Proceedings of the 40th IEEE
Conference on Decision and Control
Orlando, Florida USA, December 2001

An Improved Forward Algorithm for Optimal Control of a Class of Hybrid Systems[1]

Ping Zhang and Christos G. Cassandras
Department of Manufacturing Engineering
Boston University, Boston, MA 02215
pzhang@bu.edu, cgc@bu.edu

Abstract

This paper considers optimal control problems for a class of hybrid systems motivated by the structure of manufacturing environments that integrate process and operations control. We derive a new property of the optimal state trajectory structure which holds under a modified condition on the cost function. This allows us to develop a low-complexity, scalable algorithm for explicitly determining the optimal controls, which is more efficient than the best algorithm to date, known as the Forward Algorithm [1]. A numerical example is included to illustrate the efficacy of the proposed algorithm, and to compare it with the Forward Algorithm.

1 Introduction

An interesting class of hybrid systems (i.e., systems combining *time-driven* and *event-driven* dynamics) is motivated by the structure of many manufacturing systems. In these systems, discrete entities (referred to as *jobs*) move through a network of work centers which process the jobs so as to change their physical characteristics according to certain specifications. Each job is associated with a *temporal* state and a *physical* state. The temporal state of a job evolves according to event-driven dynamics and includes information such as the service time or departure time of the job. The physical state evolves according to time-driven dynamics and describes some measures of the "quality" of the job such as temperature, weight and chemical composition. The interaction of time-driven with event-driven dynamics leads to a natural tradeoff between temporal requirements on job completion times and physical requirements on the quality of the completed jobs. Such modeling frameworks and optimal control problems have been considered in [2, 3]. In this paper, under a modified condition on the nature of the cost function, we further exploit the special structure of the optimal sample path in the singe-stage hybrid system framework. Based on the Forward Algorithm in [1], an improved Forward Algorithm is presented. Instead of increasing the number of jobs by 1 at every step, this algorithm may increase the number of jobs by more than 1 at every step. This property makes it more efficient than the Forward Algorithm.

2 Hybrid System Framework

The single-stage hybrid system framework we consider is as follows. A sequence of N jobs is assigned by an external source to arrive for processing at known times $0 \leq a_1 \leq \cdots \leq a_N$. We denote these jobs by C_i, $i = 1, \cdots, N$. The jobs are processed first-come, first-served (FCFS) by a work-conserving and non-preemptive server. The processing time is $s(u_i)$, which is a function of a control variable u_i. In general, the control is time varying over the course of the processing time s_i. The optimal control problem, denoted by $\mathbf{P}$, has the following form:

$$\min_{u_1,\cdots,u_N} \{J = \sum_{i=1}^{N} \{\theta_i(u_i) + \phi_i(x_i)\}\} \tag{1}$$

$$\text{s.t.} \quad x_i = \max(x_{i-1}, a_i) + u_i, \quad i = 1, \cdots, N.$$

The optimal solution of $\mathbf{P}$ is denoted by u_i^* for $i = 1, \cdots, N$ and the corresponding departure times are denoted by x_i^* for $i = 1, \cdots, N$. We make the following assumptions: 1) $\theta_i(\cdot)$ is continuously differentiable, strictly convex and monotone decreasing for $u > 0$ and the following limit holds: $\lim_{u \to 0^+} \theta_i(u) = \infty$. 2) $\phi_i(\cdot)$ is continuously differentiable, strictly convex and its minimum is obtained at a finite point.

3 Optimal Control Properties

We begin with the following definitions. A job C_i is *critical* if it departs at the arrival time of the next job, i.e., $x_i = a_{i+1}$. Considering a contiguous job subset

[1]This work was supported in part by the National Science Foundation under Grant ACI-98-73339 and EEC-0088073, by AFOSR under contract F49620-01-0056, by the Air Force Research Laboratory under contract F30602-99-C-0057, and by EPRI/ARO under contract WO8333-03.

0-7803-7061-9/01/$10.00 © 2001 IEEE

$\{C_k, \cdots, C_n\}$, $1 \leq k \leq n \leq N$ on the optimal sample path, the subset is said to be a *block* if 1) $x_{k-1} \leq a_k$ and $x_n \leq a_{n+1}$; 2) The subset contains no critical jobs. *A busy period* is a contiguous set of jobs, $C_k, \cdots, C_n$ for $1 \leq k \leq n \leq N$ such that the following three conditions are satisfied: i) $x_{k-1} < a_k$; ii) $x_n < a_{n+1}$; iii) $x_i \geq a_{i+1}$, for every $i = k, \cdots, n-1$. *A busy-period structure* is a partition of the jobs $C_1, \cdots, C_N$ into busy periods. The jth busy period consists of jobs $C_{k(j)}, \cdots, C_{n(j)}$ where $k(1) = 1$, $k(j) = n(j-1)+1$ and $n(M) = N$.

Consider the following optimization problem for $C_k, \cdots, C_n$, which is denoted by $Q(k,n)$:

$$\min_{u_k, \cdots, u_n} \{J(k,n) = \sum_{i=k}^{n} \{\theta_i(u_i) + \phi_i(a_k + \sum_{j=k}^{i} u_j)\}\} \quad (2)$$

s.t. $\quad x_i = a_k + \sum_{j=k}^{i} u_j \geq a_{i+1}, \quad i = k, \cdots, n-1.$

The solution of $Q(k,n)$ and the corresponding departure times are denoted by $u_j^*(k,n)$ and $x_j^*(k,n)$ for $j = k, \cdots, n$. There is one important property of $Q(k,n)$ that is captured in the following theorem (whose proof is given in [1]).

Theorem 1 Jobs $C_k, \cdots, C_n$ constitute a single busy period on the optimal sample path if and only if the following conditions are satisfied: 1) $a_k > x_{k-1}^*$; 2) $x_i^*(k,i) \geq a_{i+1}$, for all $i = k, \cdots, n-1$; and 3) $x_n^*(k,n) < a_{n+1}$.

4 Improved Forward Algorithm

In this section, we further exploit a property of the optimal state trajectory for the optimal control problem **P**, which is based on modifying Assumption 2) as follows: $\phi_i(\cdot)$ is continuously differentiable, strictly convex and monotone increasing. This is in fact a common condition for a large class of problems, when one wishes, for example, to penalize the departure time of jobs or their system time. The proof of the main result below is omitted, but may be found in [4].

Theorem 2 If for the problem $Q(1,N)$, job C_i is not a critical job, i.e. $x_i^*(1,N) > a_{i+1}$, then for the problem **P**, job C_i cannot be the last job of any busy period, i.e., for any $k = 1, \cdots, N, n(k) \neq i$.

This theorem suggests that we can get some information about the structure of a busy period from the solution $x_i^*(1,N)$ of $Q(1,N)$. In particular, only critical jobs of $Q(1,N)$ may end a busy period. Therefore, instead of adding only one job at every step in the Forward Algorithm, we can add more than one job at every step and make the algorithm more efficient. The price is that we need solving $Q(1,N)$ firstly.

5 Example

Let us consider the following problem:

$$\min_{u_1, \cdots, u_5} J = \sum_{i=1}^{5} \{u_i^{-1} + x_i^2\} \quad (3)$$

subject to $x_i = \max(a_i, x_{i-1}) + u_i$, for the arrival sequence $\{1, 1.4, 1.5, 1.55, 1.6, 3., 3.1, 3.2, 3.3, 3.5\}$.

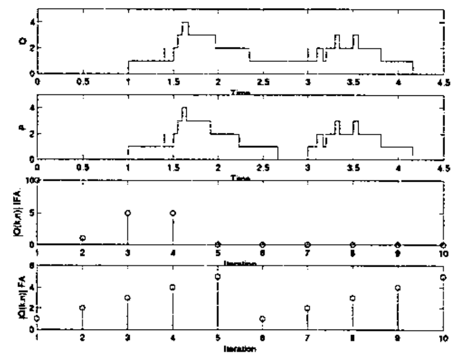

Figure 1: The first two plots show the queue length of $Q(1,N)$ and **P**. The second two present the complexity of $Q(k,n)$ using the improved Forward Algorithm and the Forward Algorithm respectively.

For this example, as seen in Figure 1, the cumulative dimensionality over all sub-problems is 21, requiring a total of 4 steps, while the corresponding numbers using the Forward Algorithm are 30 sub-problems with 10 total steps.

References

[1] Y. C. Cho, C. G. Cassandras, and D. L. Pepyne. Forward decomposition algorithms for optimal control of a class of hybrid systems. *Intl. Journal of Robust and Nonlinear Control*, 2001. To appear.

[2] D. L. Pepyne and C. G. Cassandras. Modeling, analysis, and optimal control of a class of hybrid systems. *Journal of Discrete Event Dynamic Systems: Theorey and Applications*, 8(2):175–201, 1998.

[3] C. G. Cassandras, D. L. Pepyne, and Y. Wardi. Optimal control of a class of hybrid systems. *IEEE Trans. on Automatic Control*, 46(3):398–415, 2001.

[4] P. Zhang and C. G. Cassandras. An improved forward algorithm for optimal control of a class of hybrid systems. *submitted to IEEE Trans. on Automatic Control*, 2001.