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Optimal Control for Steel Annealing Processes as Hybrid Systems

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Abstract

This paper formulates and solves an optimal control problem for steel annealing manufacturing processes involving one or more furnaces integrated with plant-wide planning and scheduling operations. We use a hybrid system framework to capture the tradeoff between metallurgical quality requirements and timely product delivery. The resulting nonconvex and nondifferentiable problem is solved by decomposing it into several smaller and simpler constrained convex optimization subproblems. Although the number of such subproblems appears to be combinationally large in the number N of jobs to be completed, we use a recently developed approach for identifying at most $2N-1$ such problems and provide some explicit numerical results.

Key words : Hybrid system, steel annealing process, nonconvex optimization

1 Itroduction

In steel manufacturing environments, physical process operations are integrated with plant-wide planning and scheduling operations. Individual steel "parts" (i.e., ingots or strips) undergo various operations to achieve certain metallurgical properties that define the 'quality' of the finished products. In particular, the *steel annealing process* is an important step for achieving a wide range of high-strength products with unique properties from a limited number of compositions. This step involves slowly heating and cooling strips to some desired temperatures.

Before heating and cooling each roll of strips in steel annealing processes, a higher level controller determines the furnace reference temperature (more generally, a "furnace heating profile") which the strip should follow, as well as the amount of time that this strip is held in a furnace. In fact, the goal of achieving *high quality* for the strip in a steel annealing process is clearly in conflict

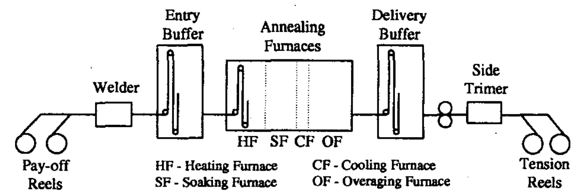

Figure 1: A steel annealing process line.

with the plant-wide planning and scheduling objective of *timely delivery* of finished products to clients. A hybrid system framework may be designed to deal with this type of conflict [1, 2]. In this framework, the generic term "job" refers to the processing task and the term "server" is used to describe the workcenter or device that processes the task. In steel annealing processes, the jobs and server correspond to the strips and the furnace respectively.

To represent the hybrid nature of the model, each job is characterized by a *physical state* and a *temporal state*. The physical state represents the physical characteristics of interest and evolves according to time-driven dynamics (e.g., differential equations) while the job is being processed by a server. The temporal state represents processing start and stop times and evolves according to discrete-event dynamics (queueing dynamics in this case). In steel annealing processes considered here, the physical state is associated with the strip temperature (evolving according to the furnace temperature), the line speed, and possibly some other properties of the strips. On the other hand, the temporal state is associated with the arrival time and the processing time of the strip in the furnace.

Figure 1 shows a typical annealing process line consisting of a number of furnaces in tandem for continuous heating, soaking, cooling, etc. [3, 4, 5]; the heating furnace is normally the one with the greatest influence on the production rate and strip quality. The entire length of an annealing line is about 1 Km $\sim$ 1.5 Km and a heating furnace, in particular, is about 400 m $\sim$ 500 m. Raw material, (e.g., a cold-rolled strip) is put on a pay-off reel on the entry side of the line and runs through the line with a certain line speed. Usually, it takes a few minutes for a roll of strips to go through a furnace. On

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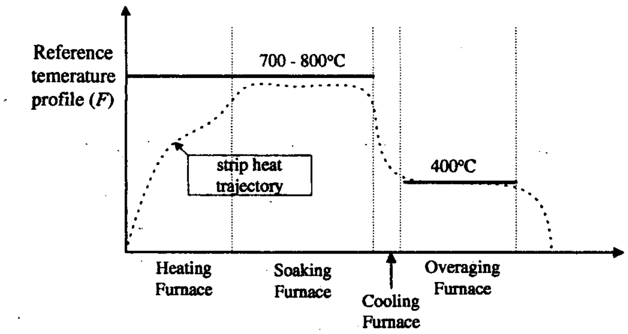

Figure 2: An example of the furnace reference temperature profile.

the delivery side, the strip is cut into a product length by a shear machine and rolled again. The strip temperature in each furnace is controlled by fuel flow rates associated with the furnace reference temperature profiles given to each strip; these variables are measured at each furnace. Typical reference temperatures through a line such as that of Fig. 1 are shown in Fig. 2.

In this paper, an optimal control problem for such steel annealing manufacturing processes involving one or more furnaces is formulated and solved in a hybrid system framework. To solve the resulting nonconvex and nondifferentiable problem, A backward recursive algorithm developed in [6] is adopted. This paper provides some explicit numerical results for both a single furnace and multiple furnace models.

2 Single Heating Furnace Model

In this paper, we will first consider a single heating furnace model described in the above hybrid system framework. The physical state of a strip in the steel annealing process is denoted by $z(t)$ and represents the temperature at each point of the strip as it evolves through the heating furnace. The strip temperature is basically dependent on the *line speed* u, which usually remains constant during the process, and the *furnace reference temperature* F, which is pre-designed at a plant-wide planning level. The thermal process in the heating furnace can be represented by a nonlinear heat-transfer equation describing the dynamic response of each strip temperature so that the temporal change in heat energy at a particular location is equal to the transport heat energy plus the radiation heat energy [3] as follows:

$$\frac{dz(t)}{dt} = -K_1 u + K_2[F^4 - z(t)^4] \quad t \geq t_0 \qquad (1)$$

where

$$K_1 = \frac{F - z(t_0)}{L}, \quad K_2 = \frac{2\sigma_{sb}\phi_s}{60 d_s 10^{-3}\tau}$$

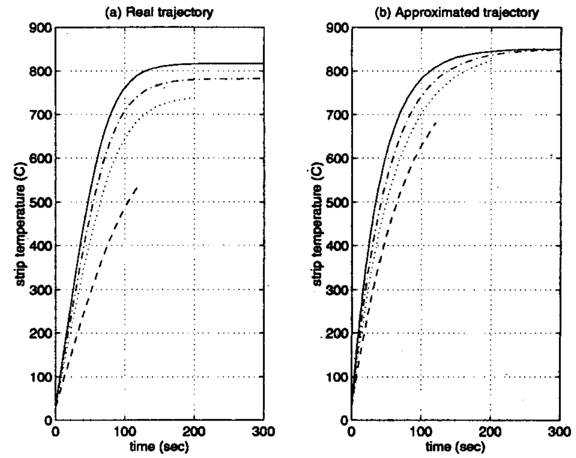

Figure 3: Real and approximated trajectories of strip temperature.

and

L	:	furnace length [m];
t_0	:	heating start time;
σ_{sb}	:	Stefan – Boltsmann constant ($= 4.88 \times 10^{-8}$ [kcal/m$^2 \cdot$ h $\cdot$ deg^4]);
ϕ_s	:	coefficient of radiative heat absorption, $0 < \phi_s < 1$ (determined as 0.17 from actual data);
d_s	:	strip specific heat [kcal/m$^3 \cdot$ deg];
τ	:	strip thickness [mm]

Since (1) is in nonlinear differential form, it is hard to represent solutions in an explicit form. Figure 3(a) shows the temperature trajectories of (1) in the following environment: $L = 500$ [m], $d_s = 4.98 \times 10^4$ [kcal/$m^3$$\cdot$deg], $\phi_s = 0.17$, $\tau = 0.71$ [mm], $u = 100, 200, 300, 500$ [m/min] and $z(t_0) = 30°C$. The curve in Fig. 3(a) turns out to be close to an exponential, hence the model can be approximated as follows:

$$\frac{dz(t)}{dt} = \frac{1}{f(u)}(F - z(t)), \quad t \geq t_0. \qquad (2)$$

where $f(u)$ is an arbitrary function appropriately chosen to achieve a desired level of accuracy. In this paper, $f(u)$ is taken to be a monotone increasing polynomial function of u, i.e., $f(u) = \sum_{r=0}^{m} c_k u^r$ for some $m = 1, 2, \cdots$. This approximation has been employed in a real heating furnace model for a steel annealing process [5]. The approximated trajectory of the strip temperature in the same environment as Fig. 3(a) is shown in Fig. 3(b), where $f(u) = 5.4u + 29$.

Let us now consider the temporal state of strips viewed as jobs denoted by $C_1, \cdots, C_N$. The temporal state of the ith strip is denoted by x_i and y_i. x_i represents the time when the job starts processing at the furnace and y_i represents the time when the job completes processing and departs from the system. In earlier work, the starting time of the $(i+1)$-th job is identical to the completion time of the ith job (i.e. $x_{i+1} = y_i$) for all $i = 1, 2, \cdots$ because each job is supposed to be a "lump". On the other hand, the fact is not true in the steep annealing process because each job of the steel annealing process is a continuous strip of a typical length, not a lump. Hence, the temporal behavior of a steel annealing process cannot be represented by a single "max-plus" equation.

Letting a_i be the arrival time of the ith strip, the event-driven dynamics describing the evolution of the temporal states x_i and y_i are given by the following two equations (3)-(4):

$$\begin{aligned} x_i &= \max(a_i, x_{i-1}) + s_1(u_i) \qquad (3) \\ y_i &= x_i + s_2(u_i) \qquad (4) \\ &\text{subject to } u_{min} \leq u_i \leq u_{max}, i = 1, \cdots, N. \end{aligned}$$

where $s_1(u_i)$ is the elapsed time for the whole body of the strip to enter the furnace, which is dependent on the length of the strip and $s_2(u_i)$ is the processing time for the each point of the strip to run through the furnace, which is dependent on the length of the furnace. u_{min} and u_{max} are the minimum and maximum allowable line speed respectively, and assume that $x_0 = -\infty$.

Since each strip C_i of length h_i runs through the furnace of length L at a constant speed, u_i, $s_1(u_i)$ and $s_2(u_i)$ are determined by

$$s_1(u_i) = \frac{h_i}{u_i} \text{ and } s_2(u_i) = \frac{L}{u_i}, \quad i = 1, \cdots, N. \qquad (5)$$

In this system, we have two control objectives : (i) to reduce temperature errors with respect to the furnace reference temperature, and (ii) to reduce the entire processing time for timely delivery to clients using acceptable levels of line speed, u_i. Thus, the optimal control problem of interest, denoted by $\mathbf{P}$, is

$$\mathbf{P} : \min_{\{u_1, \cdots, u_N\}} J = \sum_{i=1}^{N} [\theta(u_i) + \phi(y_i)] \qquad (6)$$

subject to (2)-(4).

The function $\phi(y_i)$ in (6) is the cost related to jobs departing at time y_i. For example, $\phi(y_i) = (y_i - d_i)^2$ defines a cost where departing after the due date d_i incurs a tardiness cost and completing the job before its due date incurs an inventory (backlog) cost.

The function $\theta(u_i)$ in (6) is selected so as to penalize the deviation of the ith strip temperature from the reference temperature, F_i. In earlier work [1], it was assumed that the final state, $z(s_i)$, is fixed. In this paper, the final state is not a fixed value, but depends on both the furnace reference temperature, F_i and the line speed. Accordingly, we set, for $i = 1, \cdots, N$:

$$\theta(u_i) = |F_i - z(L/u_i)|^2 + \beta \int_0^{L/u_i} (F_i - z(t))^2 dt, \quad (7)$$

where L/u_i is the time required for each point of the strip to stay in the furnace and β is a weighting factor.

To be consistent with the previous discussion regarding tradeoffs between the temporal and physical requirements in hybrid systems, $\theta(u_i)$ should have a certain monotonicity property in $u_{min} \leq u_i \leq u_{max}$ which can be guaranteed by appropriately selecting the weighting factor β in (7) as follows:

Lemma **1** if $\beta = \frac{k}{f(u_i)}$ for any $0 \leq k < 2$, then $\theta(u_i)$ is a monotone increasing function in $u_{min} \leq u_i \leq u_{max}$.

Proof : By the approximated differential linear equation of (2) and the initial state $z(0) = 0$ w.l.o.g, we obtain the temperature trajectory of each strip i on the line speed u_i:

$$z(t) = F_i(1 - e^{-t/f(u_i)}). \qquad (8)$$

Replacing (7) with (8) and $\beta = \frac{k}{f(u)}$ yields

$$\begin{aligned} \theta(u_i) &= F_i^2 \left[e^{-2L/u_i f(u_i)} - \beta \frac{f(u_i)}{2} (e^{-2L/u_i f(u_i)} - 1) \right] \\ &= F_i^2 \left[e^{-2L/u_i f(u_i)} - \frac{k}{2} (e^{-2L/u_i f(u_i)} - 1) \right] \\ &= F_i^2 \left[(1 - \frac{k}{2}) e^{-2L/u_i f(u_i)} + \frac{k}{2} \right]. \end{aligned}$$

Since $f(u_i)$ is monotone increasing in $u_{min} \leq u_i \leq u_{max}$, $\theta(u_i)$ is also monotone increasing under $0 \leq k < 2$. ■

This monotonicity is an important requirement for developing the optimal control algorithm presented in the next section.

3 Computing Optimal Controls

The function J in the optimal control problem $\mathbf{P}$ is neither convex nor differentiable. Consequently, general-purpose algorithms, based on non-differentiable calculus [7], for computing its minimum may require overwhelming computing resources. There is, however, enough special structure to be exploited and, consequently, simplify the computation of explicit solutions. In this paper, we adopt the *backward recursive algorithm* recently proposed in [6] to efficiently solve the optimal control problem. The main feature of this algorithm is that the solution of the optimal control problem $\mathbf{P}$ in (6) is computed by solving a finite sequence of differentiable, constrained convex programming problems. In addition, the number of these convex problems

Table 1: Backward recursive algorithm

ALGO 1:
Initialize : Solve $P(N,N)$ by solving $Q(N,N)$. Set $k = N$.
Main Loop :
While($k > 1$) {
Solve $P(k-1,N)$ using **ALGO 2.**
Set $k \leftarrow k-1$.
}

ALGO 2:
Initialize : Set m=0 and set $B_1 = \bar{B}_0 = \{C_{k-1}\}$.
Solve $Q(k-1,k-1)$ to obtain u_{k-1}, and compute $x_{k-1} = a_{k-1} + h_{k-1}/u_{k-1}$.
Set $k(0) = n(0) = k-1$.
Main loop :
While ($x_{n(m)} \geq a_{n(m)+1}$) {
Set $B_1 \leftarrow B_1 \cup \bar{B}_{m+1}$.
Solve the problem $Q(k-1, n(m+1))$ to compute $u_{k-1}, \cdots, u_{n(m+1)}$,
and compute $x_{n(m+1)} = a_{k-1} + \sum_{i=k-1}^{n(m+1)} h_i/u_i$.
$m \leftarrow m+1$.
}
Output : Set $u_i = \bar{u}_i$ for all $i = n(m)+1, \cdots, N$.

is not combinationally explosive as it would appear. In fact, as shown in [6], this number is bounded by $2N-1$, where N is the number of jobs processed.

The derivation of the backward recursive algorithm is based on the following fundamental observation: Even though the overall optimization problem **P** is nonconvex, the optimal state trajectory can be decomposed into segments called "blocks" such that the controls within each block can be obtained by solving a much simpler convex optimization problem with terminal constraints on the departure time of the last job in the block. A busy period on the optimal state trajectory (i.e., a period during which the server is busy and is preceded and followed by idle periods of strictly positive length) defines such a "block". More generally, a busy period may contain two or more blocks if it contains a job indexed by i such that $x_i = a_{i+1}$. The ith job would then end one block and the next one would start with job $i+1$.

For example, consider a busy period consisting of jobs $C_k, \cdots, C_n$ and suppose that this busy period contains two blocks, $C_k, \cdots, C_{\hat{k}}$ and $C_{\hat{k}+1}, \cdots, C_n$. Then, by the idle period decoupling property (Lemma 3.1 in [6]) and the partial coupling property (Lemma 3.2 in [6]), we can determine the optimal controls for the jobs in this busy period by solving two independent convex problems. The first for jobs, $C_k, \cdots, C_{\hat{k}}$ with terminal constraint $x_{\hat{k}} = a_{\hat{k}+1}$ and the second for jobs $C_{\hat{k}+1}, \cdots, C_n$ with no terminal constraint on the departure time x_n.

In short, if we can identify the busy period structure of the optimal solution, and the block structure within the busy period, then we can decompose the solution of a computationally difficult non-convex optimization problem into a collection of simpler, convex optimization problems.

To outline the backward recursive algorithm as applied to our steel annealing optimal control problem **P**, it is necessary to define two sub-problems as follows.

First, consider a problem, denoted by $P(k,n)$, where (k,n) ranges over the set $\{1 \leq k \leq n \leq N\}$. $P(k,n)$ is the same as **P** except that the jobs, $C_1, \cdots, C_N$ are replaced by the subset $C_k, \cdots, C_n$ (note that $P(1,N)$ is identical to **P**). Then, $P(k,n)$ has the following form:

$$P(k,n): \min_{u_k,\cdots,u_n} \left\{ J(k,n) = \sum_{i=k}^{n} \theta(u_i) + \phi(y_i) \right\}$$

subject to

$$x_i = \max(x_{i-1}, a_i) + \frac{h_i}{u_i}, y_i = x_i + \frac{L}{u_i},$$
$$\text{and } u_{min} \leq u_i \leq u_{max} \quad i = k, \cdots, n$$

and the boundary condition $x_{k-1} < a_k$.

A second optimization problem denoted by $Q(k,n)$ is the following:

$$\min_{u_k,\cdots,u_n} \left\{ J(k,n) = \sum_{i=k}^{n} \theta(u_i) + \phi(y_i) \right\} \qquad (9)$$

subject to

$$x_i = a_k + \sum_{j=k}^{i} \frac{h_i}{u_i} \geq a_{i+1},$$
$$u_{min} \leq u_i \leq u_{max}, \quad i = k, \cdots, n-1,$$

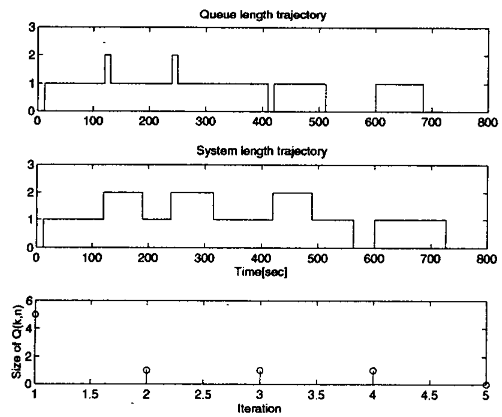

Figure 4: An optimal state trajectory and computational complexity data.

where

$$y_i = a_k + \sum_{j=k}^{i} \frac{L + h_i}{u_i}$$

Since the cost functional is continuously differentiable and strictly convex, $Q(k,n)$ is also a convex problem with linear constraints and has a unique solution at a finite point.

The backward recursive algorithm for solving the optimal control problem **P** is described in **ALGO 1** and **ALGO 2** in Table 1.

3.1 Typical numerical results

A typical result from the application of the backward recursive algorithm to the optimal control problem **P** for a single heating furnace and the environment given in Fig. 3 is shown in Fig. 4. Here, we have used $\phi(y_i) = (y_i)^2$ in (6), $\beta = 1/f(u_i)$ and $h_i = 1000$ m for all $i = 1, \cdots, 5$ in (7), the arrival sequence is $\{0.2\ 2\ 4\ 7\ 10\}$[min], furnace reference temperature $F = 600$ °C and furnace length $L = 500$[m], $u_{min} = 100, u_{max} = 700$ [m/min]. The computational complexity data shown are in terms of the number of problems of the form $Q(k,n)$ that were involved in each of the 5 iterations required to solve **P**.

In Fig 4, 'queue length' and 'system length' represent the number of jobs waiting at the queue and the number of jobs remaining in the system (queue + server) respectively. These trajectories are determined by the arrival time a_i, process starting time x_i, and completion time y_i of each job.

4 Multiple Furnace Model

The algorithm has been extended to solve the optimal control problem involving multiple furnaces consisting of heating, soaking, cooling furnaces (see Fig. 1), where heating and cooling processes are co-existent and typical features of the steel annealing process are exploited.

Consider an annealing process consisting of M furnaces. The furnace reference temperatures, $F_1, \cdots, F_M$, and the furnace lengths, $L_1, \cdots, L_M$, for M furnaces are given. It is assumed that the thermal dynamics in each furnace are governed by the approximated model of (2) with corresponding initial states.

Letting $u_{i,j}$ be the line speed of the ith strip at the jth furnace, the starting time x_i is described as

$$x_i = \max(a_i, x_{i-1}) + \frac{h_i}{u_{i,1}}, \quad 1 \le i \le N \qquad (10)$$

where $x_0 = -\infty$. The completion time of the ith strip through the multiple furnace model y_i is described as

$$y_i = x_i + \frac{L_1}{u_{i,1}} + \sum_{k=2}^{M} \frac{h_i + L_k}{u_{i,k}}, \quad 1 \le i \le N \qquad (11)$$

The optimal control problem of interest in the multiple furnace system, denoted by $\mathbf{P}_M$, is

$$\mathbf{P}_M \ : \ \min_{\{u_1,\cdots,u_N\}} J = \sum_{i=1}^{N} \left[\left(\sum_{j=1}^{M} \theta(u_{i,j}) \right) + \phi(y_i) \right] \qquad (12)$$

subject to (2), (10)-(11) and $u_{min} \le u_i \le u_{max}$ for $i = k, \cdots, n$.

As in the single furnace model, consider a problem, denoted by $P_M(k,n)$, where (k,n) ranges over the set $\{1 \le k \le n \le N\}$. $P_M(k,n)$ is the same as $\mathbf{P}_M$ except that the jobs, $C_1, \cdots, C_N$ are replaced by the subset $C_k, \cdots, C_n$ (note that $P_M(1,N)$ is identical to $\mathbf{P}_M$). Then, $P_M(k,n)$ has the following form:

$$P_M(k,n) : \min_{u_k,\cdots,u_n} \left\{ \sum_{i=k}^{n} \left[\left(\sum_{j=1}^{M} \theta(u_{i,j}) \right) + \phi(y_i) \right] \right\}$$

subject to $u_{min} \le u_i \le u_{max}$, for $i = k, \cdots, n$ and (10)-(11) and the boundary condition $x_{k-1} < a_k$.

A second optimal problem denoted by $Q_M(k,n)$ is the following:

$$Q_M(k,n) : \min_{u_k,\cdots,u_n} \left\{ \sum_{i=k}^{n} \left[\left(\sum_{j=1}^{M} \theta(u_{i,j}) \right) + \phi(y_i) \right] \right\} \qquad (13)$$

subject to $u_{min} \le u_i \le u_{max}$ for $i = k, \cdots, n$ and

$$x_i = a_k + \sum_{j=k}^{i} \frac{h_i}{u_{i,1}} \ge a_{i+1}, \quad i = k, \cdots, n-1, \qquad (14)$$

where

$$y_i = a_k + \sum_{q=k}^{i} \sum_{j=1}^{M} \frac{L_j + h_q}{u_{q,j}} .$$

Table 2: Simulation result of a multiple furnace system (M=3)

Job Index	1	2	3	4	5	6
arrival time [sec]	48	480	720	960	1440	1680
$u^*_{i,1}$ [m/min]	307.69	396.29	397.60	346.47	465.30	498.89
$u^*_{i,2}$ [m/min]	280.73	293.90	315.51	405.94	485.81	546.91
$u^*_{i,3}$ [m/min]	318.88	388.81	363.06	700.00	700.00	700.00
x_i^*[sec]	406.13	785.92	1089.72	1333.74	1642.90	1869.41
y_i^*[sec]	585.88	938.59	1240.19	1464.49	1748.67	1967.58

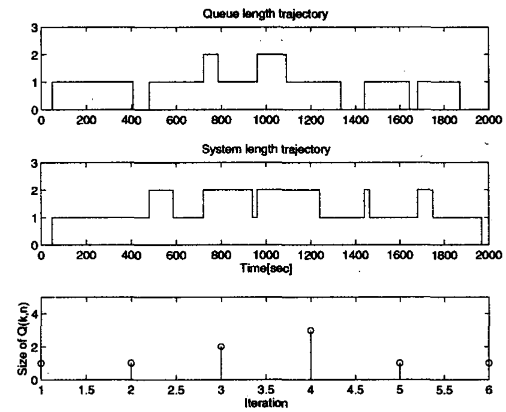

Figure 5: An optimal state trajectory and computational complexity data for a multiple furnace (M=3).

Since the cost functional is continuously differentiable and strictly convex, $Q_M(k,n)$ is also a convex problem with linear constraints and has a unique solution at a finite point. The backward recursive algorithm for solving the optimal control problem for multiple furnace system, $\mathbf{P}_M$, can be described as in **ALGO 1** and **ALGO 2** in Table 1. We omit the detailed algorithm here.

4.1 Typical numerical results

A typical result from the application of the backward recursive algorithm to the optimal control problem $\mathbf{P}_M$ for a triple-furnace ($M = 3$) and the environment given in Fig. 3 is shown in Table 2 and Fig. 5. Here, we have used $\phi(y_i) = y_i^2$ in (6), $\beta = 1/f(u_i)$ and $h_i = 600$ m for all $i = 1, \cdots, 6$ in (10), the arrival sequence is $\{0.8\ 8\ 12\ 16\ 24\ 28\}$[min], furnace reference temperature $F = \{500\ 400\ 200\}[°C]$, furnace length $L = \{400\ 300\ 200\}$[m] and $u_{min} = 100, u_{max} = 700$ [m/min].

5 Conclusion

This paper has formulated and solved an optimal control problem for steel annealing manufacturing processes in the hybrid framework suggested from earlier work [1, 2, 6]. We showed that the tradeoff between metallurgical quality requirements and timely product delivery can be analyzed in this hybrid system framework. The resulting nonconvex and nondifferentiable problem is solved by an efficient backward recursive algorithm [6]. Also, the problem for multiple furnace models is solved by the extension of the single furnace case. Ongoing work is aimed at extending our approach to systems with an infinite number of sequential jobs or with uncertainties in arrival times of jobs.

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