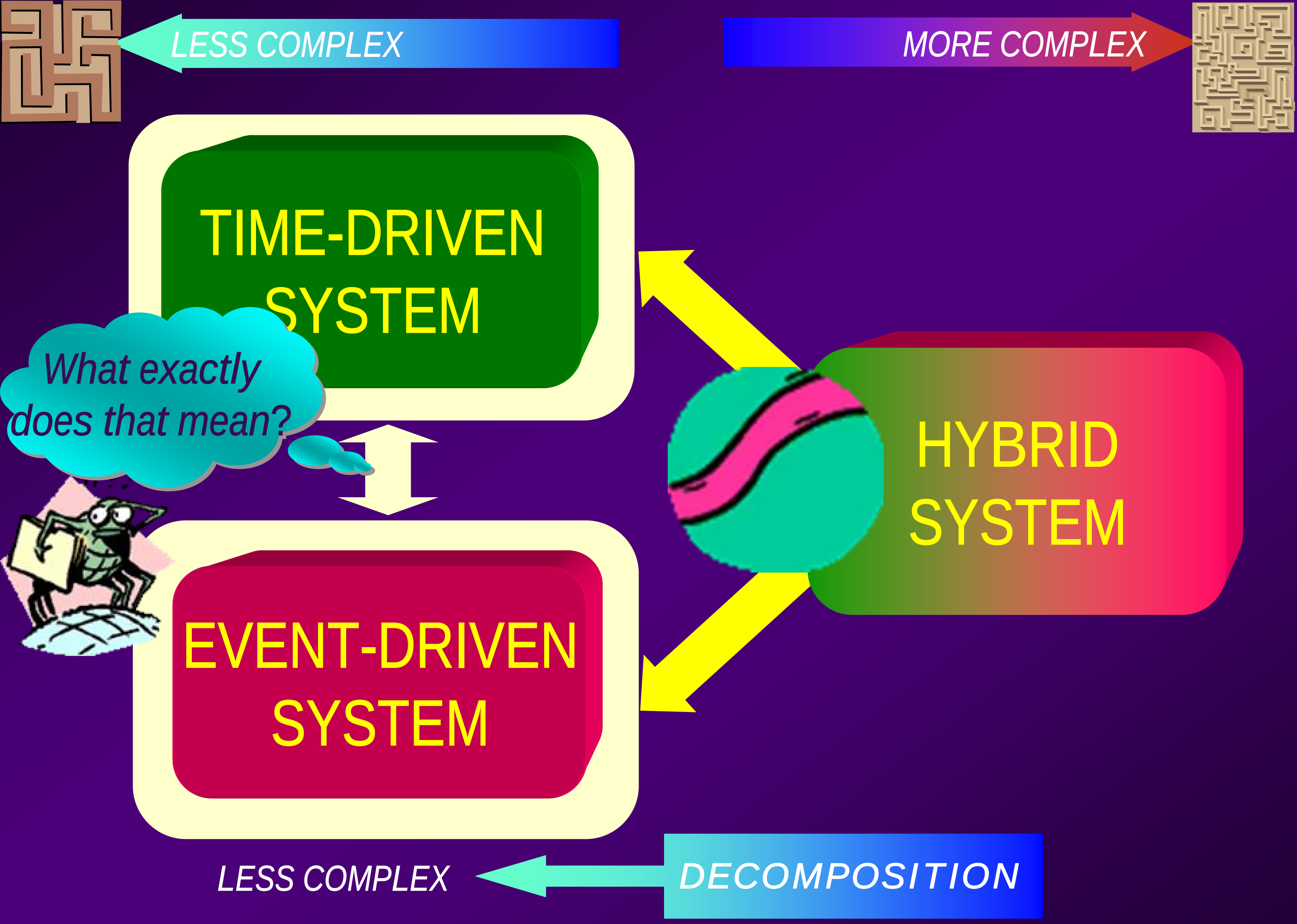
CONTINUED

Throughout a manuf. process, each part is characterized by

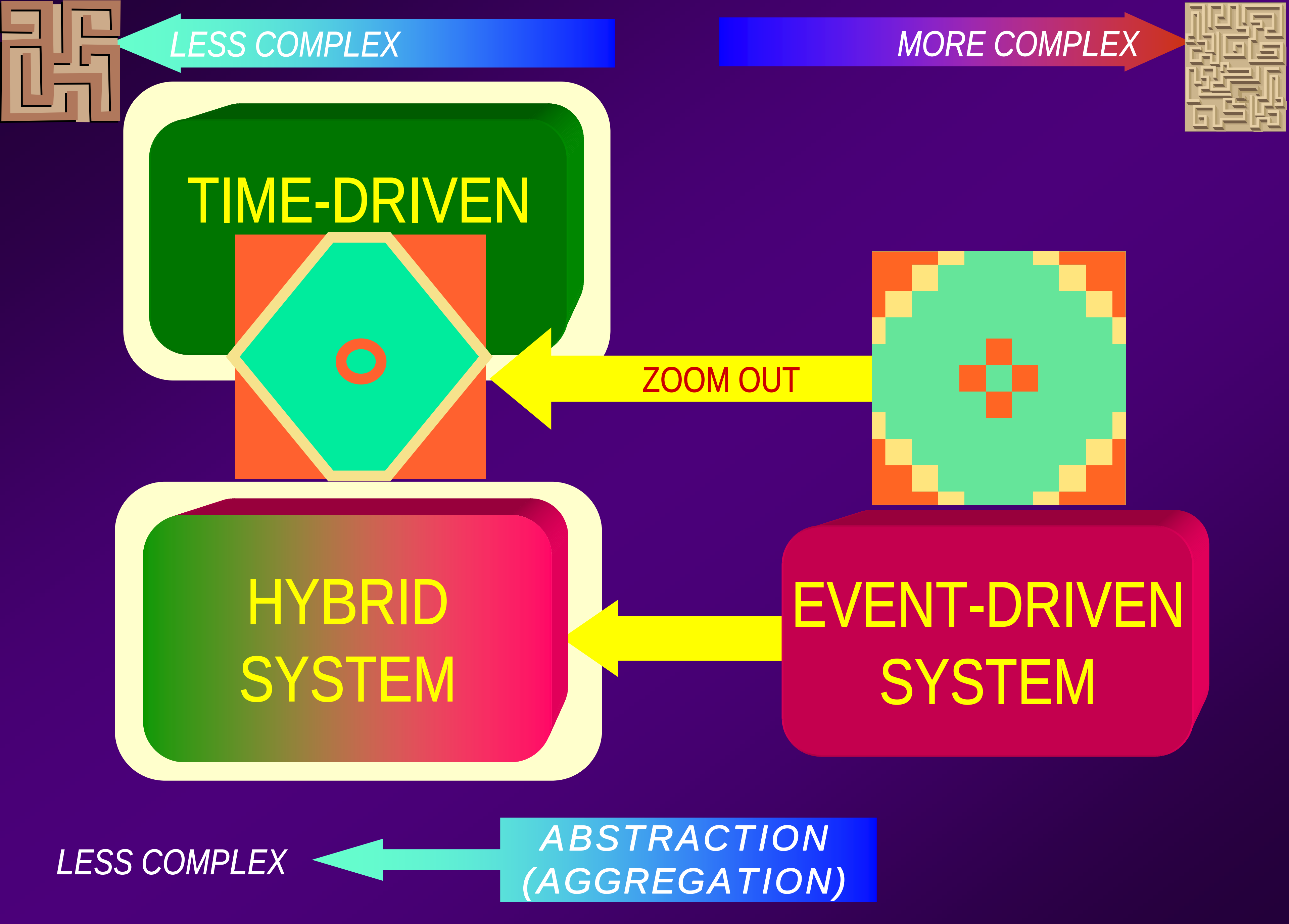
- A PHYSICAL state (e.g., size, temperature, strain)
- A TEMPORAL state (e.g., total time in system, total time to due-date)

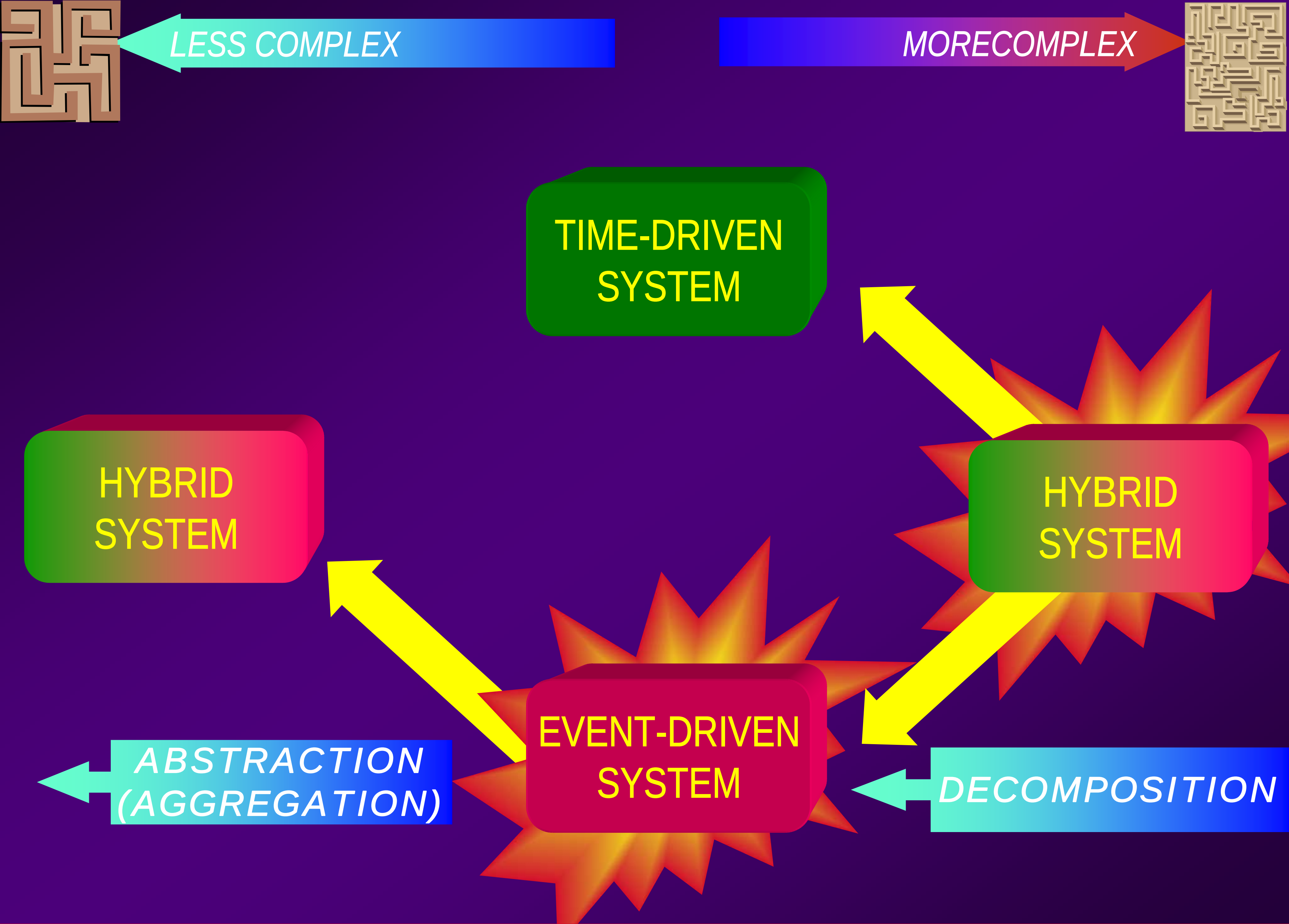


DECOMPOSITION


LESS COMPLEX
MORE COMPLEX
TIME-DRIVEN SYSTEM
What exactly does that mean?
EVENT-DRIVEN SYSTEM
HYBRID SYSTEM
LESS COMPLEX
DECOMPOSITION

ABSTRACTION (AGGREGATION)

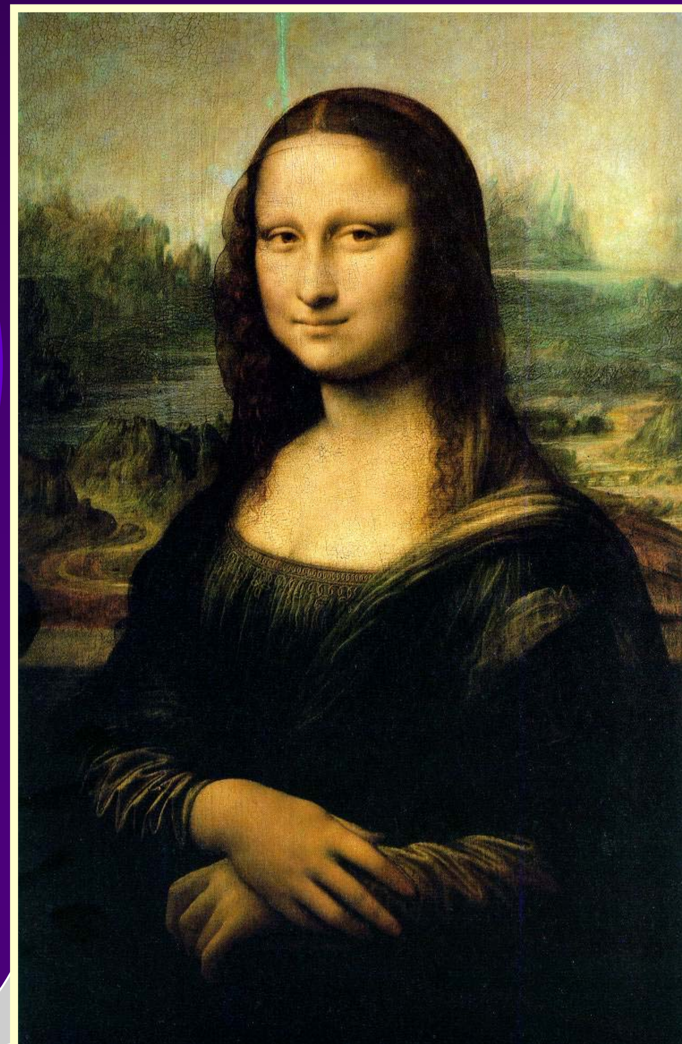

LESS COMPLEX
MORE COMPLEX
TIME-DRIVEN
ZOOM OUT
HYBRID SYSTEM
EVENT-DRIVEN SYSTEM
LESS COMPLEX
ABSTRACTION (AGGREGATION)


LESS COMPLEX
MORECOMPLEX
TIME-DRIVEN SYSTEM
HYBRID SYSTEM
HYBRID SYSTEM
EVENT-DRIVEN SYSTEM
ABSTRACTION (AGGREGATION)
DECOMPOSITION

WHAT IS THE RIGHT ABSTRACTION LEVEL ?

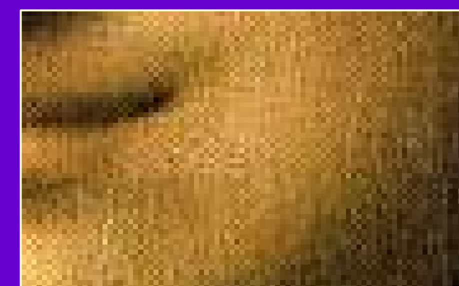
TOO FAR…

model not detailed enough



JUST RIGHT…

good model



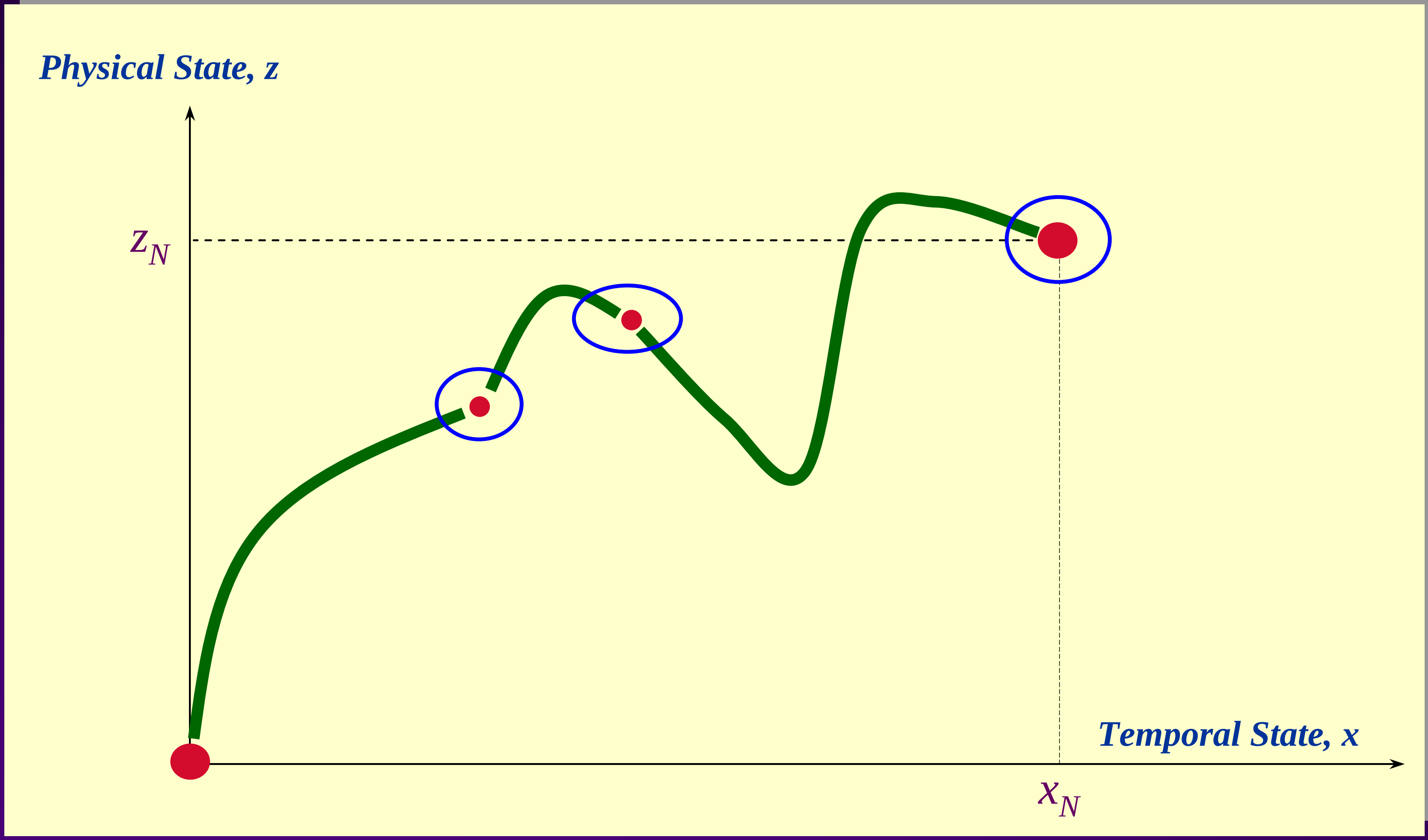
TOO CLOSE…

too much undesirable detail

CREDIT: W.B. Gong

DECOMPOSITION IN OPTIMAL CONTROL

OPTIMAL CONTROL PROBLEMS



- Get to desired final physical state z_N in minimum time x_N, subject to N-1 switching events
- Minimize: - deviations from N desired physical states $(z_i - q_i)^2$
  - deviations from target desired times $(x_i - \tau_i)^2$

OPTIMAL CONTROL PROBLEMS

Temporal state

$$\min_{\mathbf{u}} \sum_{i=1}^{N} \int_{x_{i-1}}^{x_i} L_i(z_i(t), u_i(t))dt$$

Physical state

Time-driven Dynamics

$$s.t. \begin{cases} \dot{z}_i = g_i(z_i, u_i, t) \\ x_{i+1} = f_i(x_i, u_i, t) \end{cases}$$

Event-driven Dynamics

OPTIMAL CONTROL PROBLEMS

CONTINUED

$$\min_{\mathbf{u}} \sum_{i=1}^{N} \left[\int_{x_{i-1}}^{x_i} L_i(z_i(t), u_i(t))dt + \psi_i(x_i) \right]$$

Cost under $u_i(t)$ *over* $[x_{i-1}, x_i]$

$$\phi_i(x_i, x_{i-1})$$

Cost of switching time x_i

$$\min_{\mathbf{u}} \sum_{i=1}^{N} \left[\phi_i(x_i, x_{i-1}) + \psi_i(x_i) \right]$$

Let: $s_i = x_i - x_{i-1}$

Time spent at ith operating mode

Assume: $\phi_i(x_i, x_{i-1}) = \phi_i(s_i)$

HIERARCHICAL DECOMPOSITION

$$\min_{\mathbf{u}} \sum_{i=1}^{N} \left[\phi_i(s_i) + \psi_i(x_i) \right]$$

s.t.

$$\dot{z}_i = g_i(z_i, u_i, t)$$

$$x_{i+1} = f_i(x_i, u_i, t)$$

HIGHER LEVEL PROBLEM:

$$\min_{\mathbf{s}} \sum_{i=1}^{N} \left[\phi_i^*(s_i) + \psi_i(x_i) \right]$$

s.t.

$$x_{i+1} = f_i(x_i, s_i, t)$$

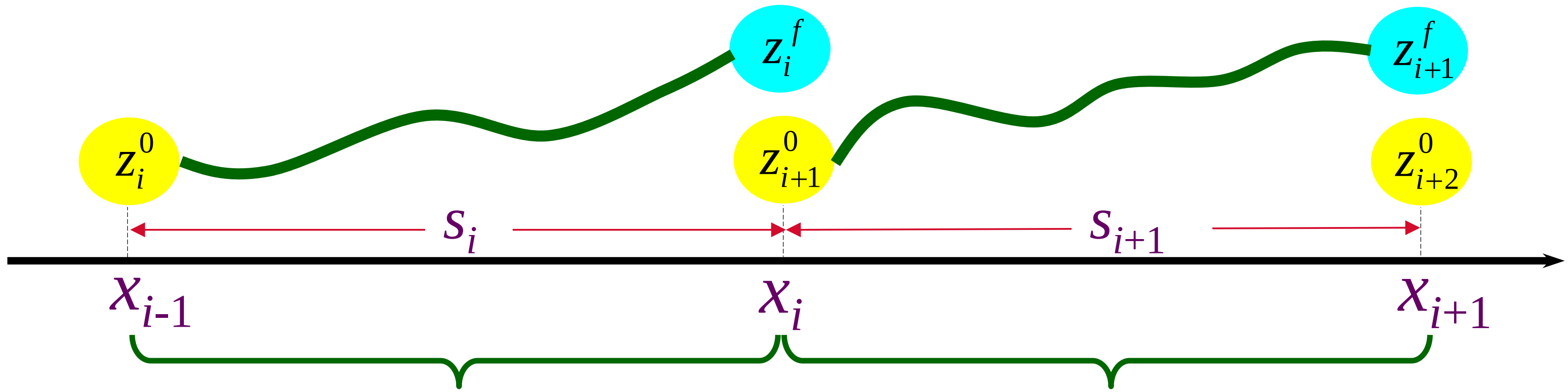
LOWER LEVEL PROBLEMS:

$$\min_{u_i} \phi_i(s_i) = \int_0^{s_i} L_i(z_i(t), u_i(t))\,dt$$

s.t.

$$\dot{z}_i = g_i(z_i, u_i, t)$$

FIXED s_i



$$\min_{u_i(z_i^0, z_i^f, s_i)} \phi_i(s_i) \qquad \min \phi_{i+1}(s_{i+1})$$

$$u_i^*(z_i^0, z_i^f, s_i)$$

$$\theta_i(z_i^0, z_i^f, s_i) = \min_{u_i} \phi_i(z_i, u_i, s_i)$$

"ROUTINE" OPTIMAL CONTROL PROBLEM!

$$\min_{\mathbf{z}^0, \mathbf{z}^f, \mathbf{s}} \sum_{i=1}^{N} \left[\theta_i(z_i^0, z_i^f, s_i) + \psi_i(x_i) \right]$$

s.t.

$$x_{i+1} = f_i(x_i, u_i, t)$$

***REALLY* CHALLENGING PROBLEM!**

Typical example:

$$x_{i+1} = \max(x_i, a_{i+1}) + s_i(z_i, u_i)$$

Decomposition works well in this case…

…but we still have to solve
all LOWER-LEVEL problems
and a HIGHER-LEVEL problem

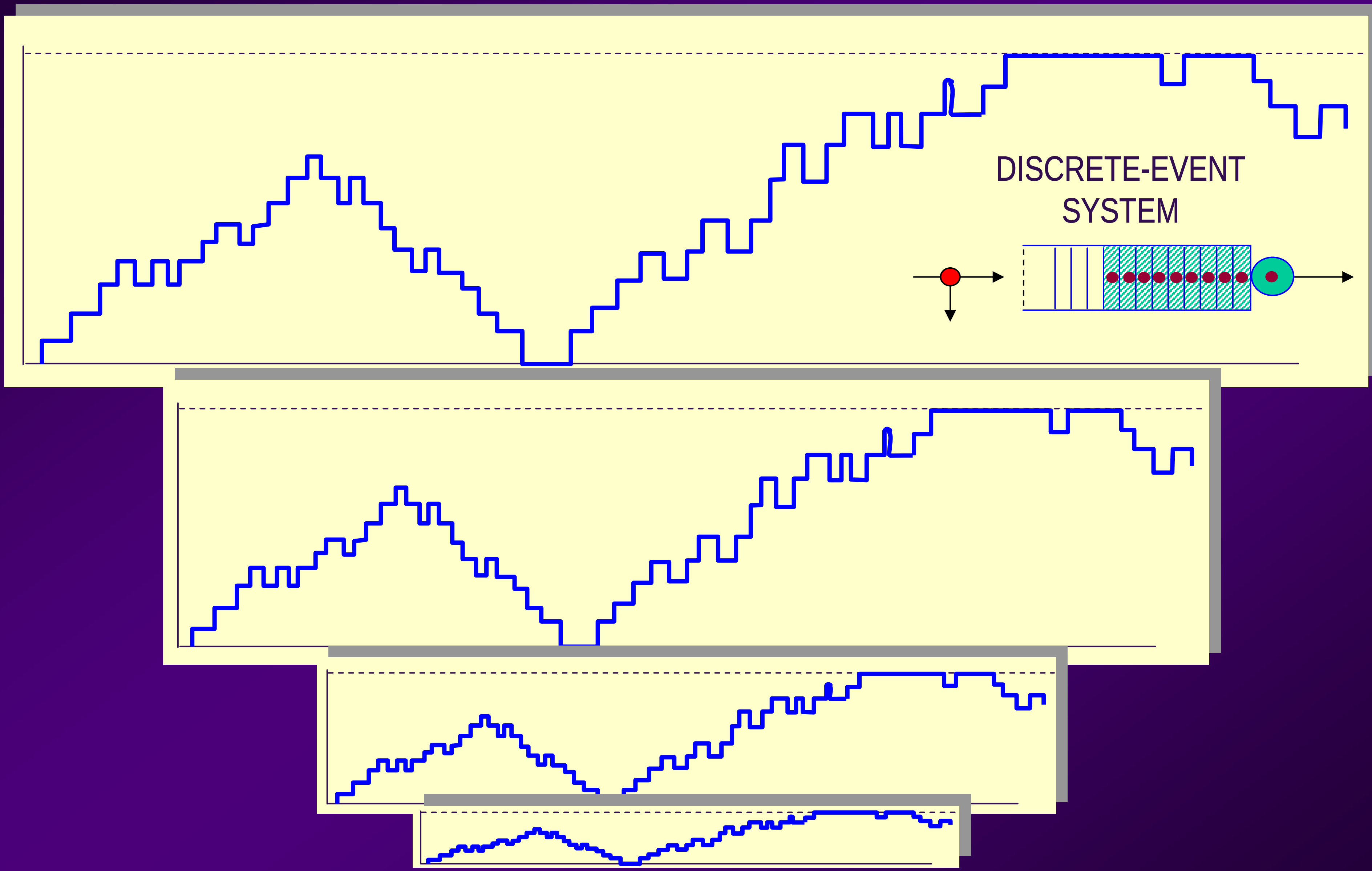
[Gokbayrak and Cassandras, 2000] *[Xu and Antsaklis, 2000]*

…and there is also the issue of
selecting among many possible
modes to switch to

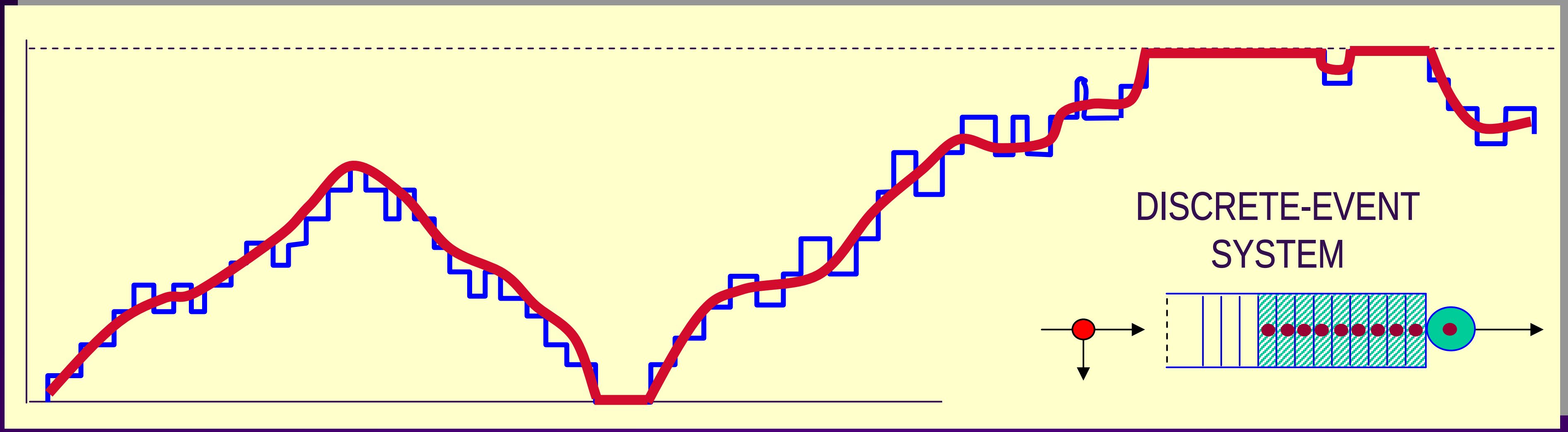
[Bemporad et al, 2000]

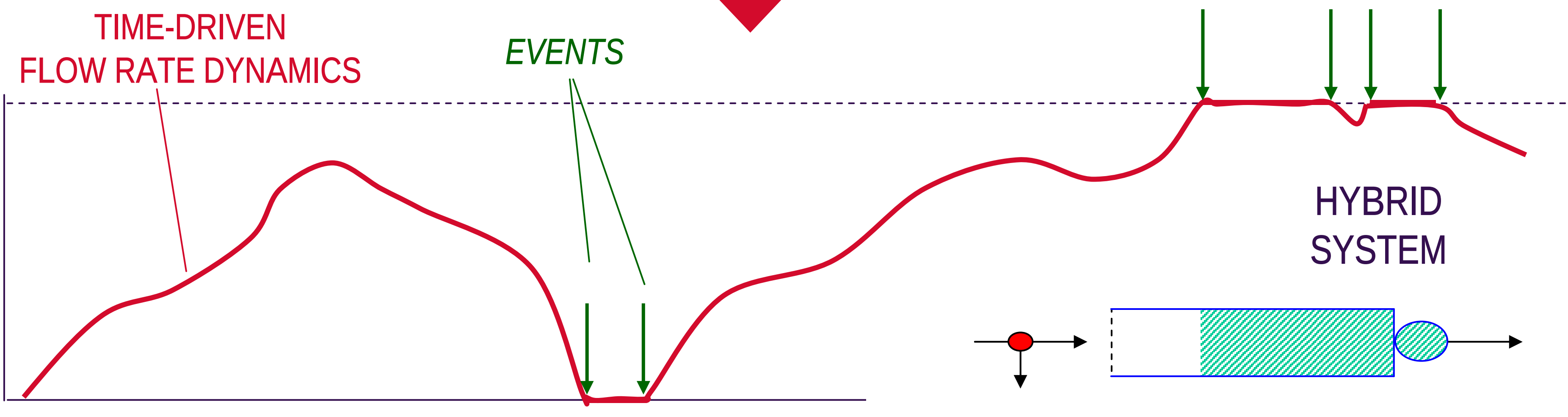
ABSTRACTION

ABSTRACTION OF A DISCRETE-EVENT SYSTEM

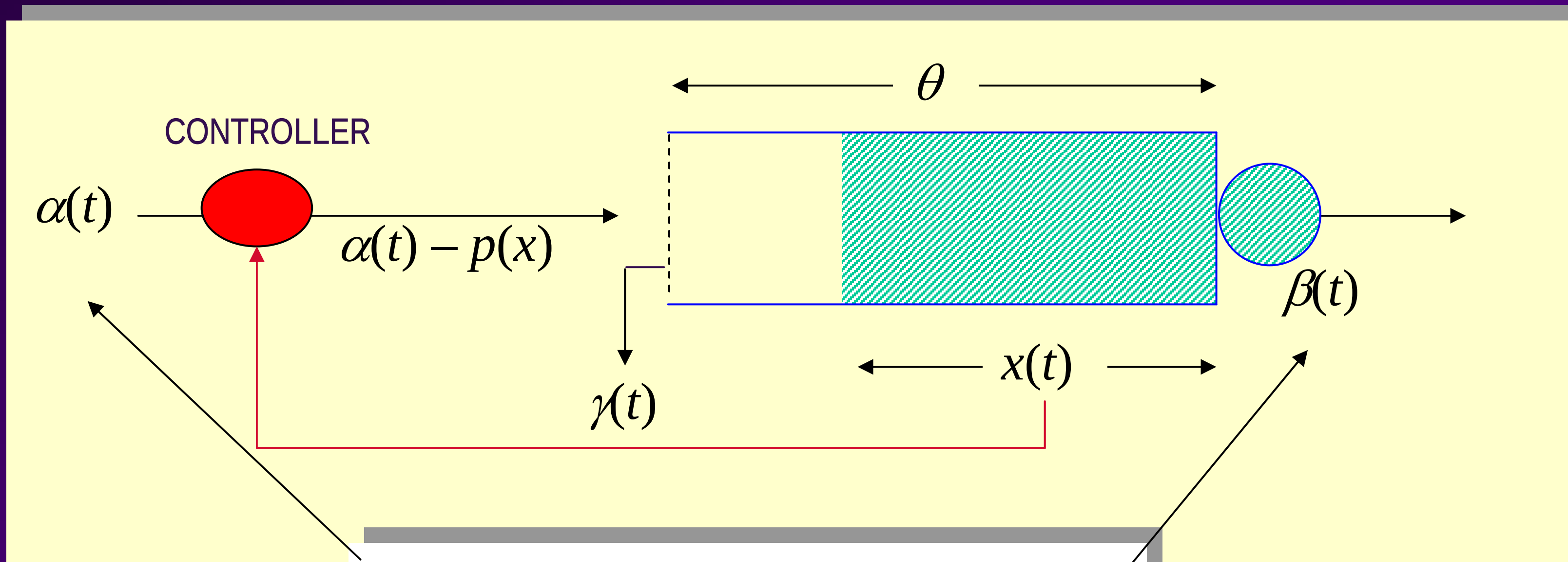


ABSTRACTION OF A DISCRETE-EVENT SYSTEM





STOCHASTIC FLOW MODELS (SFM)



$\alpha(t)$, $\beta(t)$: *arbitrary stochastic processes (piecewise continuously differentiable)*

$$\frac{dx}{dt} = \begin{cases} 0 & x(t) = 0,\ \lambda(t) - p(0) \le 0 \\ 0 & x(t) = \theta,\ \lambda(t) - p(\theta) \ge 0 \\ \lambda(t) - p(x(t)) & \text{otherwise} \end{cases}$$

$\lambda(t) = \alpha(t) - \beta(t)$

feedback

WHY SFM?

- “Lower resolution” model of “real” system intended to capture *just enough* info. on system dynamics

- Aggregates many events into simple continuous dynamics, preserves only events that cause drastic change

  $\Rightarrow$ computationally efficient

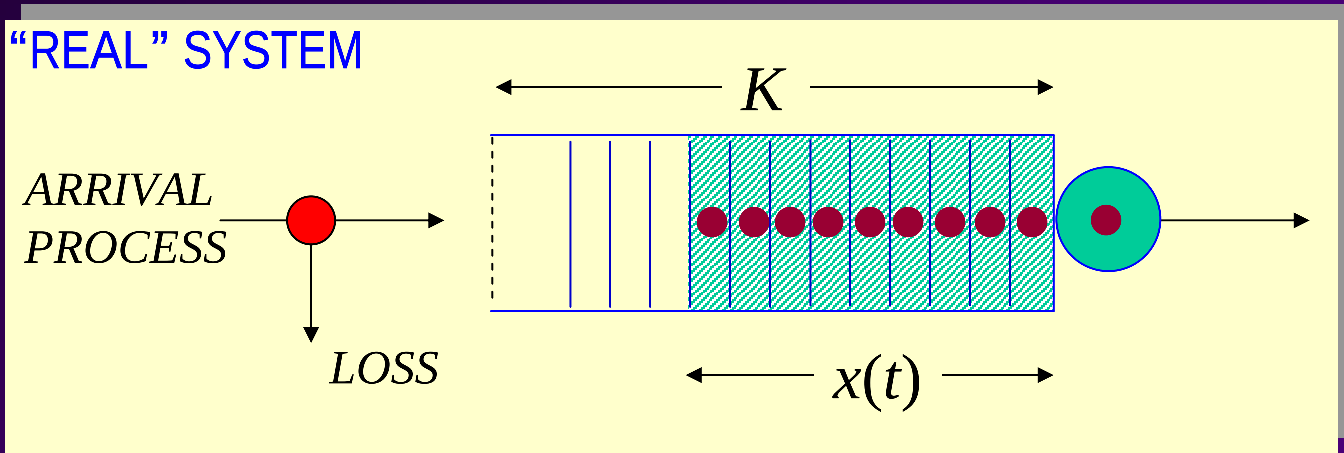
  (e.g., *orders of magnitude faster simulation*)

- If used for CONTROL purposes,

  another “BARGAIN” opportunity arises…

AN EXAMPLE OF A “BARGAIN” USING A “SURROGATE” PROBLEM

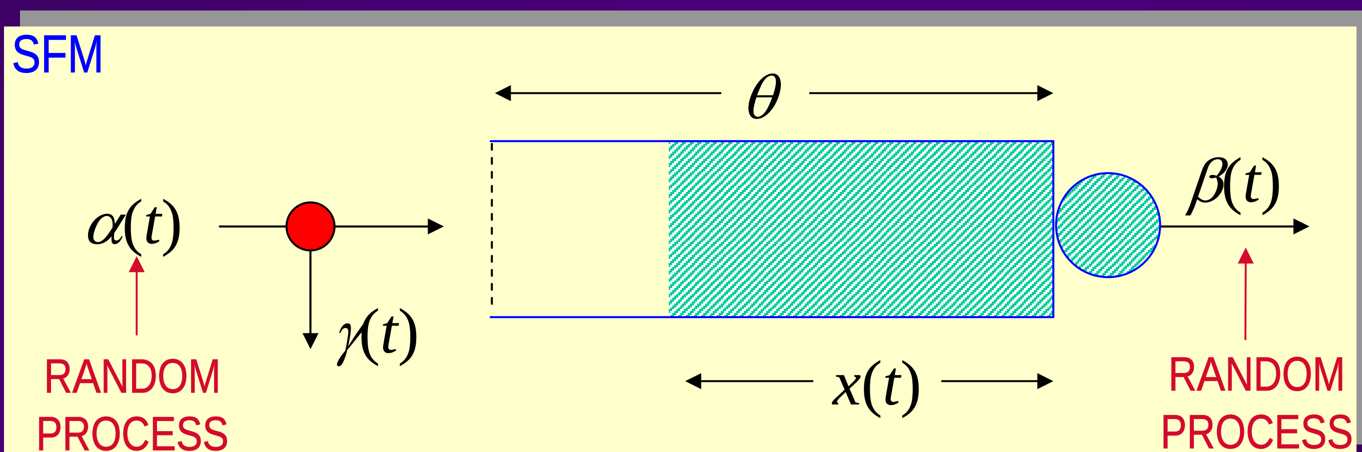
THRESHOLD BASED BUFFER CONTROL



$L(K)$: Loss Rate

$Q(K)$: Mean Queue Lenth

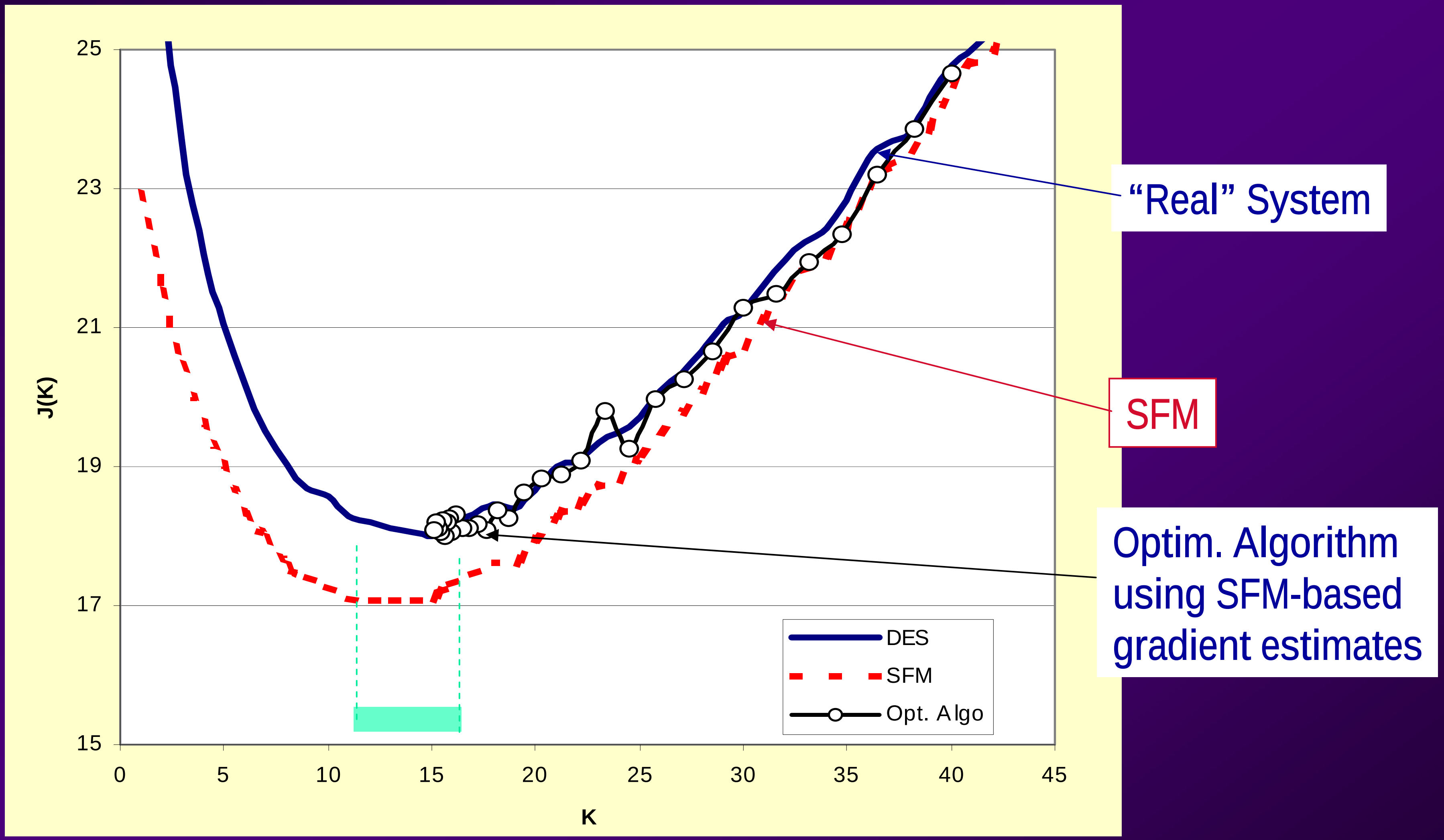
PROBLEM: Determine K to minimize $[Q(K) + R \cdot L(K)]$



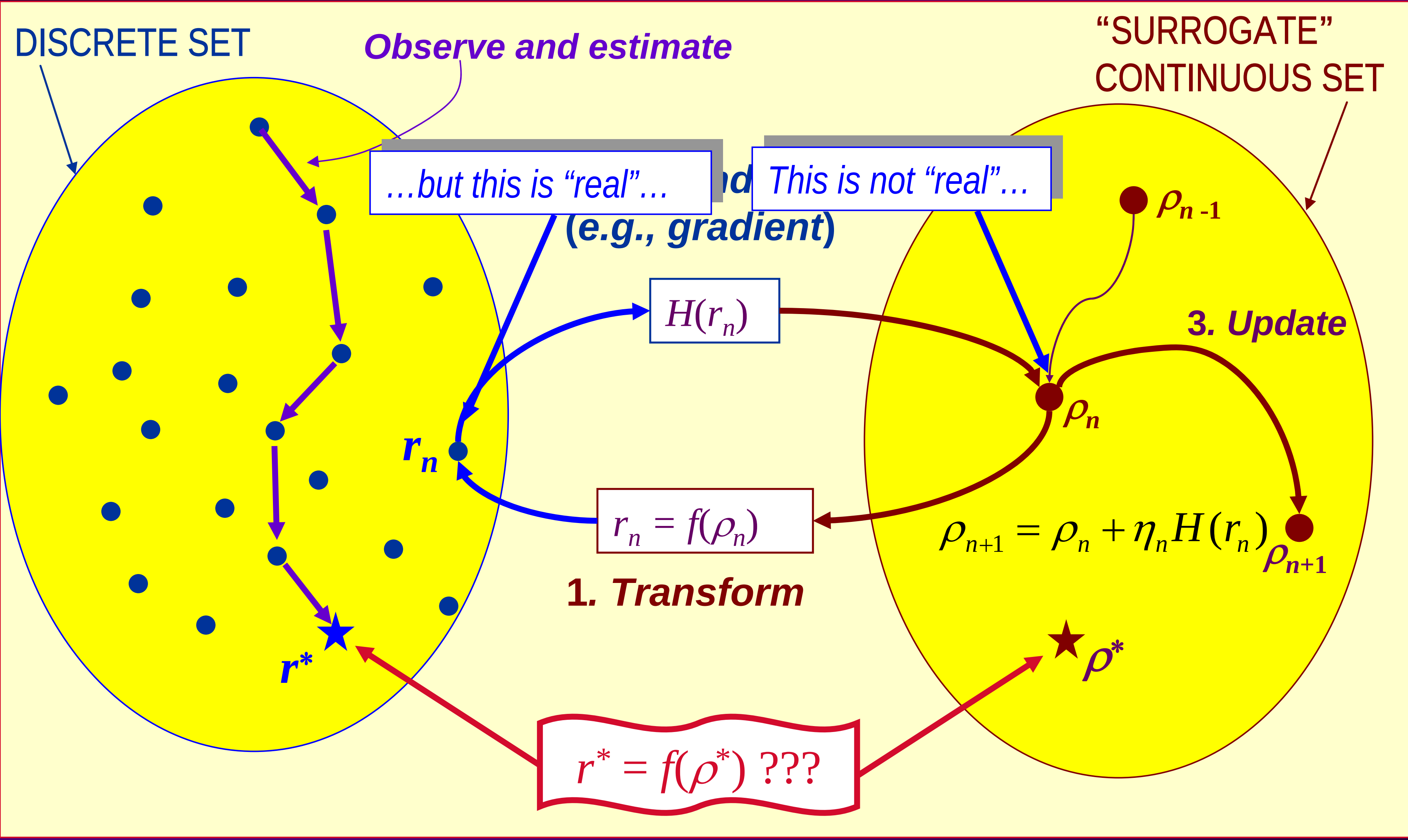
SURROGATE PROBLEM: Determine θ to minimize $[Q^{SFM}(\theta) + R \cdot L^{SFM}(\theta)]$

THRESHOLD BASED BUFFER CONTROL

CONTINUED



"SURROGATE" PROBLEM IDEA



WHEN DOES THIS *PROVABLY* WORK?

- Need some structural properties; otherwise, the NFL limit gets you!

- Similarities to Ordinal Optimization

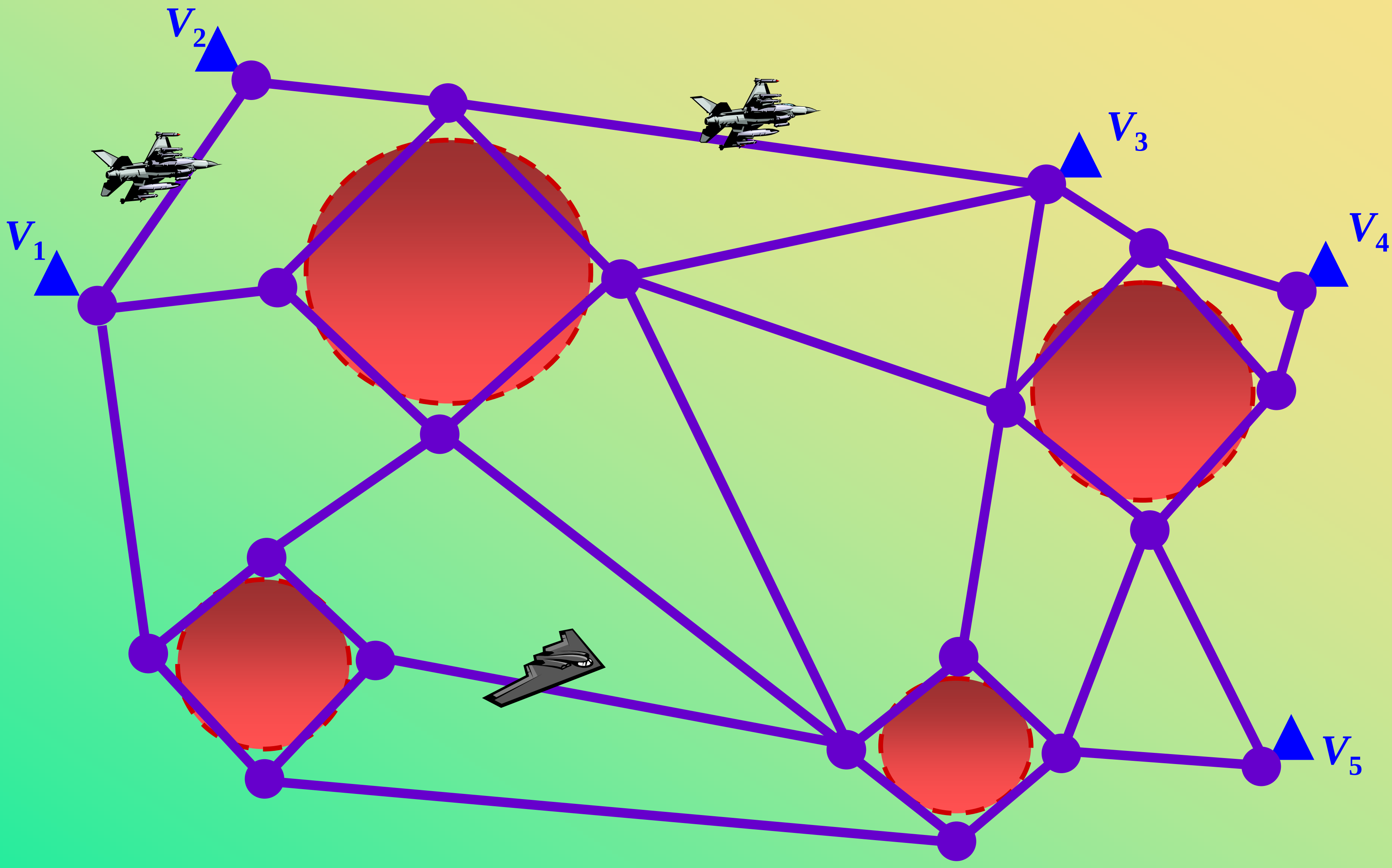
  [*Ho et al, JDEDS 1992*]

- Resource allocation problems

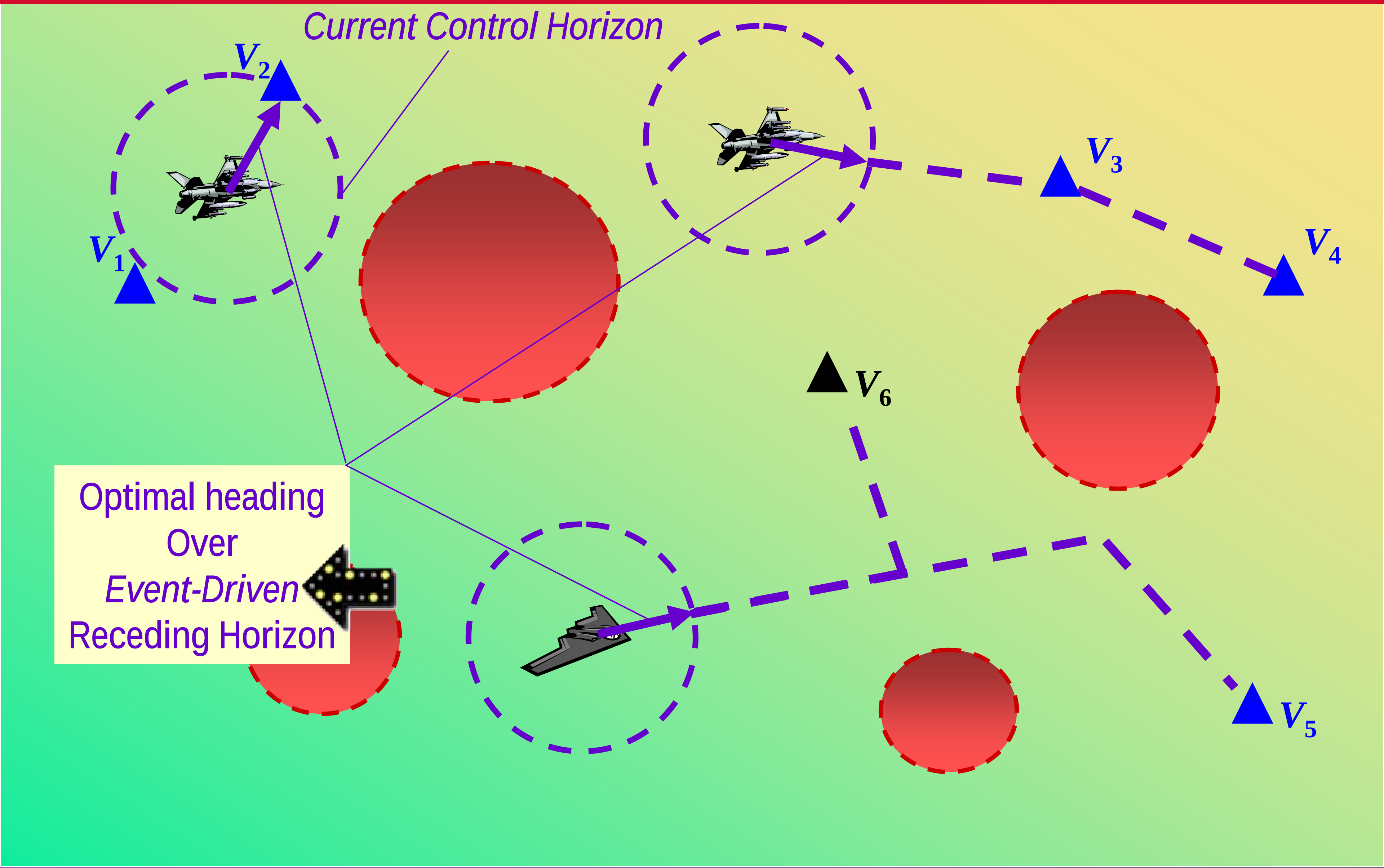
  [*Gokbayrak and Cassandras, JOTA 2002*]

- Cooperative control problems – *see Session FrA06*

BYPASSING COMPLEXITY IN *COOPERATIVE CONTROL* *CONTINUED*



BYPASSING COMPLEXITY IN *COOPERATIVE CONTROL* *CONTINUED*

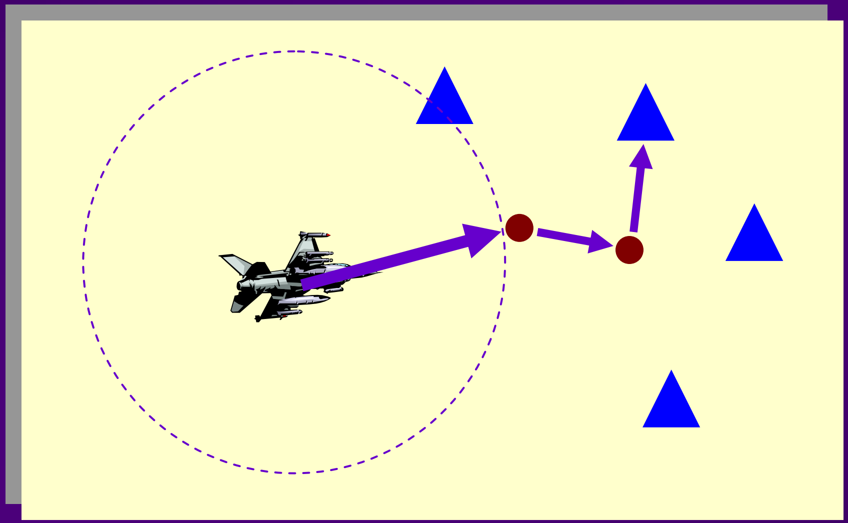


BYPASSING COMPLEXITY IN *COOPERATIVE CONTROL* *CONTINUED*

MAIN IDEA:

Replace complex *Discrete Stochastic Optimization* problem by a sequence of simpler *Continuous Optimization* problems

But how do we guarantee that vehicles actually head for desired DISCRETE POINTS?



It turns out they do!

Can replace HARD problem by several SIMPLER ones…

DANGERS OF DECOMPOSITION, ABSTRACTION

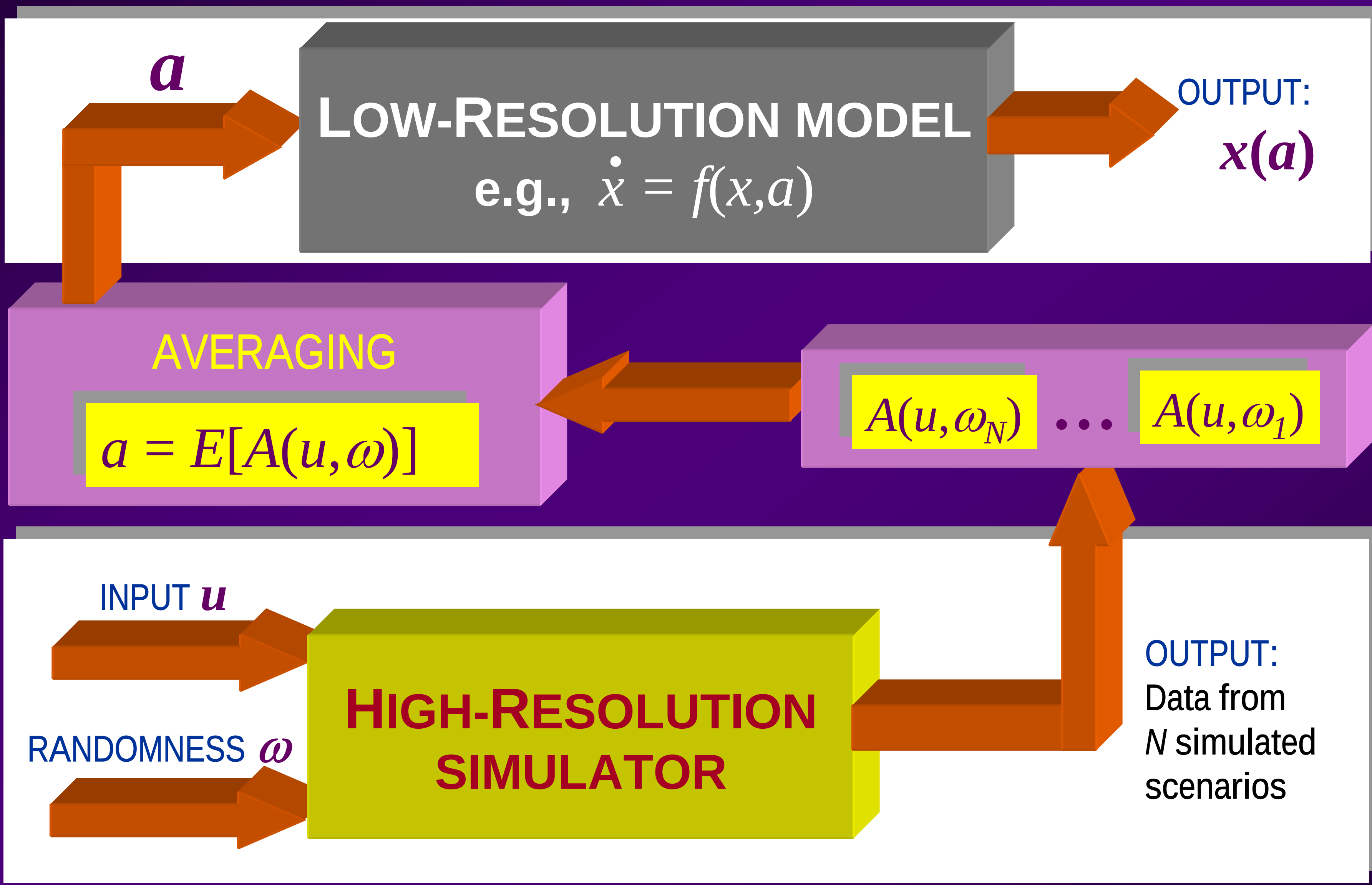
What exactly does that mean?

TIME-DRIVEN SYSTEM

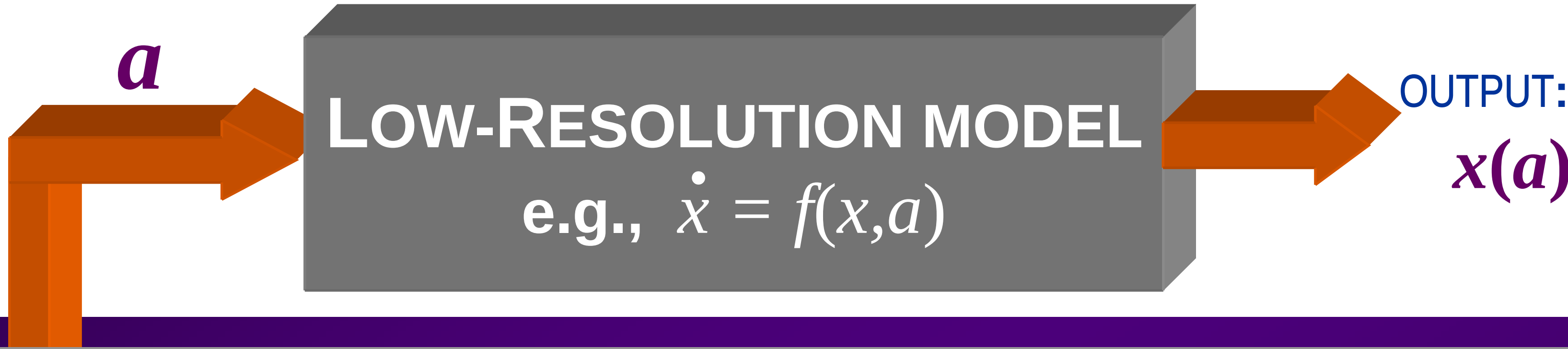
↕

EVENT-DRIVEN SYSTEM

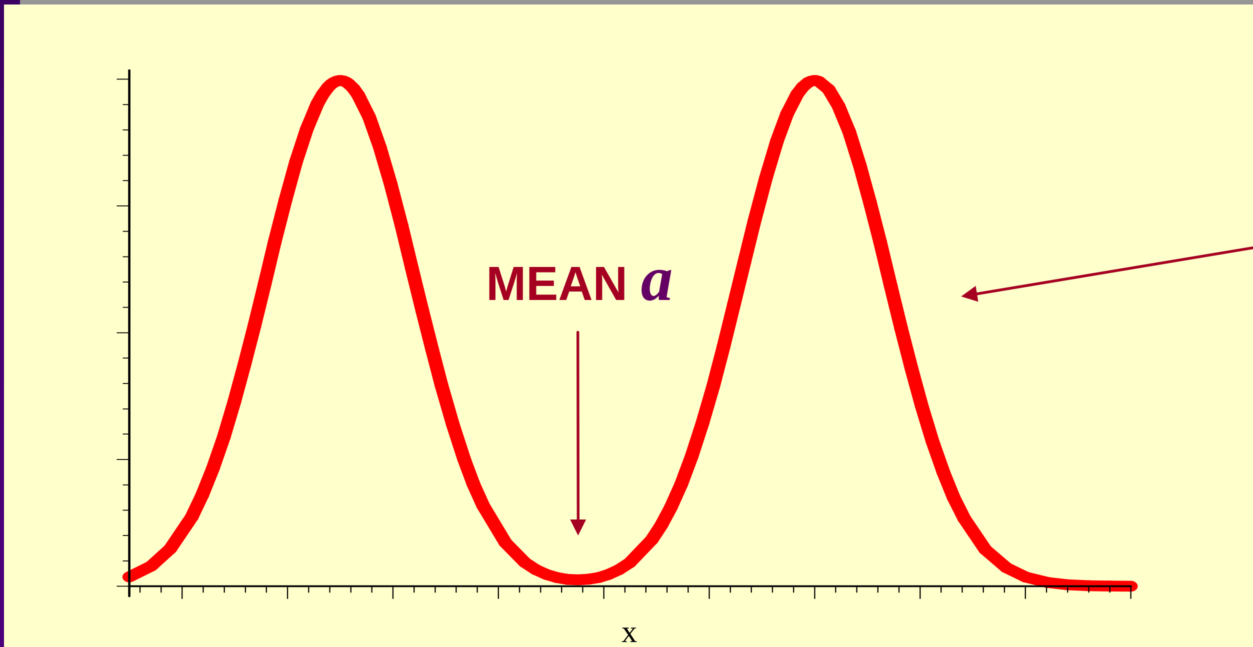
DANGERS OF DECOMPOSITION, ABSTRACTION



WHY THIS FAILS...



Average corresponds to unlikely scenario $\Rightarrow$ $x(a)$ *is way off...*



Prob. Density Function of A obtained from High Resolution model

WHY THIS FAILS...

- SIMPLE AVERAGING: $f(E[A])$

$$\not\parallel$$

- SAMPLE, THEN AVERAGE: $E[f(A)]$

If ultimate OUTPUT is $x(a) = 0$ or 1
this can result in 0 instead of 1
$\Rightarrow$ *completely wrong conclusion* !

WHAT'S THE WAY AROUND THIS?

QUESTION: To average or not to average ?

ANSWER: Average "just enough"

↓

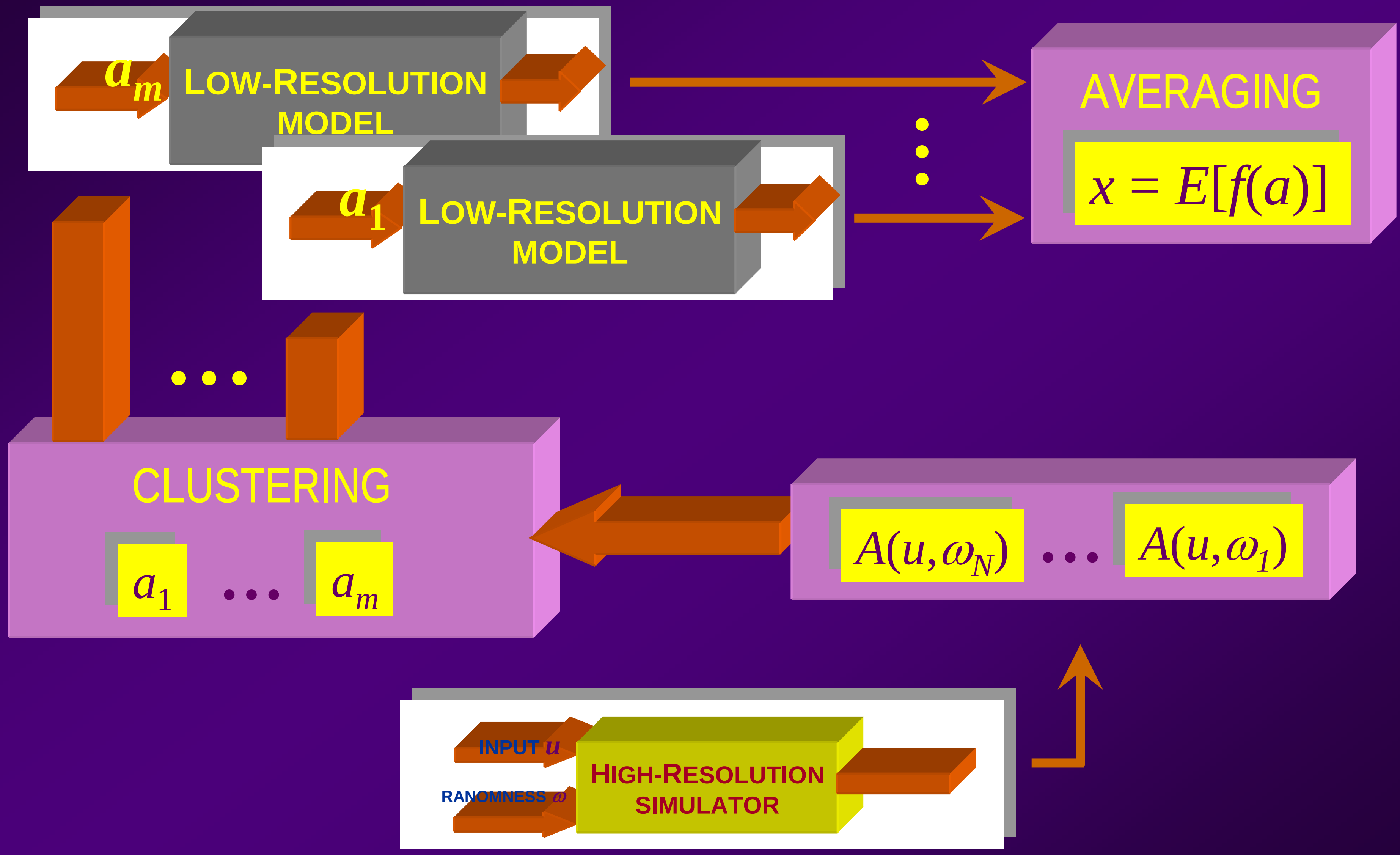
Replace AVERAGE by CONDITIONAL AVERAGES, one for each *CLUSTER*

CLUSTER = group of "similar" scenarios from High Resolution model

↓

CLUSTER ANALYSIS

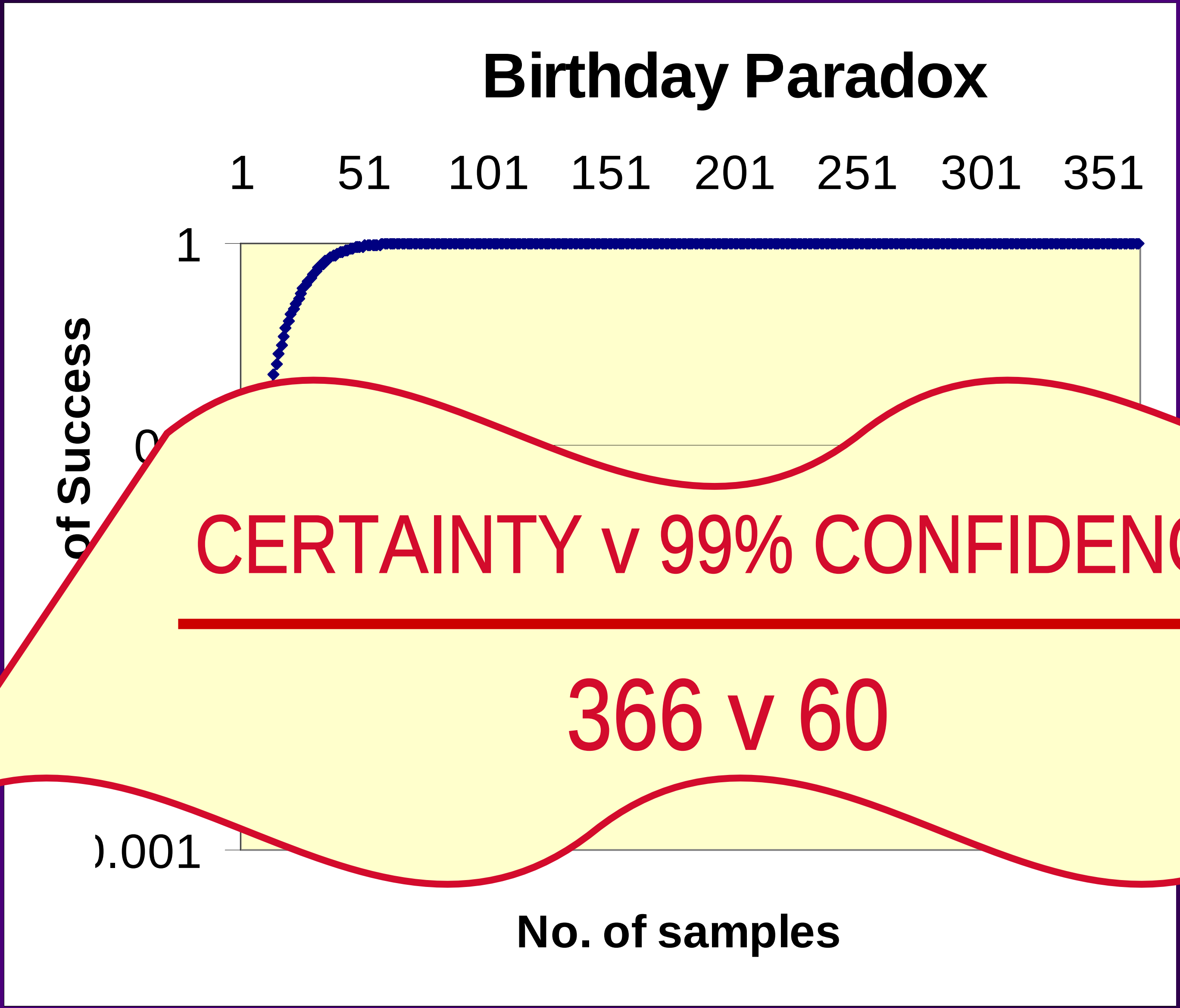
CLUSTERING



a_m
LOW-RESOLUTION MODEL
a_1
LOW-RESOLUTION MODEL
AVERAGING
$x = E[f(a)]$
CLUSTERING
a_1 … a_m
$A(u,\omega_N)$ … $A(u,\omega_1)$
INPUT u
RANOMNESS ω
HIGH-RESOLUTION SIMULATOR

HIGH PROBABLILITY
v
CERTAINTY

HIGH PROBABILITY v CERTAINTY



CERTAINTY v 99% CONFIDENCE

366 v 60

A BRAVE NEW *COMPLEX* WORLD…

Servants that Never Sleep

If you've always wished you could get reliable and affordable household help, you've got something to look forward to. In a few years, service robots will perform a wide variety of tasks. They'll clean windows, serve beverages, empty the dishwasher and more. And they'll enable older people to live at home longer.

2020

They'll carry our luggage, load our cars, help out in the kitchen, and nurse us when we're sick. They'll even entertain us, teach us how to play tennis and bring us a wealth of information regardless of where we are. In the future, intelligent robots — always friendly and never impatient — will perform a spectrum of activities that will help make life a breeze.

Eltville, Germany, fall 2020. The swift little robots rustle the leaves as they hurry up and down the rows of vines, carefully picking clusters of ripe grapes. The rush is on now with the grape harvest, because a long rainy spell is forecast to start tomorrow. The vintners can't run the risk of letting the grapes get moldy, because they're sure that 2020 will be a superb vintage! The stock price of the Rheingau Wine Investment Fund is already soaring to a record high.

"There you are at last!" Christine Dost hugs her son Peter and his young family. "I apologize, mom, but the sales rep for the new Multi-Rob stopped by. You know, that's a great gadget! A robot for everything! Cleaning windows, mopping floors, vacuuming carpets, serving beverages. There are all sorts of accessories too, anything you could ask for. Just what you always wanted, right?" But Peter's mom isn't impressed. "I've already got so many of those little helpers in the house. I don't need another one," she says. Peter grins a little at this. It's always the same with his parents — at their age you get a little set in your ways. But he's sure that when Multi-Rob →

A BRAVE NEW *COMPLEX* WORLD…

MANAGING COMPLEXITY

- Better HARDWARE and SOFTWARE help…
- System ARCHITECTURE v OPERATIONAL CONTROL
- HIGH v LOW RESOLUTION models
  (Too much detail can hurt)
- Know what PROBLEM needs to be solved,
  then develop METHODOLOGIES (otherwise, NFL limit gets y

- MODEL-DRIVEN v DATA-DRIVEN approaches
  (Embrace DATA -- and the NETWORK that gets data to you)

ACKNOWLEDGEMENTS

Thanks to several current and former students whose contributions are reflected in this talk…

THANK YOU !

+ C… lag… es…

Y… ry… H… iy…
We… Bo… g… i…
Yo… V… a… on
Be… Mo… n…
Yo… g… n… i… lidis

+ Sponsors…

NSF, AFOSR, AFRL, DARPA, NASA, EPRI, ARO, UT, ALCOA, NOKIA, GE