DISCRETE EVENT SYSTEMS

MODELING AND PERFORMANCE ANALYSIS

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OUTLINE

- Why are DISCRETE EVENT SYSTEMS important?
- Models for Discrete Event Systems (DES) and Hybrid Systems (HS)
- DES Simulation
- Control and Optimization in DES: Resource Contention problems
- “Rapid Learning”:

  Perturbation Analysis (PA) and Concurrent Estimation (CE)
- Applications
- Dealing with Complexity – Fundamental Complexity Limits
- Abstraction through Hybrid Systems:

  Stochastic Flow Models (SFM), the IPA Calculus

WHY DISCRETE EVENT SYSTEMS ?

- Many systems are naturally Discrete Event Systems (DES) (e.g., Internet)
  → *all* state transitions are event-driven
- Most of the rest are Hybrid Systems (HS)
  → *some* state transitions are event-driven
- Many systems are distributed
  → components interact asynchronously (through events)
- Time-driven sampling inherently inefficient ("open loop" sampling)
  → event-driven control

REASONS FOR *EVENT-DRIVEN* MODELS, CONTROL, OPTIMIZATION

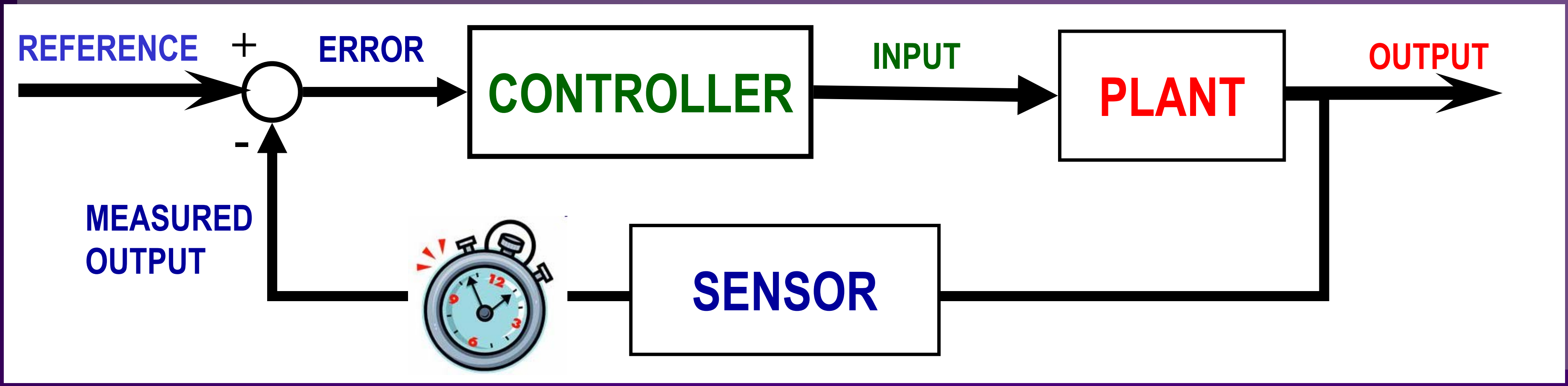
- Many systems are **stochastic**
  $\longrightarrow$ actions needed in response to random events

- Event-driven methods provide significant advantages in **computation** and **estimation** quality

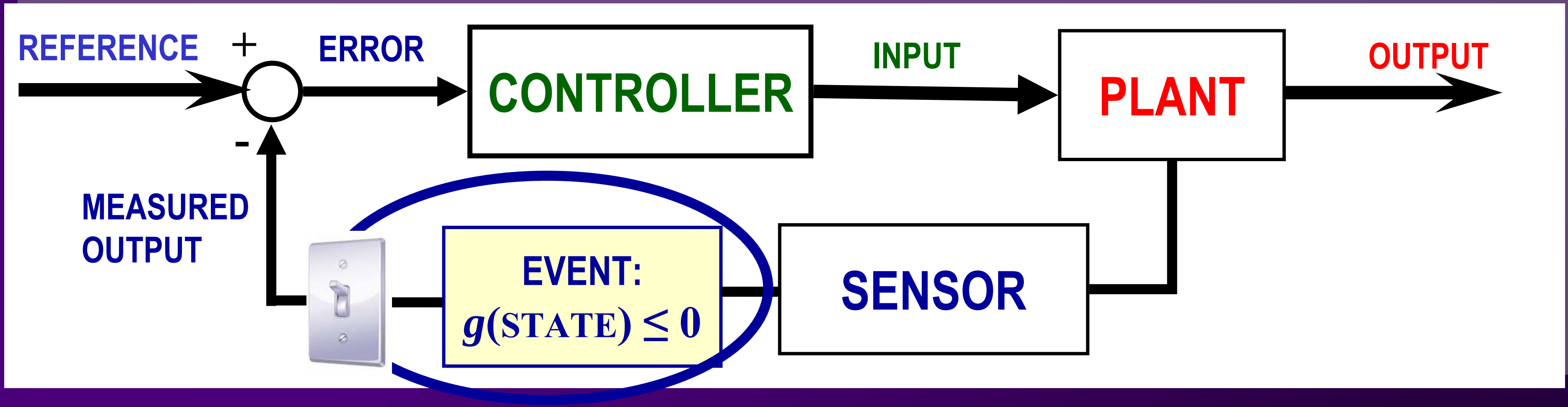
- System performance is often **more sensitive to event-driven** components than to time-driven components

- Many systems are **wirelessly networked** $\longrightarrow$ energy constrained
  $\longrightarrow$ time-driven communication consumes energy unnecessarily
  $\longrightarrow$ use **event-driven control**

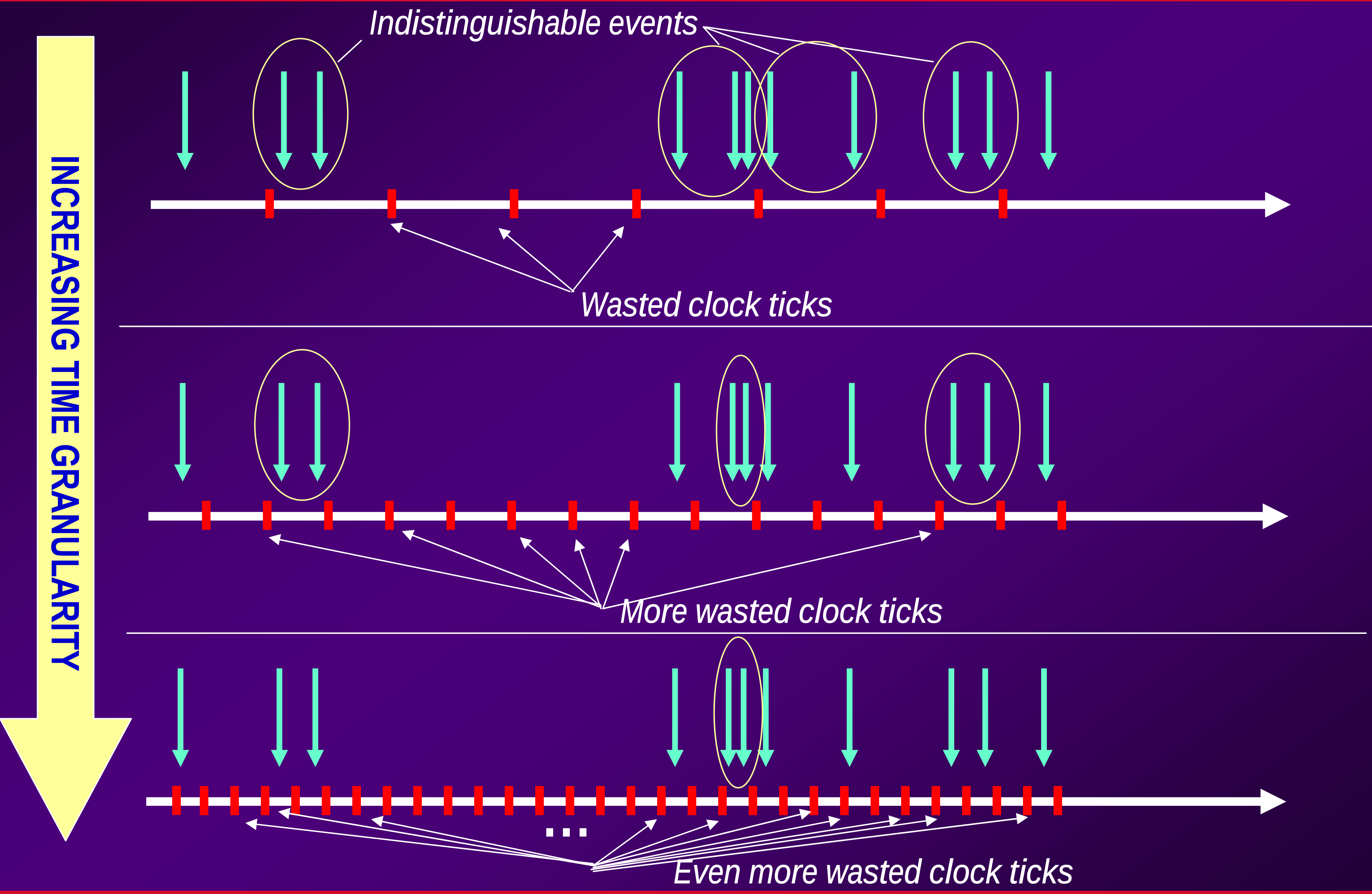
TIME-DRIVEN v EVENT-DRIVEN CONTROL



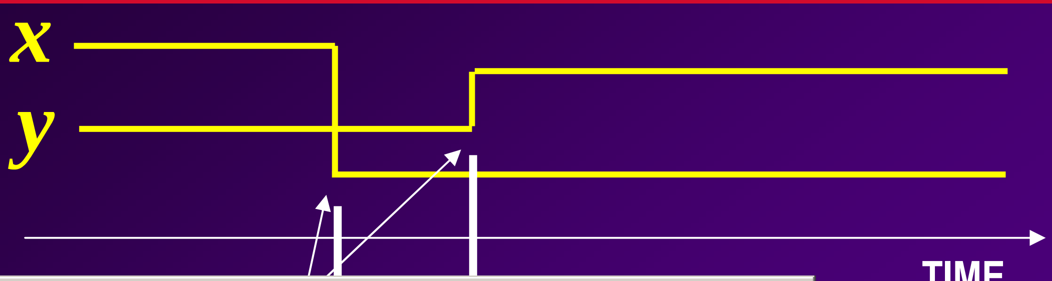
EVENT-DRIVEN CONTROL: Act *only when needed* (or on **TIMEOUT**) - not based on a clock

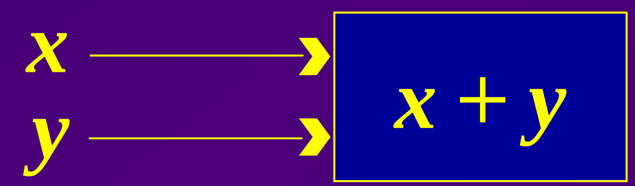


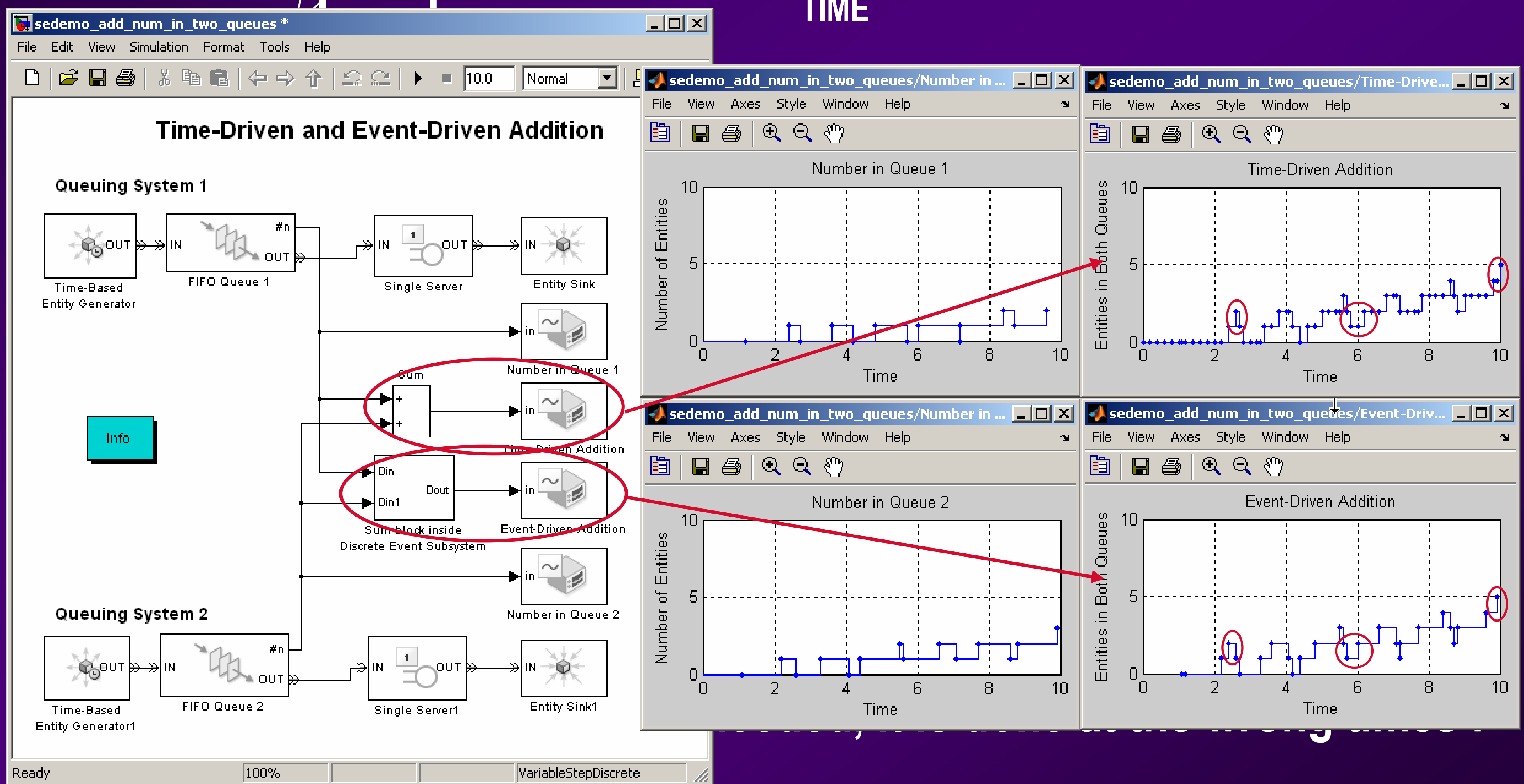
SYNCHRONOUS v ASYNCHRONOUS BEHAVIOR



SYNCHRONOUS v ASYNCHRONOUS COMPUTATION







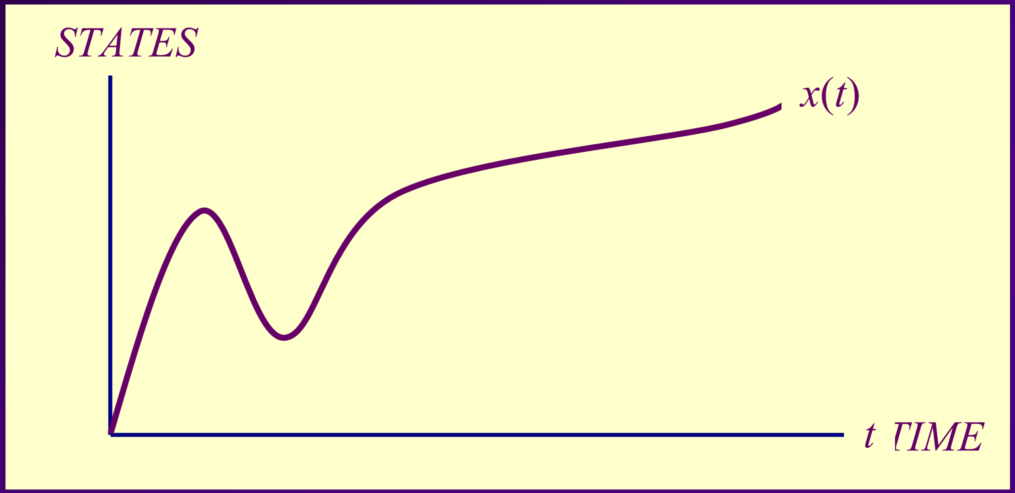
MODELING DES AND HS:
-Timed Automata
- Hybrid Automata

OTHER MODELS NOT COVERED HERE:

- Timed Petri Nets
- Max-Plus Algebra
- Temporal Logic

TIME-DRIVEN v EVENT-DRIVEN SYSTEMS

TIME-DRIVEN SYSTEM



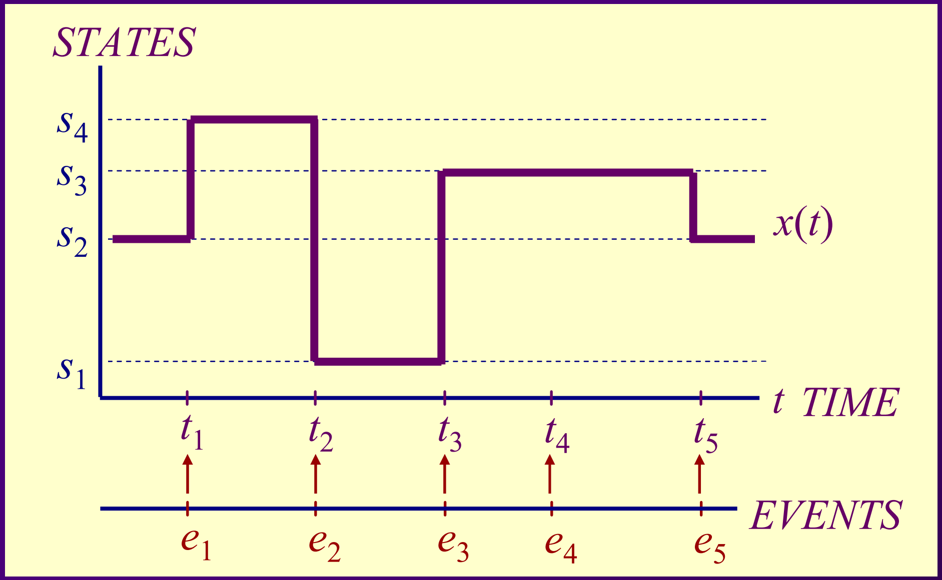
STATE SPACE:

$$X = \Re$$

DYNAMICS:

$$\dot{x} = f(x, t)$$

EVENT-DRIVEN SYSTEM



STATE SPACE:

$$X = \{s_1, s_2, s_3, s_4\}$$

DYNAMICS:

$$x' = f(x, e)$$

AUTOMATON

AUTOMATON: (E, X, Γ, f, x_0)

E : Event Set

X : State Space

$\Gamma(x)$: Set of *feasible* or *enabled* events at state x

f : State Transition Function $f : X \times E \to X$
(undefined for events $e \notin \Gamma(x)$)

x_0 : Initial State, $x_0 \in X$

$$\{e_1, e_2, \ldots\} \longrightarrow \boxed{f(x,e) = x'} \longrightarrow \{x_1, x_2, \ldots\}$$

TIMED AUTOMATON

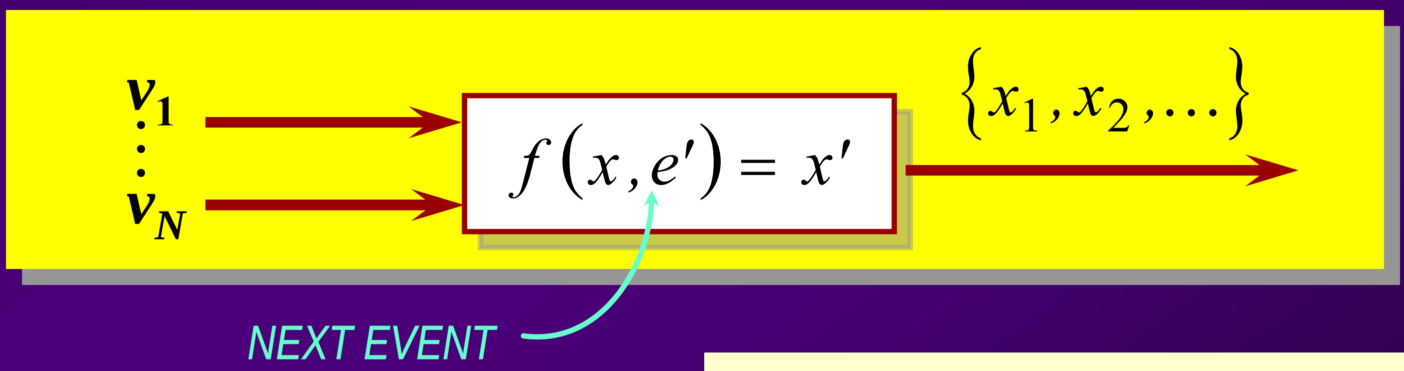
Add a ***Clock Structure V*** to the automaton: $(E, X, \Gamma, f, x_0, \boldsymbol{V})$

where:

$$\boldsymbol{V} = \{\boldsymbol{v}_i : i \in E\}$$

and $\boldsymbol{v}_i$ is a ***Clock or Lifetime sequence***: $\boldsymbol{v}_i = \{v_{i1}, v_{i2}, \ldots\}$

one for each event i



Need an *internal mechanism* to determine *NEXT EVENT e′* and hence *NEXT STATE* $x' = f(x, e')$

HOW THE TIMED AUTOMATON WORKS...

- **CURRENT STATE**

  $x \in X$ with feasible event set $\Gamma(x)$

- **CURRENT EVENT**

  e that caused transition into x

- **CURRENT EVENT TIME**

  t associated with e

Associate a
CLOCK VALUE/RESIDUAL LIFETIME y_i
with each feasible event $i \in \Gamma(x)$

HOW THE TIMED AUTOMATON WORKS...

- **NEXT/TRIGGERING EVENT** e' **:**

$$e' = \arg \min_{i \in \Gamma(x)} \{y_i\}$$

- **NEXT EVENT TIME** t' **:**

$$t' = t + y^*$$

$$\text{where:} \quad y^* = \min_{i \in \Gamma(x)} \{y_i\}$$

- **NEXT STATE** x' **:**

$$x' = f(x, e')$$

HOW THE TIMED AUTOMATON WORKS...

Determine new ***CLOCK VALUES*** y_i'
for every event $i \in \Gamma(x)$

$$y_i' = \begin{cases} y_i - y^* & i \in \Gamma(x'), i \in \Gamma(x), i \neq e' \\ v_{ij} & i \in \Gamma(x') - \{\Gamma(x) - e'\} \\ 0 & \textit{otherwise} \end{cases}$$

where: v_{ij} = new lifetime for event i

EVENT CLOCKS
ARE STATE VARIABLES

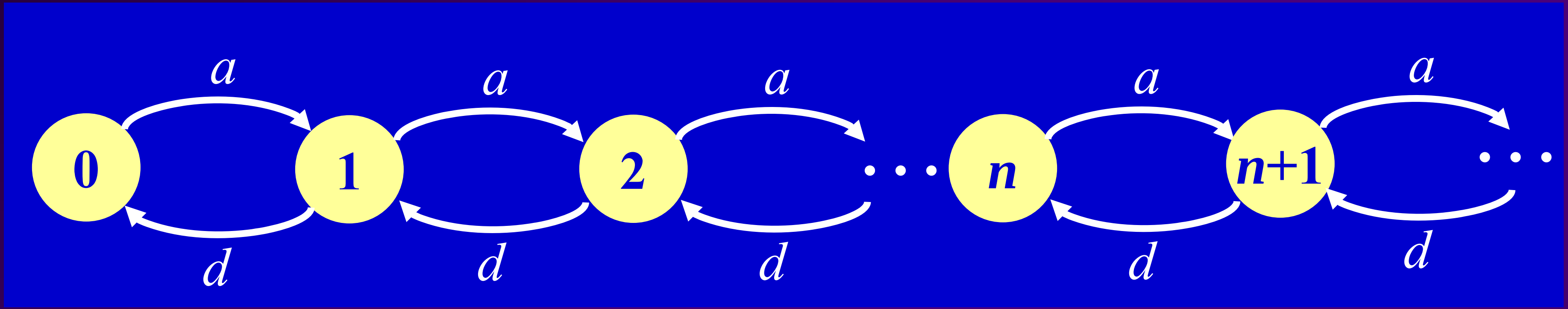
$\mathbf{v}_1$ ⋮ $\mathbf{v}_N$

$$x' = f(x, e'), \quad e' = \arg \min_{i \in \Gamma(x)} \{y_i\}$$

$$\mathbf{y}' = \mathbf{g}(\mathbf{y}, x, V)$$

$\{x_1, x_2, \ldots\}$

TIMED AUTOMATON – AN EXAMPLE



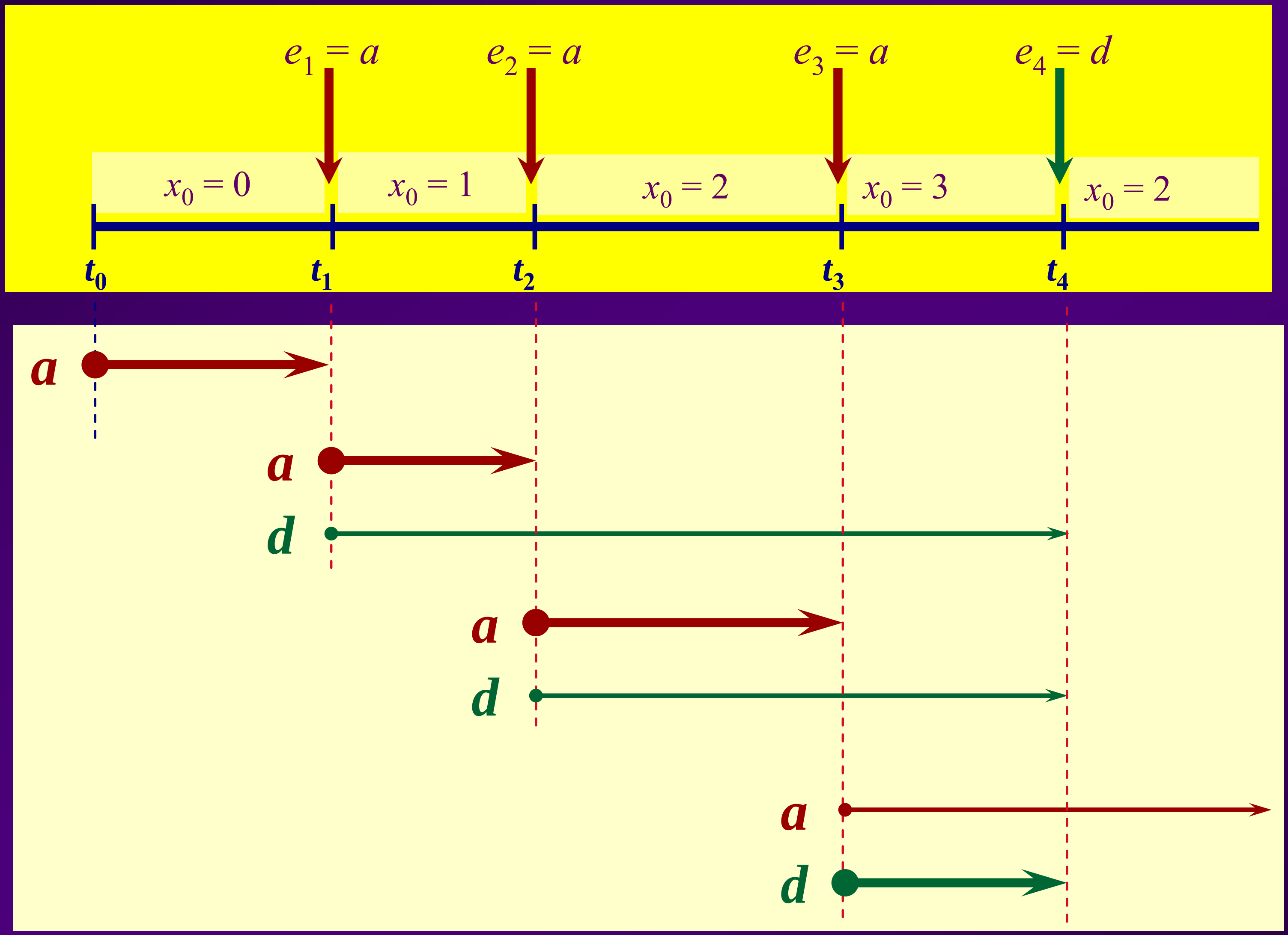
$$E = \{a, d\}$$
$$X = \{0, 1, 2, \ldots\}$$

$$\Gamma(x) = \{a, d\}, \quad \text{for all } x > 0$$
$$\Gamma(0) = \{a\}$$

$$f(x, e') = \begin{cases} x+1 & e' = a \\ x-1 & e' = d, \; x > 0 \end{cases}$$

$$\text{Given input}: \quad \mathbf{v}_a = \{v_{a1}, v_{a2}, \ldots\}, \; \mathbf{v}_d = \{v_{d1}, v_{d2}, \ldots\}$$

TIMED AUTOMATON – A STATE TRAJECTORY



STOCHASTIC TIMED AUTOMATON

- Same idea with the Clock Structure consisting of ***Stochastic Processes***

- Associate with each event *i a* ***Lifetime Distribution*** based on which $\boldsymbol{v}_i$ is generated

↓

Generalized Semi-Markov Process
(GSMP)

In a simulator, $\boldsymbol{v}_i$ is generated through a pseudorandom number generator

HYBRID AUTOMATA

$$G_h = (Q, X, E, U, f, \phi, Inv, guard, \rho, q_0, \mathbf{x}_0)$$

Q: set of discrete states (modes)

X: set of continuous states (normally R^n)

E: set of events

U: set of admissible controls

f: vector field, $f : Q \times X \times U \to X$

ϕ: discrete state transition function, $\phi : Q \times X \times E \to Q$

Inv: set defining an invariant condition (domain), $Inv \subseteq Q \times X$

$guard$: set defining a guard condition, $guard \subseteq Q \times Q \times X$

ρ: reset function, $\rho : Q \times Q \times X \times E \to X$

q_0: initial discrete state

$\mathbf{x}_0$: initial continuous state

Key features:

Transition MAY occur

Guard condition: Subset of X in which a transition from q to q' is enabled, defined through ϕ

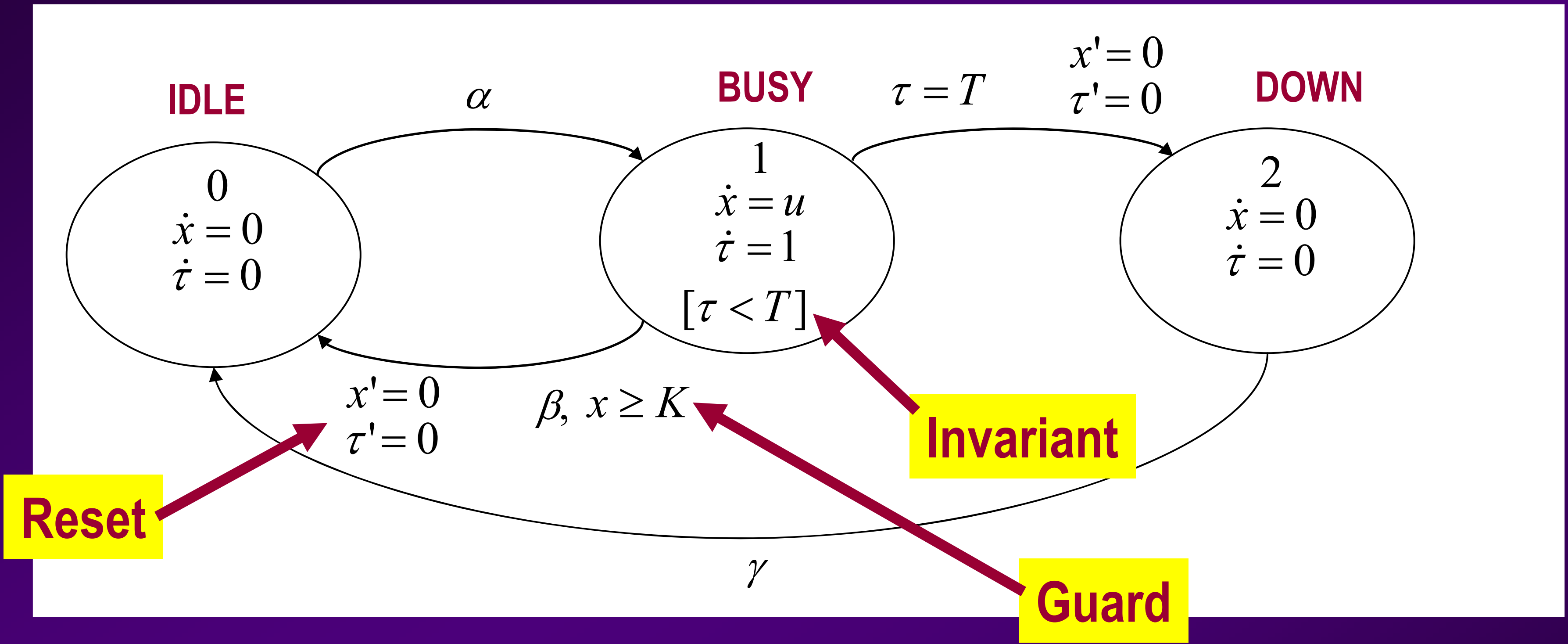
Transition MUST occur

Invariant condition: (domain) Subset of X to which x must belong in order to remain in q. If this condition no longer holds, a transition to some q' must occur, defined through ϕ

Reset condition: New value x' at q' when transition occurs from (x,q)

HYBRID AUTOMATA

Unreliable machine with timeouts

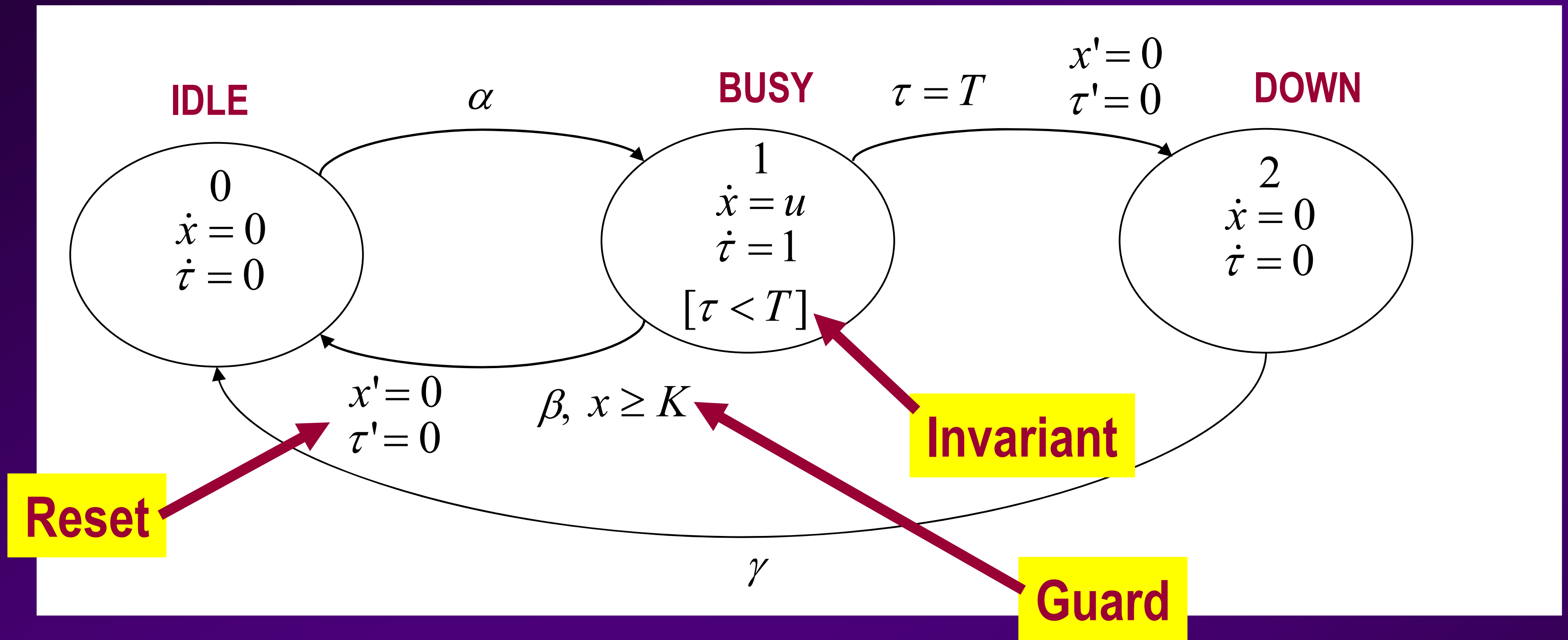


$x(t)$: physical state of part in machine

$\tau(t)$: clock

α : START, β : STOP, γ : REPAIR

HYBRID AUTOMATA



$$\phi(0; x, \tau; e) = \begin{cases} 1 & e = \alpha \\ 0 & \text{otherwise} \end{cases}$$

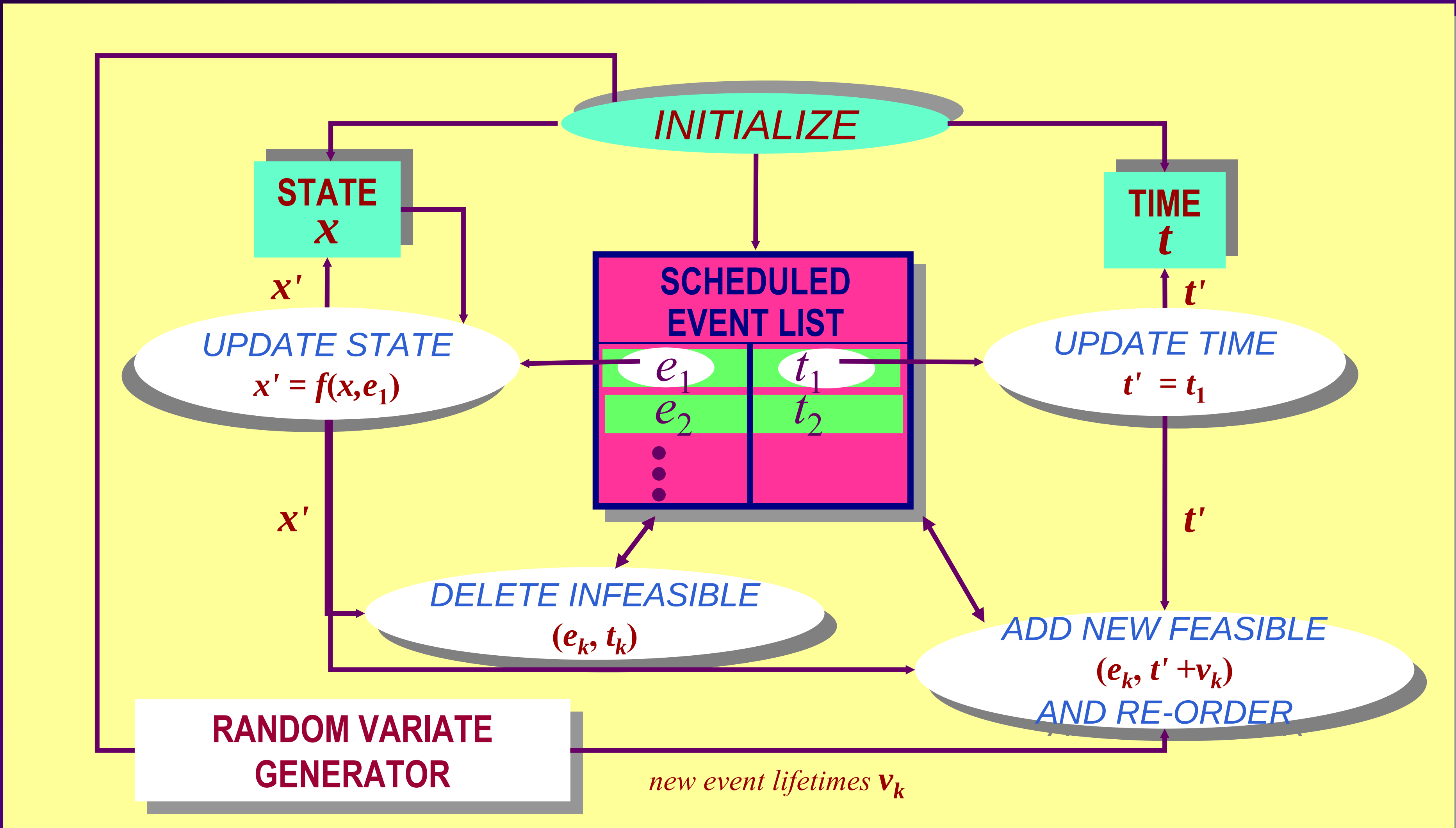
$$\phi(1; x, \tau; e) = \begin{cases} 2 & \tau = T \\ 0 & x \geq K,\ e = \beta \\ 1 & \text{otherwise} \end{cases}$$

$$\phi(2; x, \tau; e) = \begin{cases} 0 & e = \gamma \\ 2 & \text{otherwise} \end{cases}$$

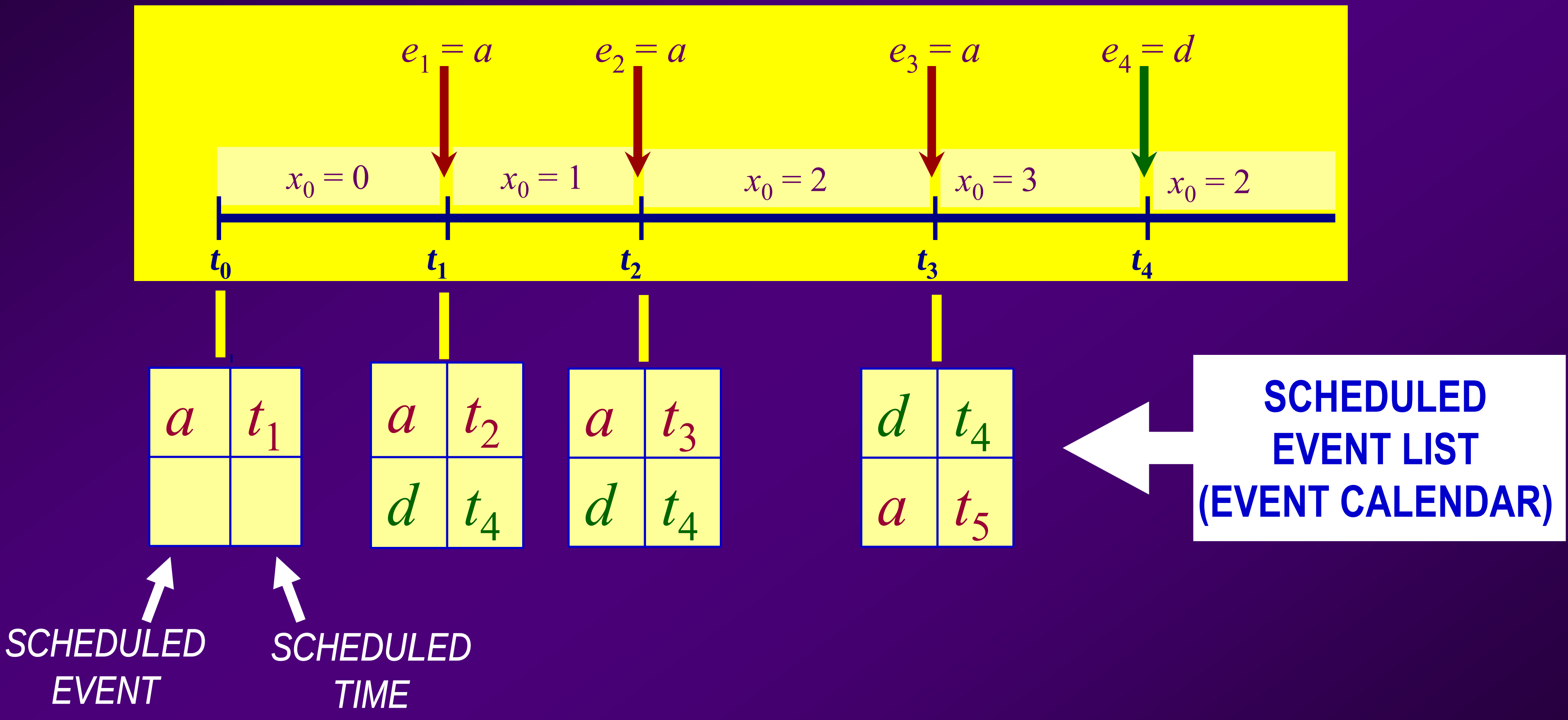
DES SIMULATION

DISCRETE EVENT SIMULATION

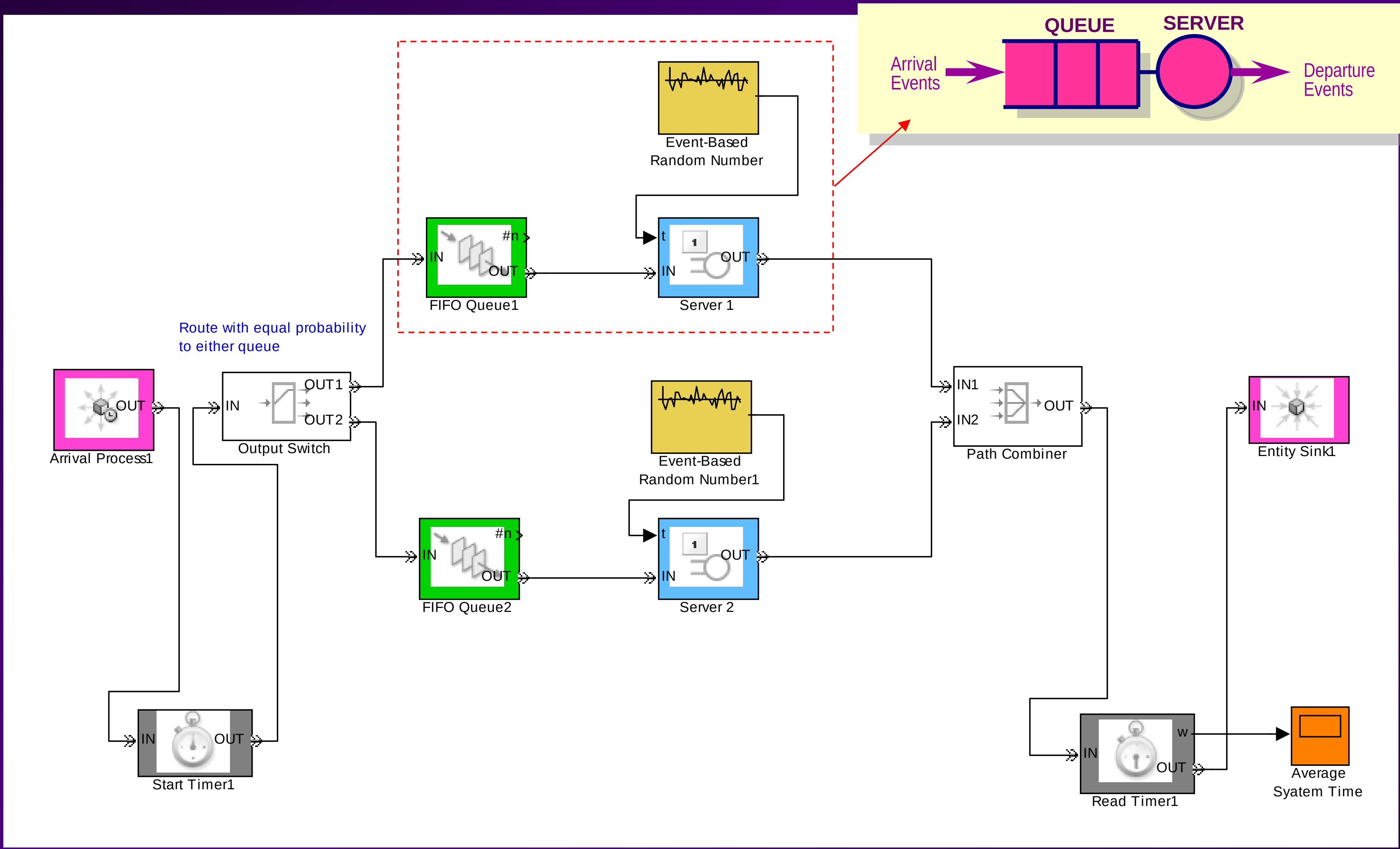
…is simply a computer-based implementation of the DES sample path generation mechanism described so far



DISCRETE EVENT SIMULATION - EXAMPLE

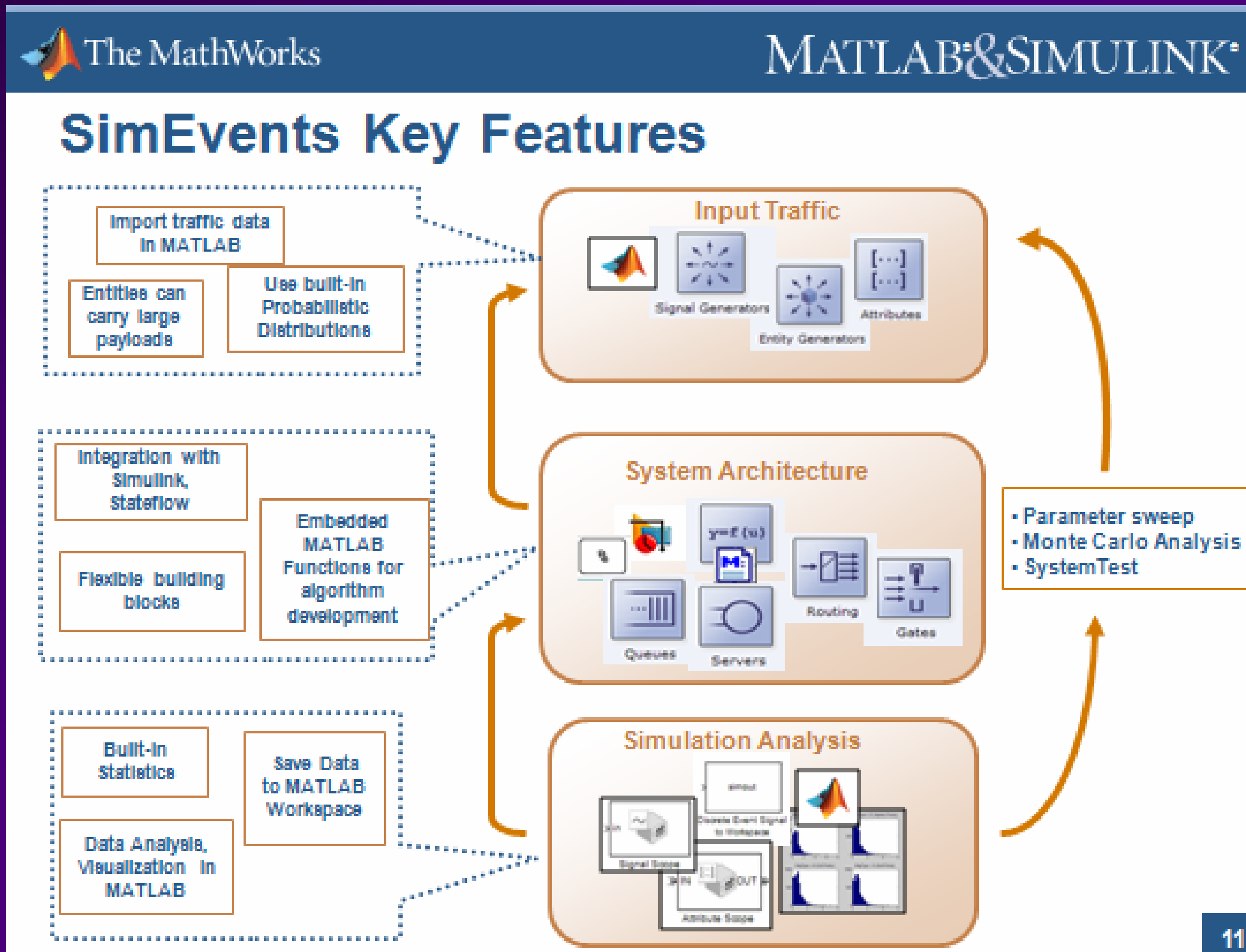


DISCRETE EVENT SIMULATION - EXAMPLE



DISCRETE EVENT SIMULATION - *SimEvents*

www.mathworks.com/products/simevents/



[Cassandras, Clune, and Mosterman, 2006]

SOFTWARE IMPLEMENTATION ISSUES

- Object-oriented design (Libraries)
- Flexibility (user can easily define new objects)
- Hierarchical (macro) capability [e.g., a *G/G/m/n* block]
- Random Variate generation accuracy
- Execution speed
- User interface
- Animation (useful sometimes – distracting/slow other times)
- Integrating with operational control software
  (e.g., *scheduling, resource allocation, flow control*)
- Web-based simulation: *Google-like capability* !?

SELECTED REFERENCES - MODELING

Timed Automata, Timed Petri Nets, Max-Plus Algebra

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– Heidergott, B., G.J. Olsder, and J. van der Woude, “Max Plus at Work – Modeling and Analysis of Synchronized Systems: A Course on Max-Plus Algebra and its Applications,” Princeton University Press, 2006

Hybrid Systems

– Bemporad, A. and M. Morari, “Control of Systems Integrating Logic Dynamics and Constraints,” Automatica, Vol. 35, No. 3, pp.407-427, 1999.
– Branicky, M.S., V.S. Borkar, and S.K. Mitter, “A Unified Framework for Hybrid Control: Model and Optimal Control Theory,”IEEE Trans. on Automatic Control, Vol. 43, No. 1, pp. 31-45, 1998.
– Cassandras, C.G., and J. Lygeros, “Stochastic Hybrid Systems,” Taylor and Francis, 2007.
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CONTROL AND OPTIMIZATION IN DES

DES CONTROL

ENABLE/DISABLE controllable events in order to achieve desired LOGICAL BEHAVIOR

- Reach desirable states
- Avoid undesirable states
- Prevent system deadlock

SUPERVISORY CONTROL

ENABLE/DISABLE controllable events in order to achieve desired PERFORMANCE

- N events of type 1 within T time units
- nth type 1 event occurs before mth type 2 event
- No more than N type 1 events after T time units

RESOURCE CONTENTION PROBLEMS IN DES

RESOURCE CONTENTION

USERS competing for limited *RESOURCES* in an event-driven dynamic environment

Examples:

- *MESSAGES* competing for *SWITCHES* in networks
- *TASKS* competing for *PROCESSORS* in computers
- *PARTS* competing for *EQUIPMENT* in manufacturing

TYPICAL GOALS:

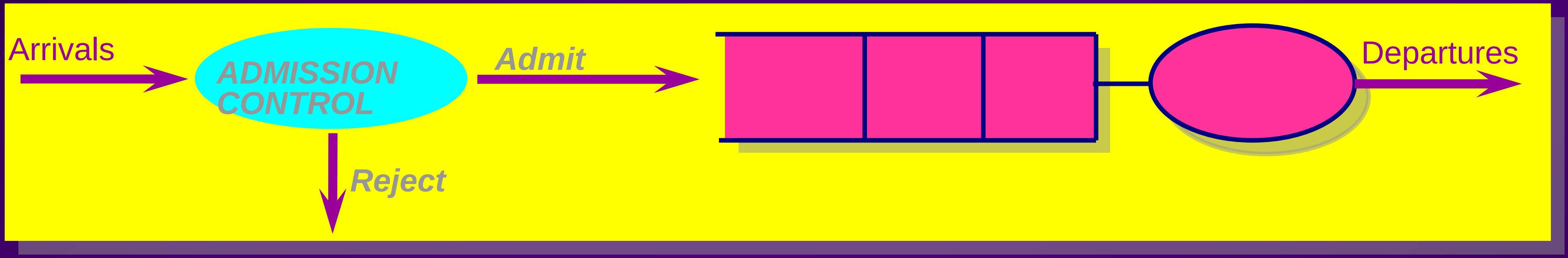
- User requests satisfied on “best effort” basis
- User requests satisfied on “guaranteed performance” basis
- Users treated “fairly”

RESOURCE CONTENTION

1. ADMISSION CONTROL

Admit or Reject user requests

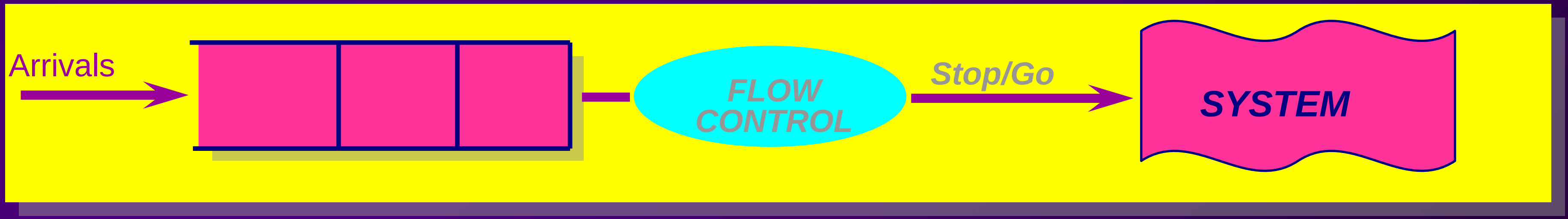
(e.g., to ensure good quality of service for admitted users)



2. FLOW CONTROL

Control *when* to accept user requests

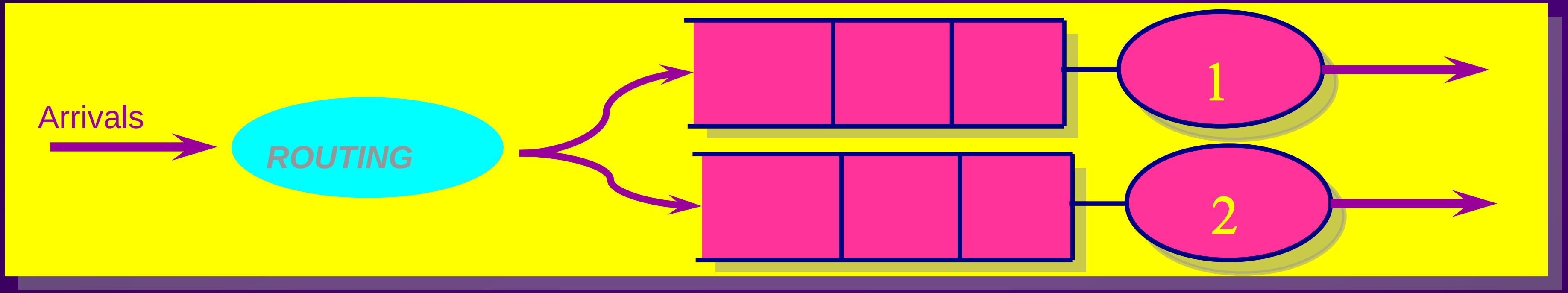
(e.g., to "smooth" bursty demand, prevent congestion)



RESOURCE CONTENTION

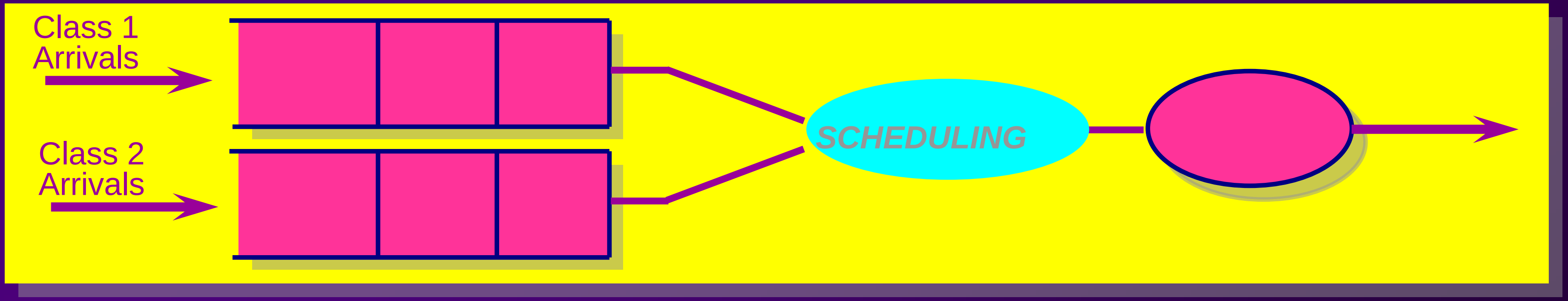
3. ROUTING

User selects a resource
(e.g., route to shortest queue)



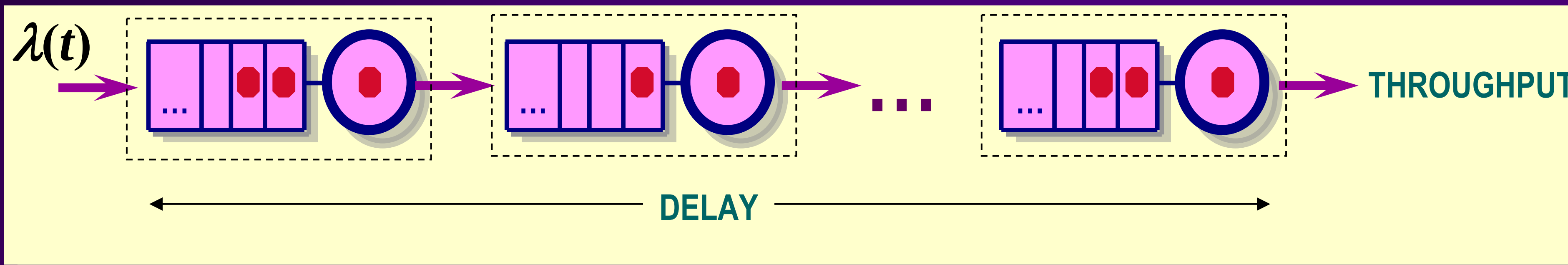
4. SCHEDULING

Resource selects user
(e.g., serve longest queue first)



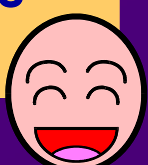
RESOURCE ALLOCATION

Manufacturing system with *N* sequential operations:

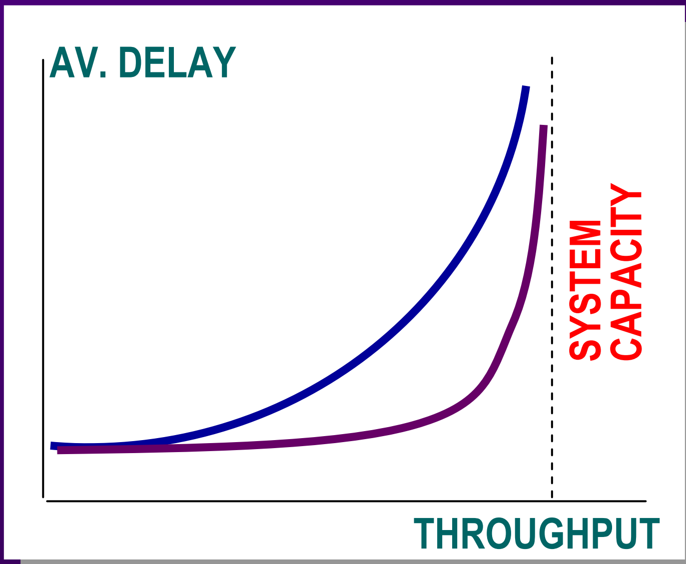


INCREASE $\lambda(t)$

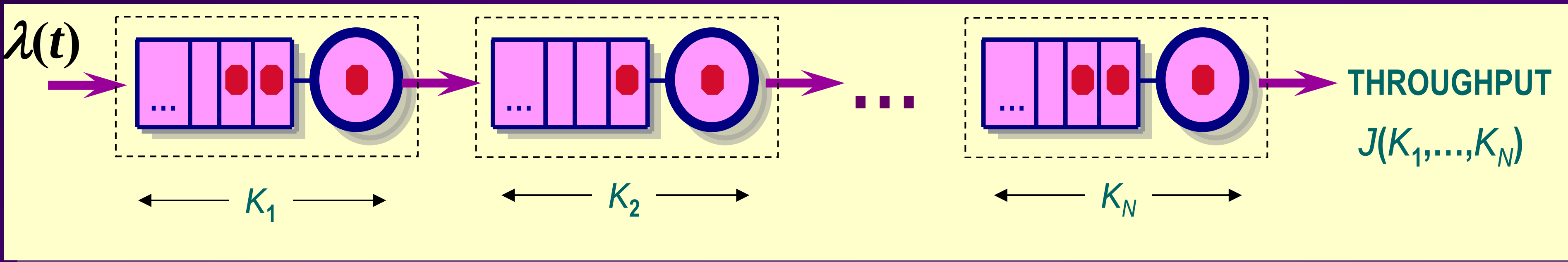
THROUGHPUT increases (GOOD)



average DELAY increases (BAD)

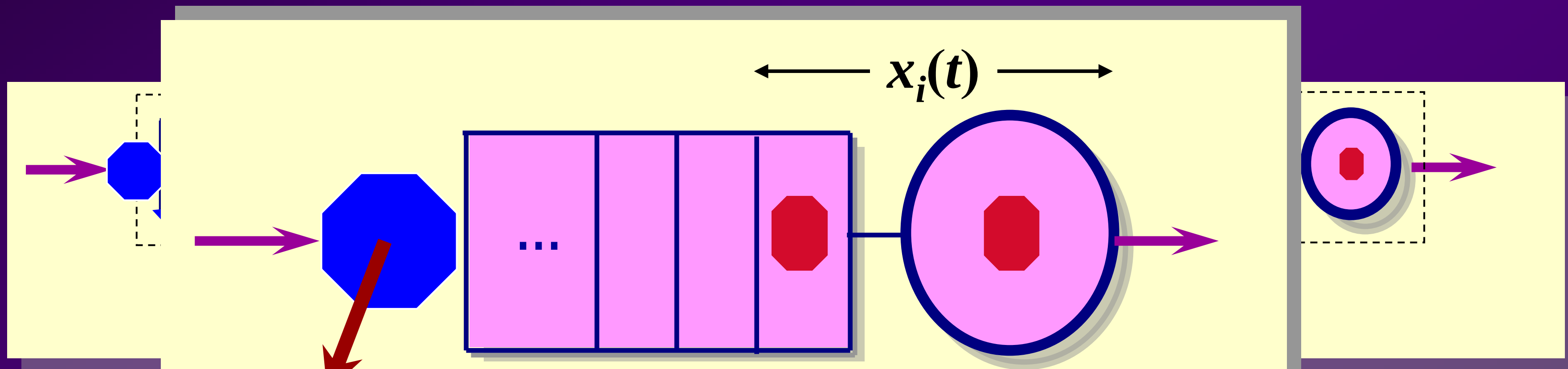


KANBAN (OR *BUFFER*) *ALLOCATION*



$$\max_{K_1,\dots,K_N} J(K_1,\dots,K_N) \text{ s.t.} \sum_{i=1}^{N} K_i = C$$

RESOURCE ALLOCATION

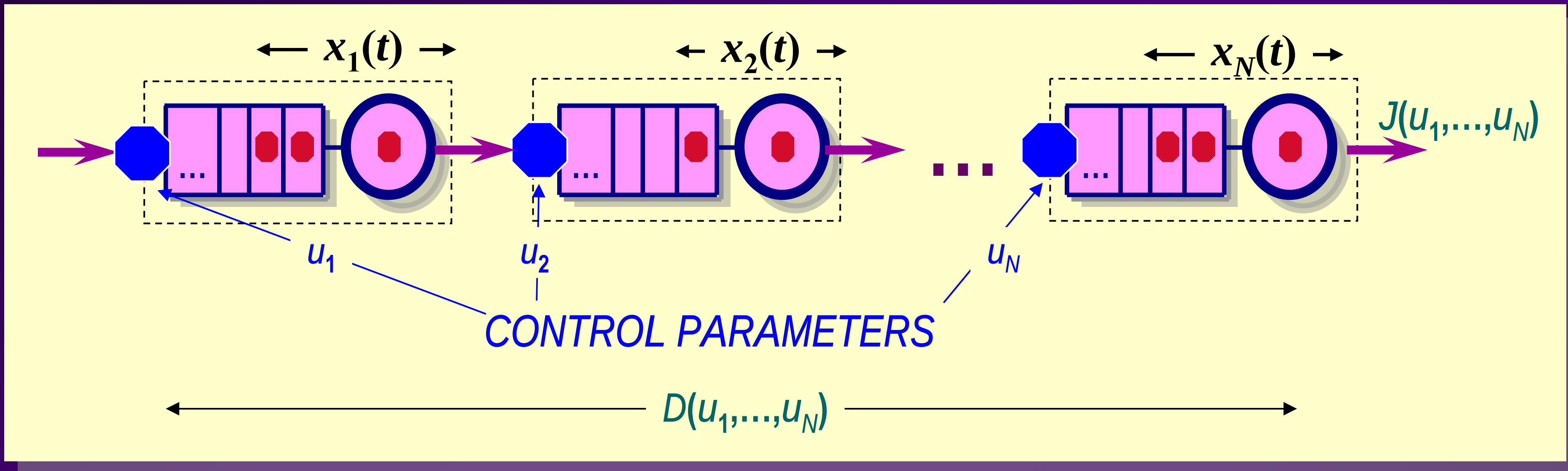


GO **if** $x_i(t) < K_i$, ***STOP*** **otherwise**

FEEDBACK

No. of *KANBAN (tickets)* allocated to stage *i*

RESOURCE ALLOCATION



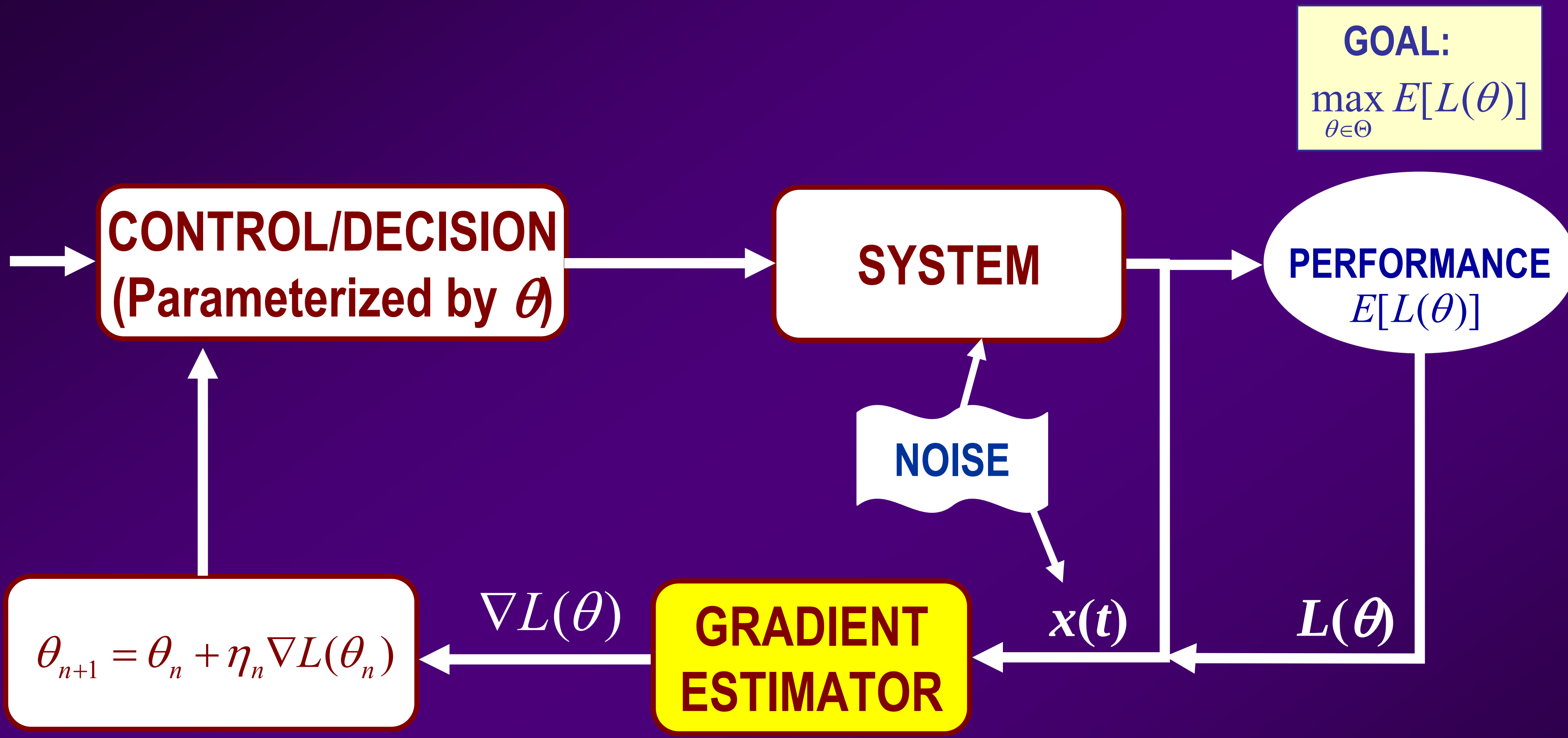
$$\max_{u_1,\ldots,u_N} J(u_1,\ldots,u_N) \;\; \text{s.t.} \begin{cases} D(u_1,\ldots,u_N) \le C \\ \text{system dynamics} \end{cases}$$

SOLUTION METHODOLOGIES

- **Queueing** models and analysis: *Descriptive*, not *prescriptive*
- **Markov Decision Processes** (MDPs):
  - Decisions planned ahead
  - Need accurate stochastic models
  - Curse of dimensionality (Dynamic Programming)
- **Approximate Dynamic Programming** (ADP) techniques
- **Data-driven** techniques:
  - Gradient Estimation
  - Rapid Learning

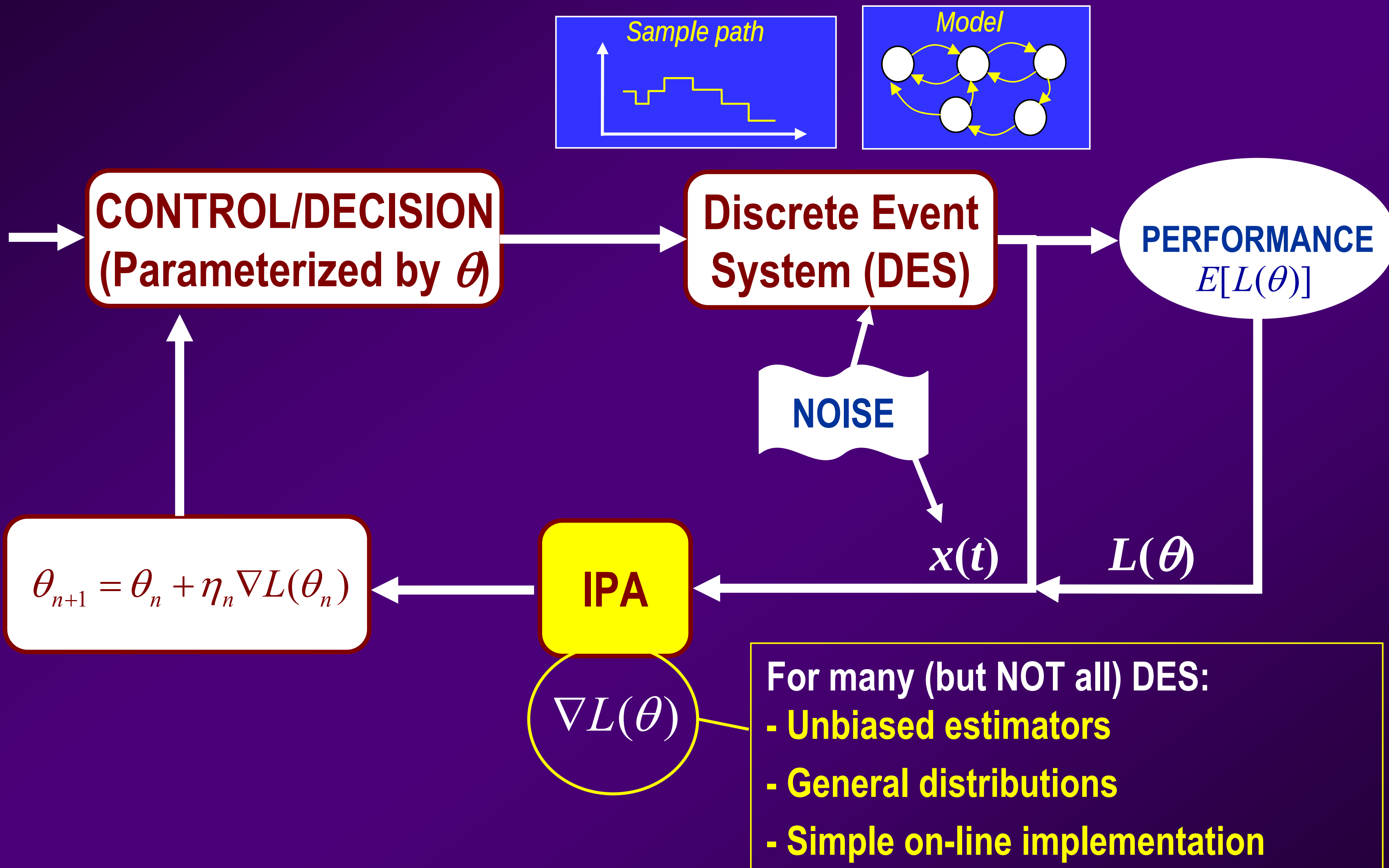
STOCHASTIC CONTROL AND OPTIMIZATION: PERTURBATION ANALYSIS, RAPID LEARNING

REAL-TIME STOCHASTIC CONTROL AND OPTIMIZATION



DIFFICULTIES:
- $E[L(\theta)]$ NOT available in closed form
- $\nabla L(\theta)$ not easy to evaluate
- $\nabla L(\theta)$ may not be a good estimate of $\nabla E[L(\theta)]$

THE CASE OF *DES*: INFINITESIMAL PERTURBATION ANALYSIS (IPA)



[*Ho and Cao, 1991*], [*Glasserman, 1991*], [*Cassandras, 1993*], [*Cassandras and Lafortune, 2008*]

RAPID LEARNING

LEARNING BY TRIAL-AND-ERROR

CONVENTIONAL TRIAL-AND-ERROR ANALYSIS (e.g., simulation)

- *Repeatedly change parameters/operating policies*
- *Test different conditions*
- *Answer multiple WHAT IF questions*

LEARNING THROUGH *CONCURRENT ESTIMATION*

Design Parameters

Operating Policies, Control Parameters

SYSTEM

Performance Measures

CONCURRENT ESTIMATION

WHAT IF…

- Parameter $p_1 = a$ were replaced by $p_1 = b$
- Parameter $p_2 = c$ were replaced by $p_2 = d$

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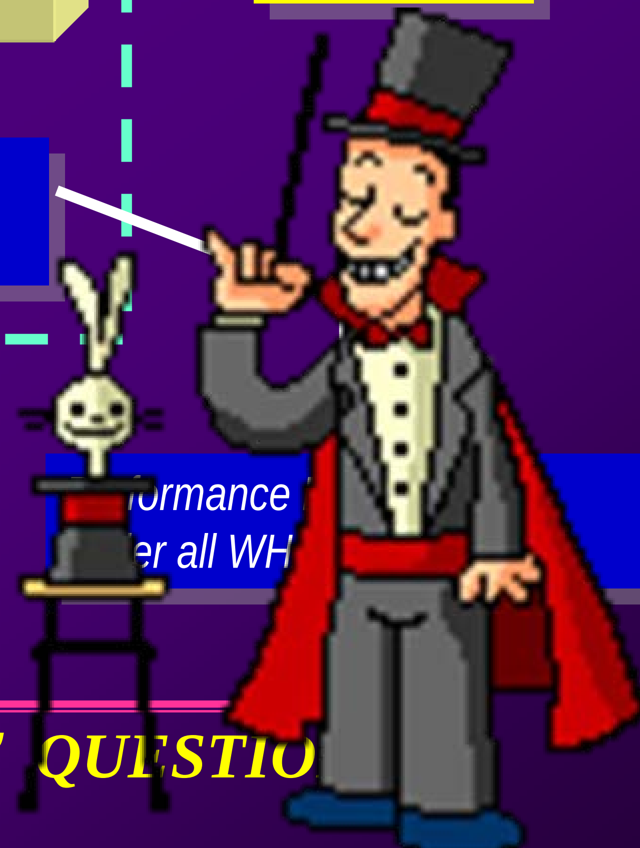
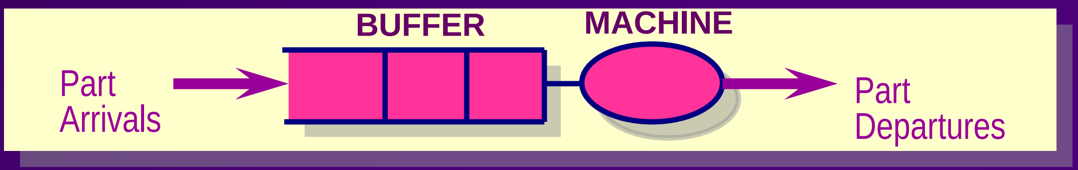
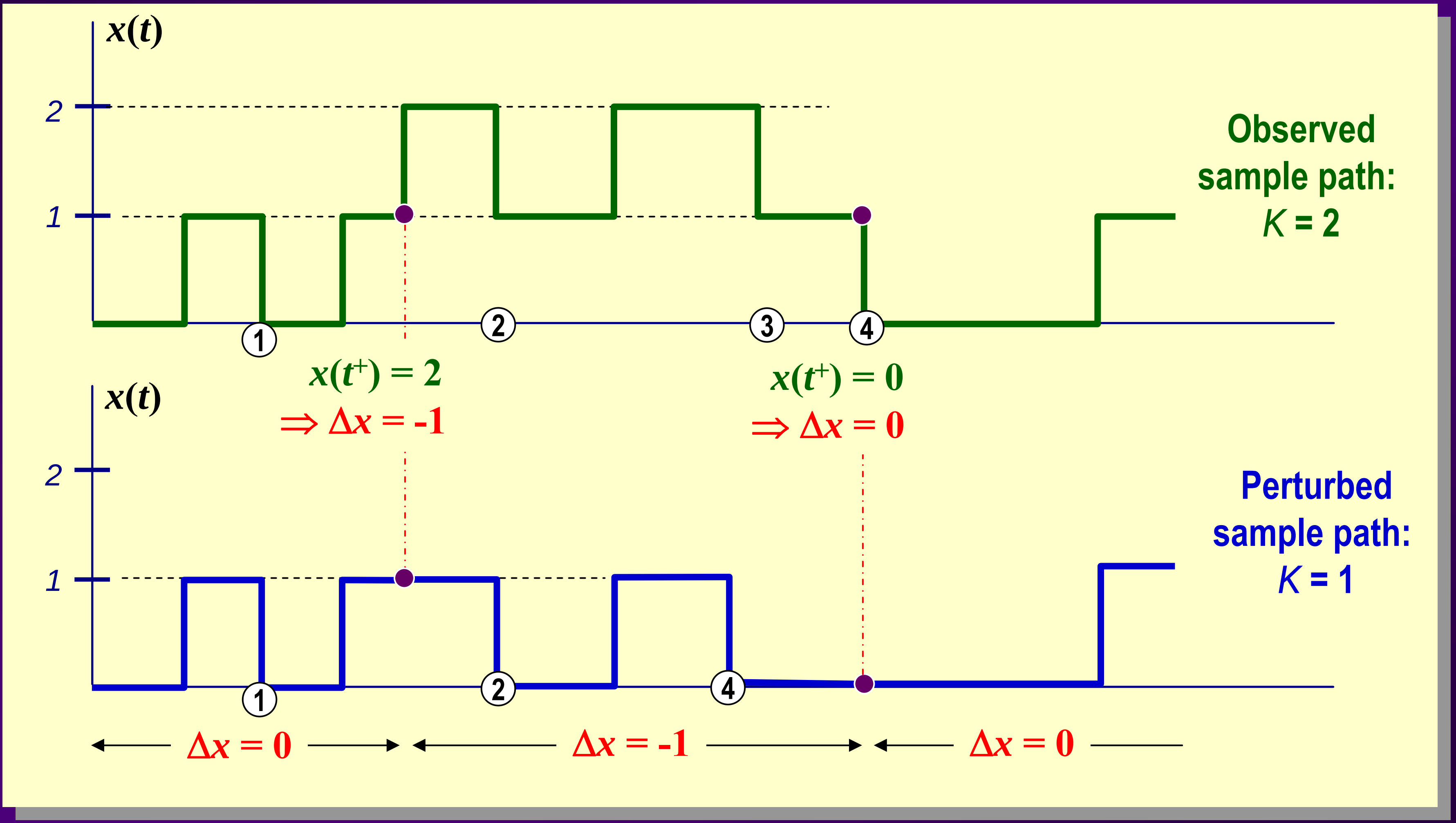


ANSWERS TO MULTIPLE "WHAT IF" QUESTIO
AUTOMATICALLY PROVIDED

LEARNING THROUGH *CONCURRENT ESTIMATION*

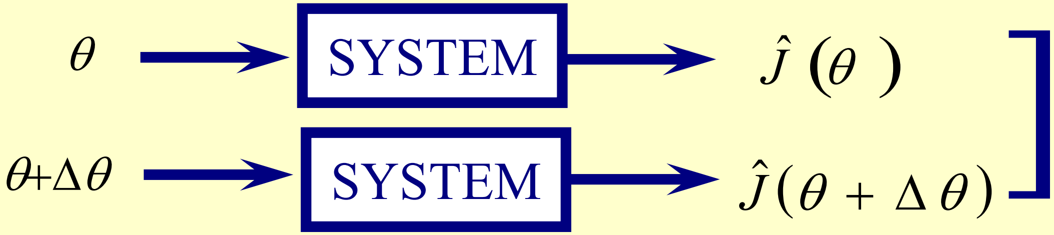




PERTURBATION ANALYSIS

IPA vs "BRUTE FORCE" DERIVATIVE ESTIMATION

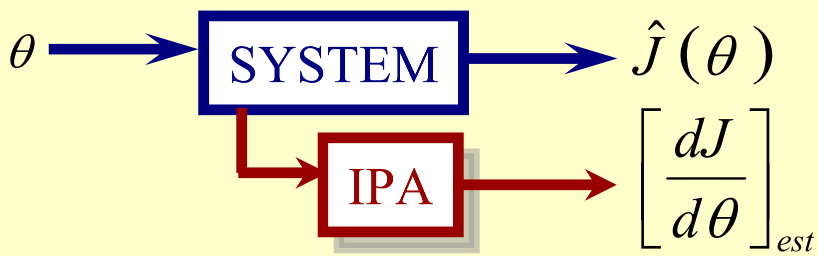
"Brute Force" Derivative Estimation:



$$\Rightarrow \left[\frac{dJ}{d\theta}\right]_{est} = \frac{\hat{J}(\theta+\Delta\theta)-\hat{J}(\theta)}{\Delta\theta}$$

DRAWBACKS:
- **Intrusive**: actively introduces perturbation $\Delta\theta$
- **Computationally costly**: 2 observation processes [$(N+1)$ for N-dim θ]
- **Inherently inaccurate**: $\Delta\theta$ large $\Rightarrow$ poor derivative approx.
  $\Delta\theta$ small $\Rightarrow$ numerical instability

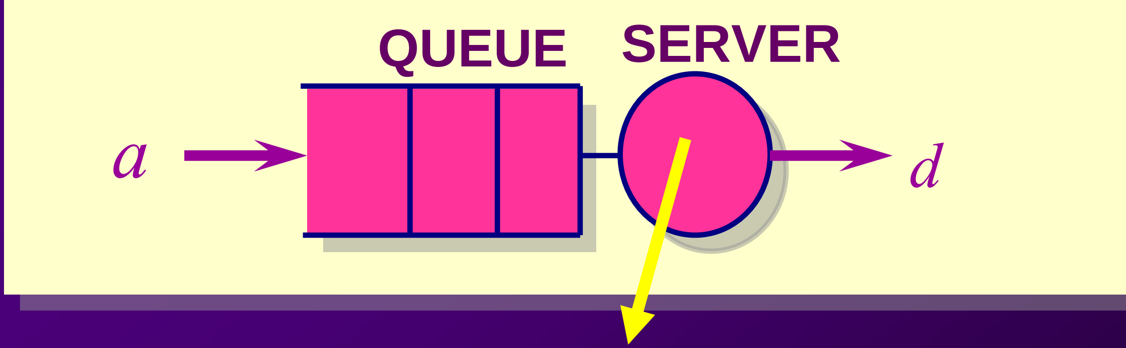
Infinitesimal Perturbation Analysis (IPA):



PERTURBATION GENERATION

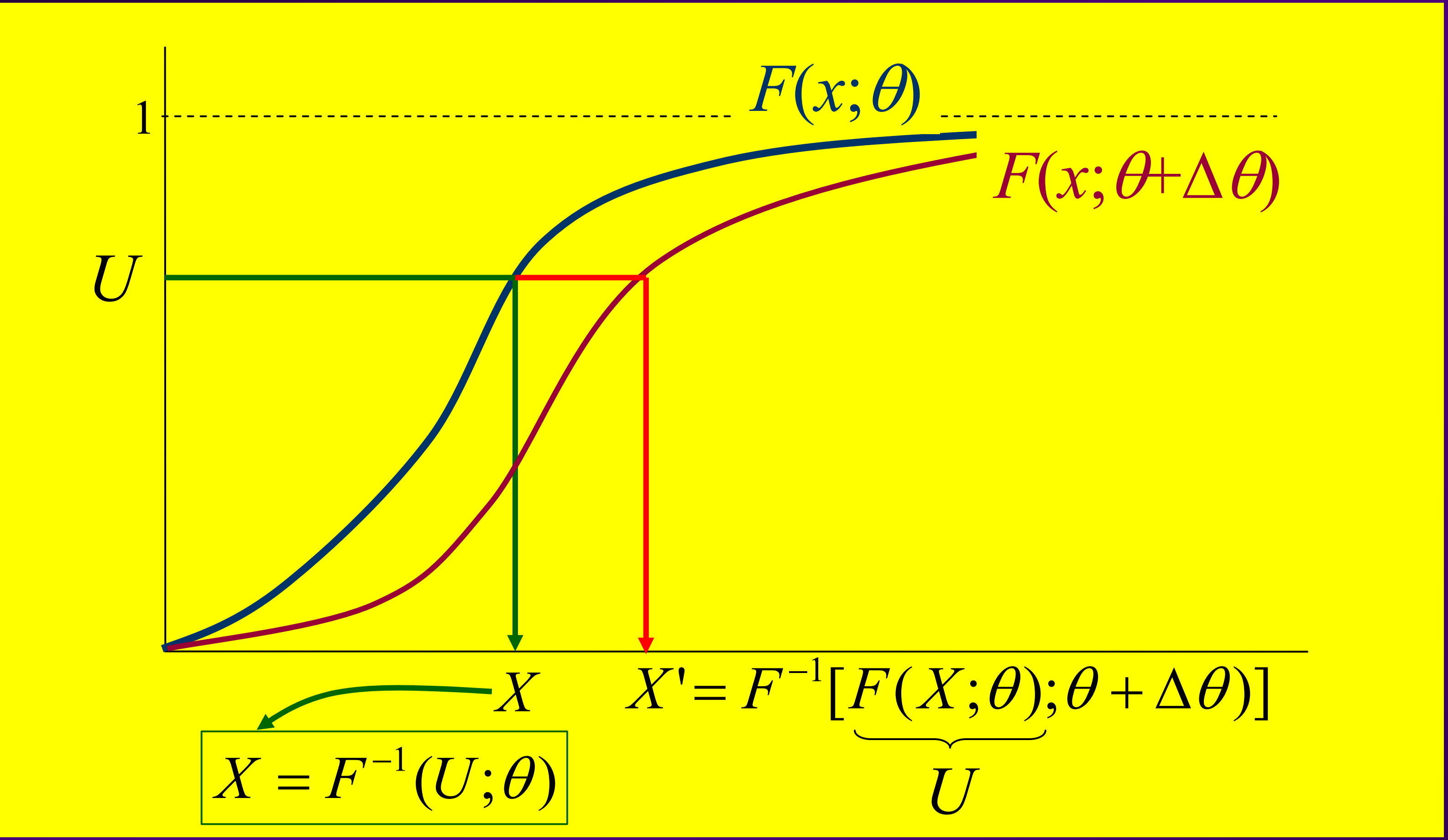
- DES parameter θ perturbed by $\Delta\theta$
- Suppose θ is a parameter of some cdf $F(x;\theta)$
- OBSERVED sample path: $X(\theta)$
- What is $\Delta X(\theta)$?
- What is $\dfrac{dX(\theta)}{d\theta}$?

EXAMPLE:



- Mean Service Time: θ
- Service Times: $X_1(\theta), X_2(\theta), \ldots$
- What happens when θ is changed by $\Delta\theta$?

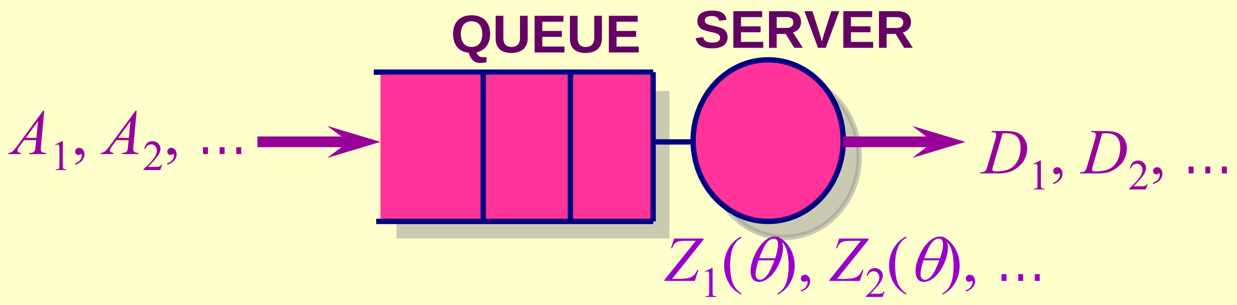
PERTURBATION GENERATION



$$\Delta X = F^{-1}[F(X;\theta);\theta+\Delta\theta)] - X$$

$$\frac{dX}{d\theta} = -\frac{\left[\partial F(x;\theta)/\partial\theta\right]_{(X,\theta)}}{\left[\partial F(x;\theta)/\partial x\right]_{(X,\theta)}}$$

PERTURBATION PROPAGATION



Suppose θ is perturbed by $\Delta\theta \;\Rightarrow\; \Delta D_k = D_k' - D_k$

Lindley Equation for any queueing system:

$$D_k = \max\{A_k, D_{k-1}\} + Z_k$$

Max-Plus Algebra
(One of many dioid algebras)
Cunningham-Greene, R.A., Minimax Algebra, 1979.

$$\Rightarrow \quad \Delta D_k = \max\{A_k, D_{k-1}'\} - \max\{A_k, D_{k-1}\} + \Delta Z_k$$
$$= \max\{A_k, D_{k-1} + \Delta D_{k-1}\} - \max\{A_k, D_{k-1}\} + \Delta Z_k$$

Iterative relationship that depends ONLY on observed sample path data !

PERTURBATION PROPAGATION

Four cases to consider… After some algebra:

$$\Delta D_k = \Delta Z_k + \begin{cases} \Delta D_{k-1} & \text{if } I_k \le 0 \text{ and } \Delta D_{k-1} \ge I_k \\ 0 & \text{if } I_k > 0 \text{ and } \Delta D_{k-1} \le I_k \\ I_k & \text{if } I_k \le 0 \text{ and } \Delta D_{k-1} \le I_k \\ \Delta D_{k-1} - I_k & \text{if } I_k > 0 \text{ and } \Delta D_{k-1} \ge I_k \end{cases}$$

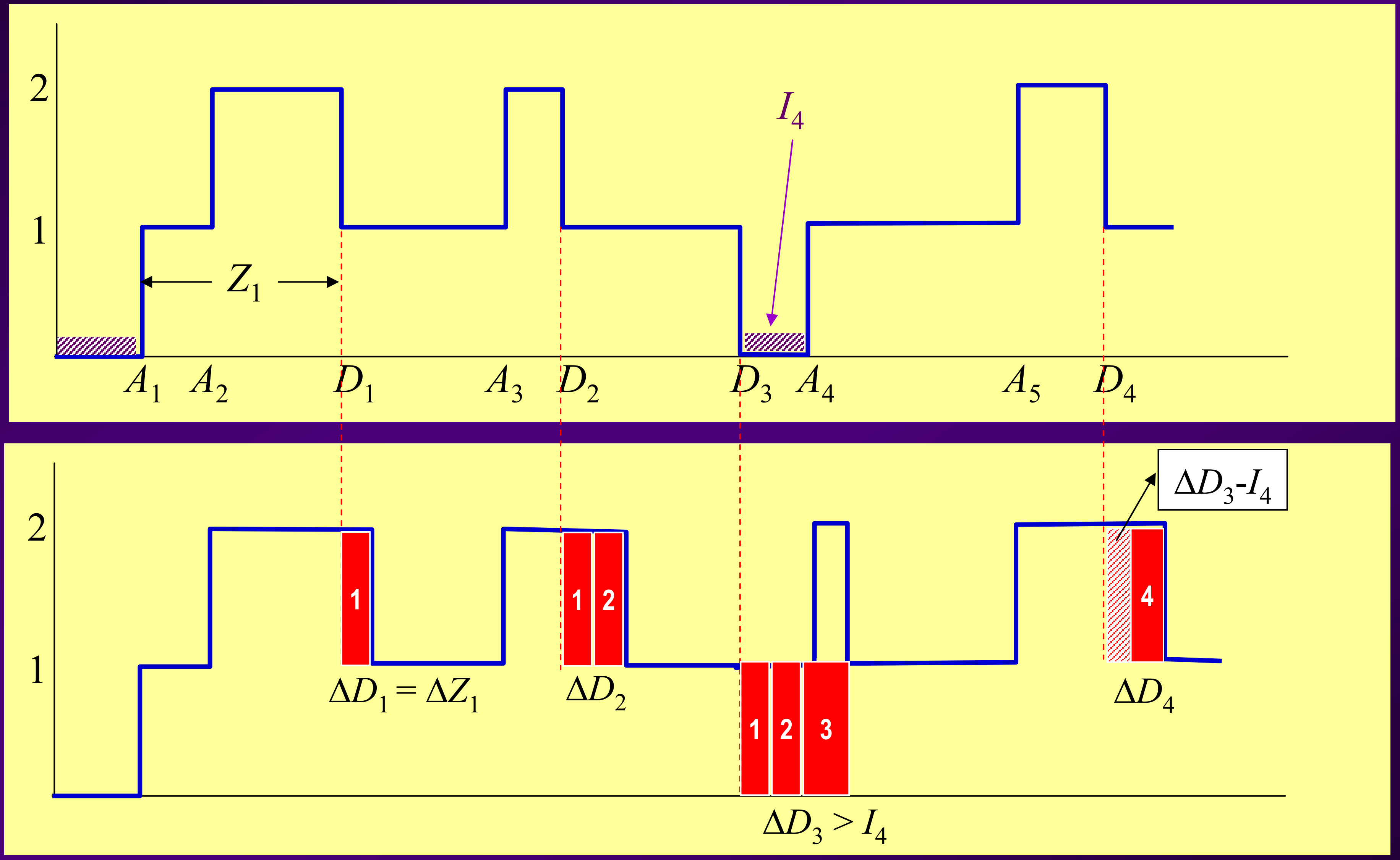
where: $I_k = A_k - D_{k-1}$ **(idle period length if > 0)**

I_k **observed**

ΔZ_k **calculated from Z_k as described earlier (PERTURBATION GENERATION)**

PERTURBATION PROPAGATION

EXAMPLE: Perturbations in mean service time $\Rightarrow \Delta Z_1(\theta), \Delta Z_2(\theta), \ldots$



INFINITESIMAL PERTURBATION ANALYSIS (IPA)

If $\Delta\theta$ is so "small" as to ensure that $\Delta D_{k\text{-}1} \leq I_k$ then

$$\Delta D_k = \Delta Z_k + \begin{cases} \Delta D_{k-1} & \text{if } I_k \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

Perturbation Generation

Perturbation Propagation

If derivatives are used, this can be rigorously shown:

$$\frac{dD_k}{d\theta} = \frac{dZ_k}{d\theta} + \begin{cases} \dfrac{dD_{k-1}}{d\theta} & \text{if } I_k \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

UNIVERSAL IPA ALGORITHM

1. INITIALIZATION: If α feasible at x_0: $\Delta_\alpha := \dfrac{dV_{\alpha,1}}{d\theta}$

Else for all other α: $\Delta_\alpha := 0$

2. WHENEVER β IS OBSERVED (including $\beta = \alpha$) :

If α activated with new lifetime V_α :

2.1. Compute $dV_\alpha / d\theta$

2.2. $\Delta_\alpha := \Delta_\beta + \dfrac{dV_\alpha}{d\theta}$

IMPLEMENTATION: Simple, *non-intrusive*, overhead negligible

See *http://people.bu.edu/cgc/IPA/*

INFINITESIMAL PERTURBATION ANALYSIS (IPA)

QUESTION: Are IPA sensitivity estimators "good" ?

- Unbiasedness
- Consistency

ANSWER: Yes, for a large class of DES that includes

- G/G/1 queueing systems
- Jackson-like queueing networks
  (e.g., no blocking allowed)

NOTE: IPA applies to REAL-VALUED parameters of some event process distribution (e.g., mean interarrival and service times)

IPA UNBIASEDNESS

Theorem. For a given sample function $L(\theta)$, suppose that

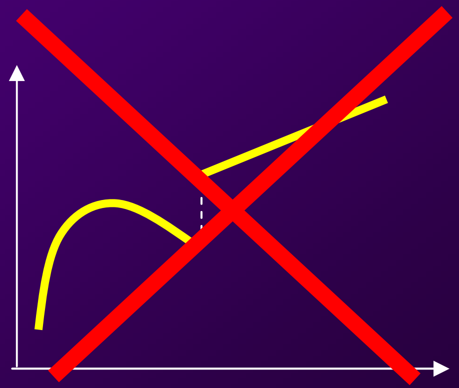
(a) the sample derivative $dL(\theta)/d\theta$ exists w.p. 1 for every $\theta \in \Theta$ where Θ is a closed bounded set, and

(b) $L(\theta)$ is Lipschitz continuous w.p. 1 and the Lipschitz constant has finite first moment.

Then,

$$\frac{d}{d\theta} E[L(\theta)] = E\left[\frac{dL(\theta)}{d\theta}\right]$$

[Rubinstein and Shapiro, 1993]



COMMUTING CONDITION

States x, y, z_1 and events α, β such that

$$p(z_1; x, \alpha) \cdot p(y; z_1, \beta) > 0$$

Then, for some z_2

$$p(z_2; x, \beta) = p(y; z_1, \beta) \text{ and } p(y; z_2, \alpha) = p(z_1; x, \alpha)$$

Moreover, for x, z_1, z_2 s.t. $p(z_1;x,\alpha) = p(z_2;x,\alpha) > 0$, $z_1 = z_2$

$$\begin{tikzcd} & z_1 \arrow[dr, "\beta"] & \\ x \arrow[ur, "\alpha"] \arrow[dr, "\beta"'] & & y \\ & z_2 \arrow[ur, "\alpha"'] & \end{tikzcd}$$

[*Glasserman, 1991*]

EXTENSIONS OF IPA

There are several generalizations, at the expense of more simulation overhead (still, very minimal)

e.g.,

- routing probabilities
- systems with real-time constraints
- some scheduling policies

IMPLEMENTATION:

Still very straightforward and *non-intrusive*

CONCURRENT ESTIMATION

THE CONSTRUCTABILITY PROBLEM

Consider a Discrete Event System (DES) operating under parameter settings $\{u_1, \ldots, u_m\}$

INPUT: $\omega =$ all event lifetimes

$u_1 =$ a parameter setting

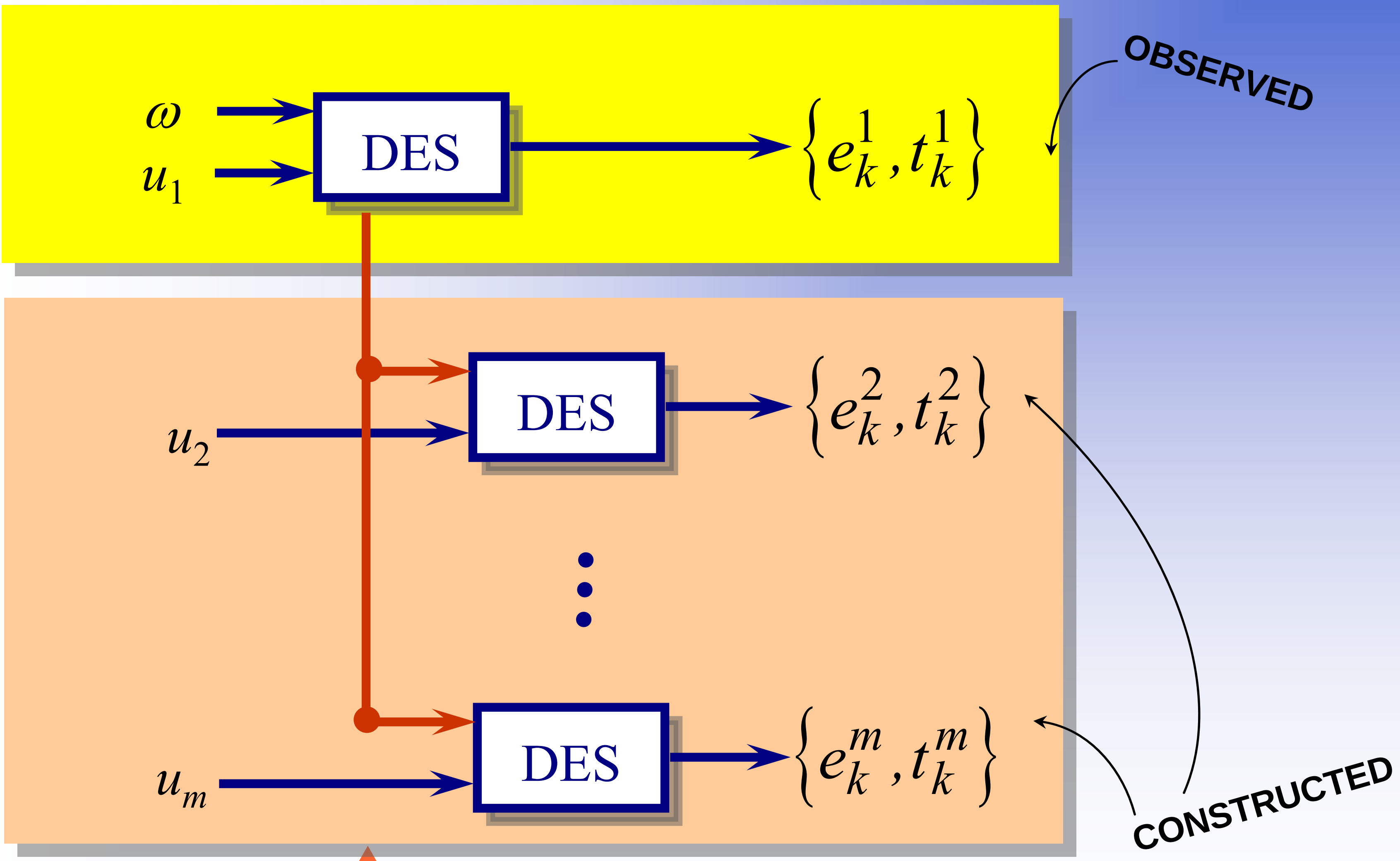
OUTPUT: $\{e_k^1, t_k^1\} =$ observed sample path under u_1

PROBLEM:

Construct sample paths under all $\{u_2, \ldots, u_m\}$ based only on

- The input ω
- The observed sample path $\{e_k^1, t_k^1\}$
- Observed state history

THE CONSTRUCTABILITY PROBLEM *CONTINUED*



THE CONSTRUCTABILITY PROBLEM *CONTINUED*

- **OBSERVABILITY**:

A sample path $\left\{e_k^j, t_k^j\right\}$ is *observable* with respect to $\left\{e_k, t_k\right\}$ if

$$\Gamma\left(x_k^j\right) \subseteq \Gamma\left(x_k\right) \quad \text{for all } k = 0,1,\ldots$$

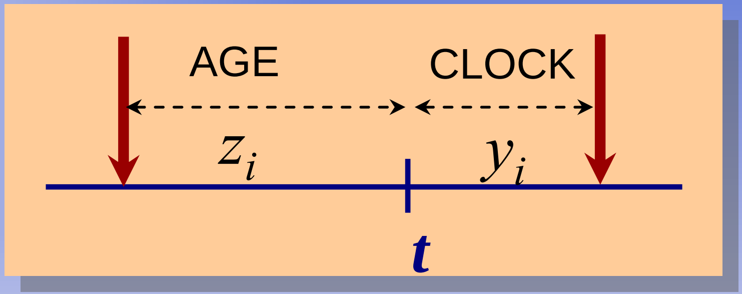
INTERPRETATION: For every state x_k^j visited in the constructed sample path, all feasible events $i \in \Gamma\left(x_k^j\right)$ are also feasible in the corresponding observed state x_k

Ref: *Cassandras and Strickland, 1989*

THE CONSTRUCTABILITY PROBLEM *CONTINUED*

Define the conditional cdf of an *event clock* given the *event age*:

$$H(t, z_i) = P[y_i \le t \mid z_i]$$



CONSTRUCTABILITY:

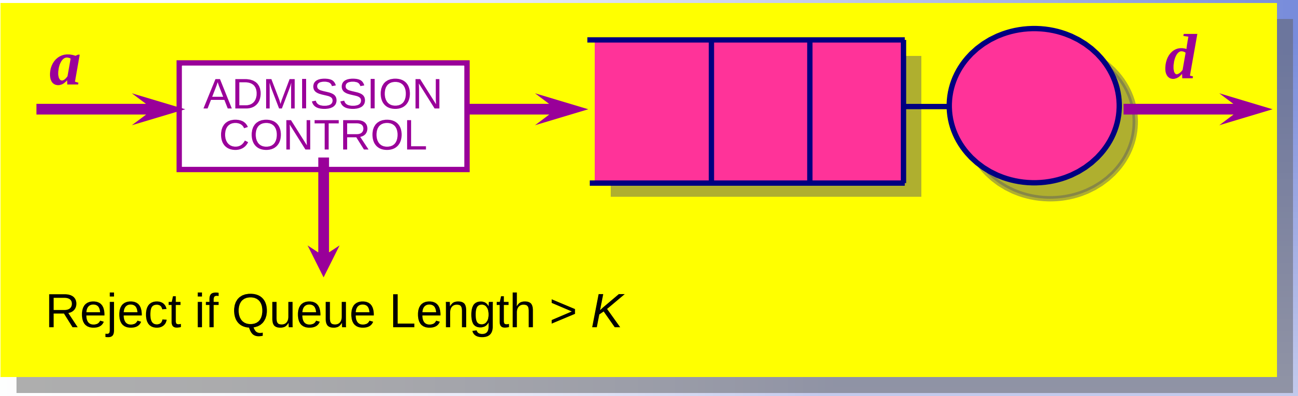
A sample path $\{e_k^j, t_k^j\}$ is *constructable* with respect to $\{e_k, t_k\}$ if

1. $\Gamma(x_k^j) \subseteq \Gamma(x_k)$ for all $k = 0, 1, \ldots$

2. $H^j(t, z_{i,k}^j) = H(t, z_{i,k})$ for all $i \in \Gamma(x_k^j)$, $k = 0, 1, \ldots$

CONSTRUCTABILITY: AN EXAMPLE

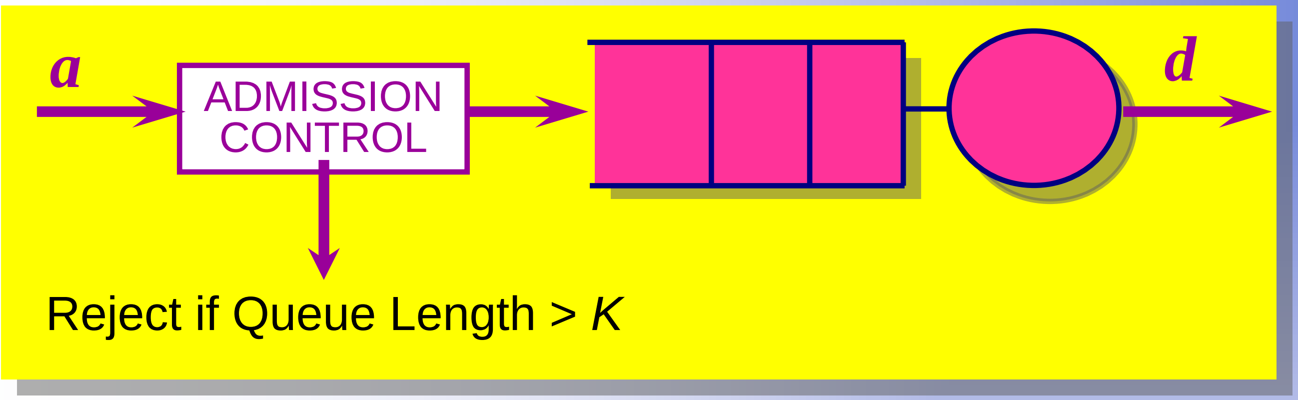
A simple design/optimization problem



PROBLEM:

- How does the system behave under different choices of *K*?
- What is the *optimal K*?

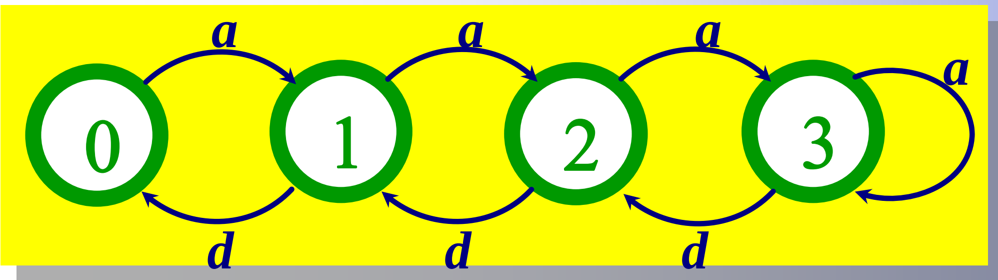
CONSTRUCTABILITY: AN EXAMPLE *CONTINUED*



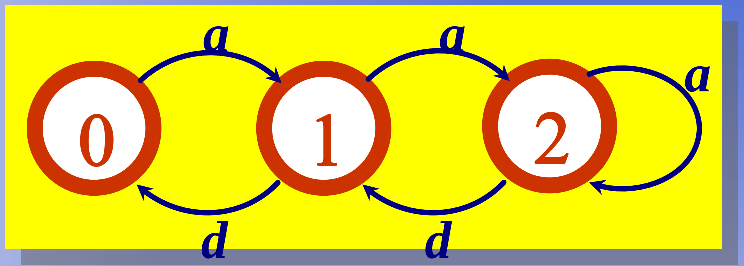
CONCURRENT ESTIMATION APPROACH:

- Choose any K
- Simulate (or observe *actual system*) under K
- Apply Concurrent Simulation to *LEARN* effect of all other feasible K

CONSTRUCTABILITY: AN EXAMPLE *CONTINUED*

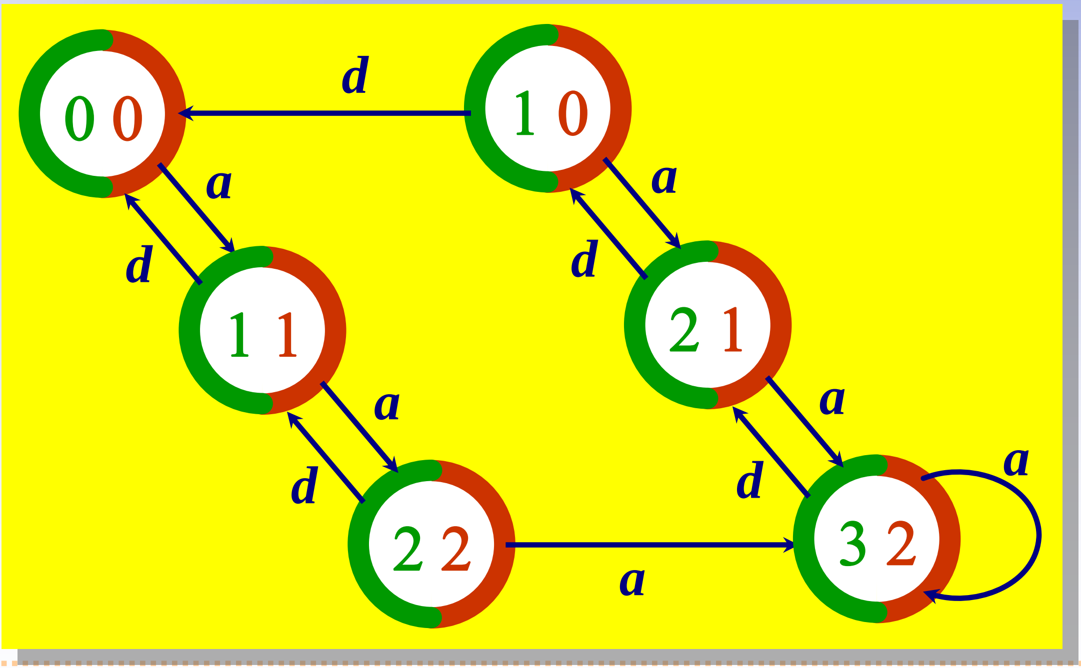


NOMINAL SYSTEM: $K = 3$

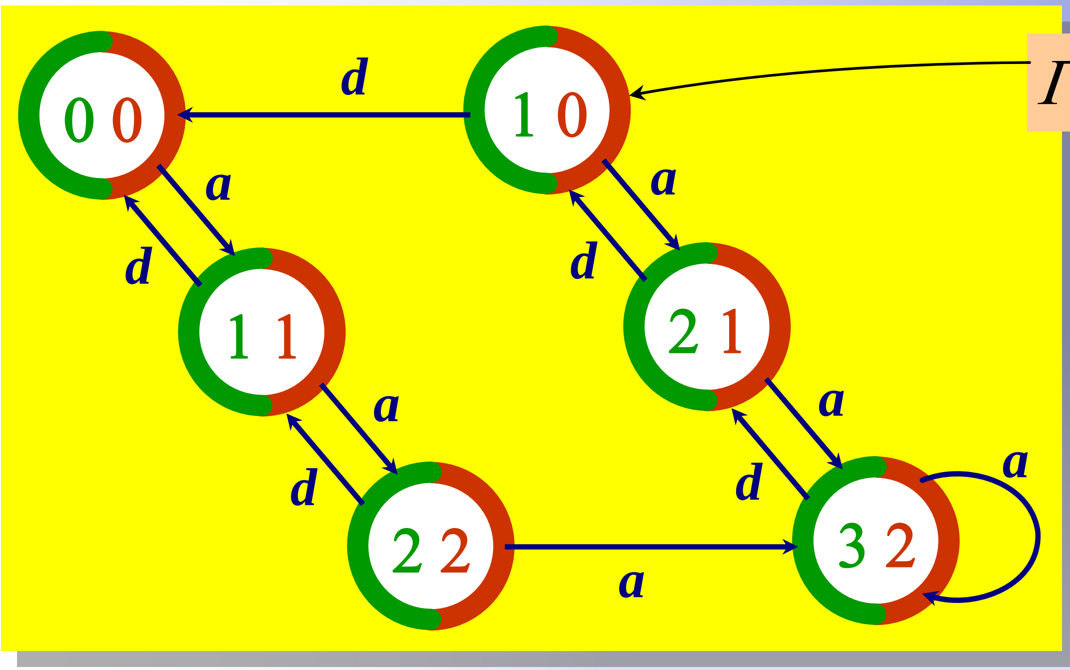


PERTURBED SYSTEM: $K = 2$

AUGMENTED SYSTEM:
Check for OBSERVABILITY



CONSTRUCTABILITY: AN EXAMPLE *CONTINUED*



$$\Gamma(0) = \{a\} \subset \Gamma(1) = \{a, d\}$$

OBSERVABILITY satisfied!

However, if roles of NOMINAL and PERTURBED are reversed, then OBSERVABILITY is ***not*** satisfied...

SOLVING CONSTRUCTABILITY

- Constructability is ***not*** easily satisfied, in general
- However, the CONSTRUCTABILITY PROBLEM can be solved a some cost

SOLUTION METHODOLOGIES:

- STANDARD CLOCK (SC) methodology for Markovian systems

Ref: *Vakili, 1991*

- AUGMENTED SYSTEM ANALYSIS (ASA) for systems with at most one non-Markovian event

Ref: *Cassandras and Strickland, 1989*

- TIME WARPING ALGORITHM (TWA) for arbitrary systems

Ref: *Cassandras and Panayiotou, 1996*

AUGMENTED SYSTEM ANALYSIS (ASA)

- Assume Markovian DES $\Rightarrow$ only OBSERVABILITY required
- Observe sample path of Σ_1
- As state sequence unfolds, check for OBSERVABILITY for every constructed $n = 2,\ldots,m$
- If OBSERVABILITY violated for some n, ***suspend*** nth sample path construction
- Wait until a state of Σ_1 is entered s.t. OBSERVABILITY satisfied for ***suspended*** nth sample path and resume construction

EVENT MATCHING ALGORITHM

INITIALIZE: $A = \{2,\ldots,m\}$ = ACTIVE sample paths

1. Observe every event e in sample path of Σ_1

2. Update state: $x'_n = f(x_n, e)$

3. For each $n = 2,\ldots,m$, check if [is OBSERVABILITY satisfied?]

3.1. If $\Gamma(x'_1) \supseteq \Gamma(x'_n)$, add n to A (if $n \in A$, leave n in A)

3.2. Else, remove n from A and leave x'_n unchanged

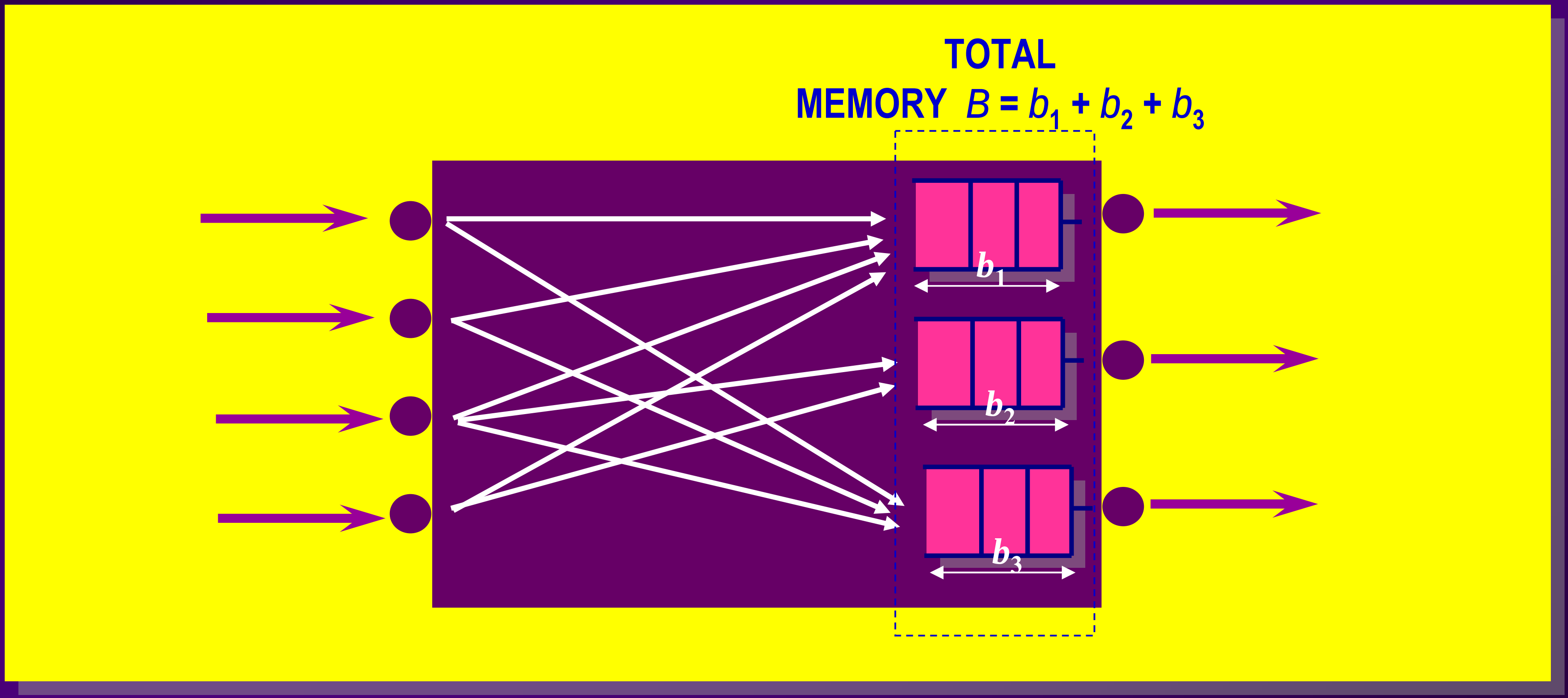
3. Return to **Step 1** for next event in sample path of Σ_1

- *Price to pay in ASA*: long suspensions possible in **Step 3.1**

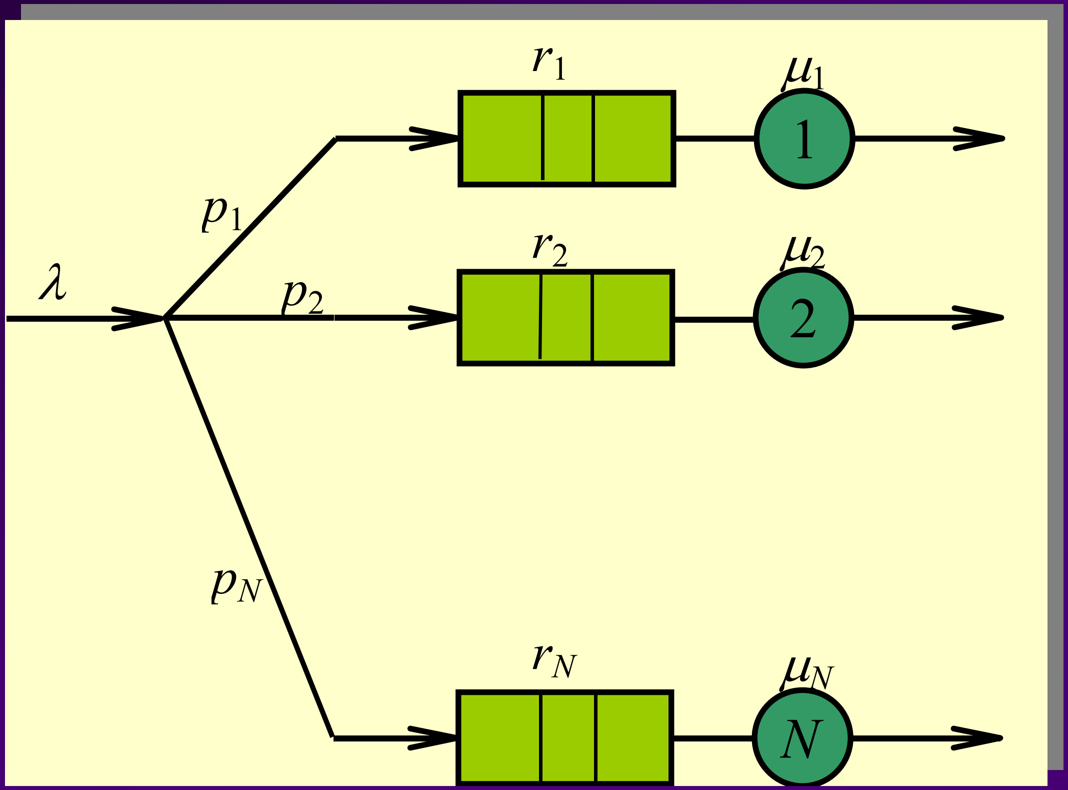
SOME APPLICATIONS

BUFFER ALLOCATION IN A SWITCH

NETWORK SWITCH MEMORY ALLOCATION OVER OUTPUT PORTS



BUFFER ALLOCATION IN A SWITCH

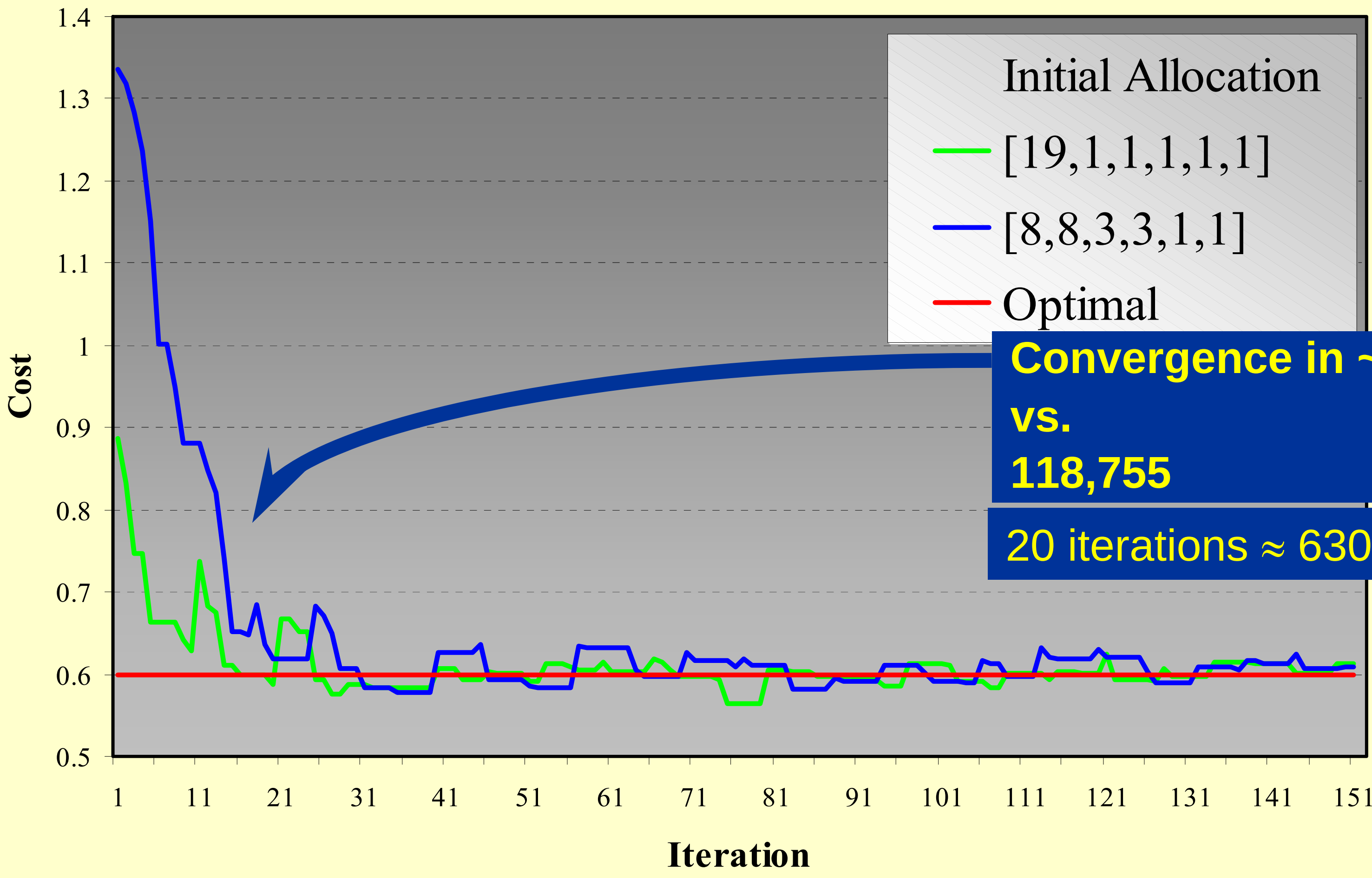


Allocate K buffer slots (RESOURCES) over N servers (USERS) so as to minimize *BLOCKING PROBABILITY*

subject to $\sum_{i=1}^{N} r_i = K$

For N=6, K=24 → Possible allocations = **118,755**

CONCURRENT ESTIMATION + OPTIMIZATION

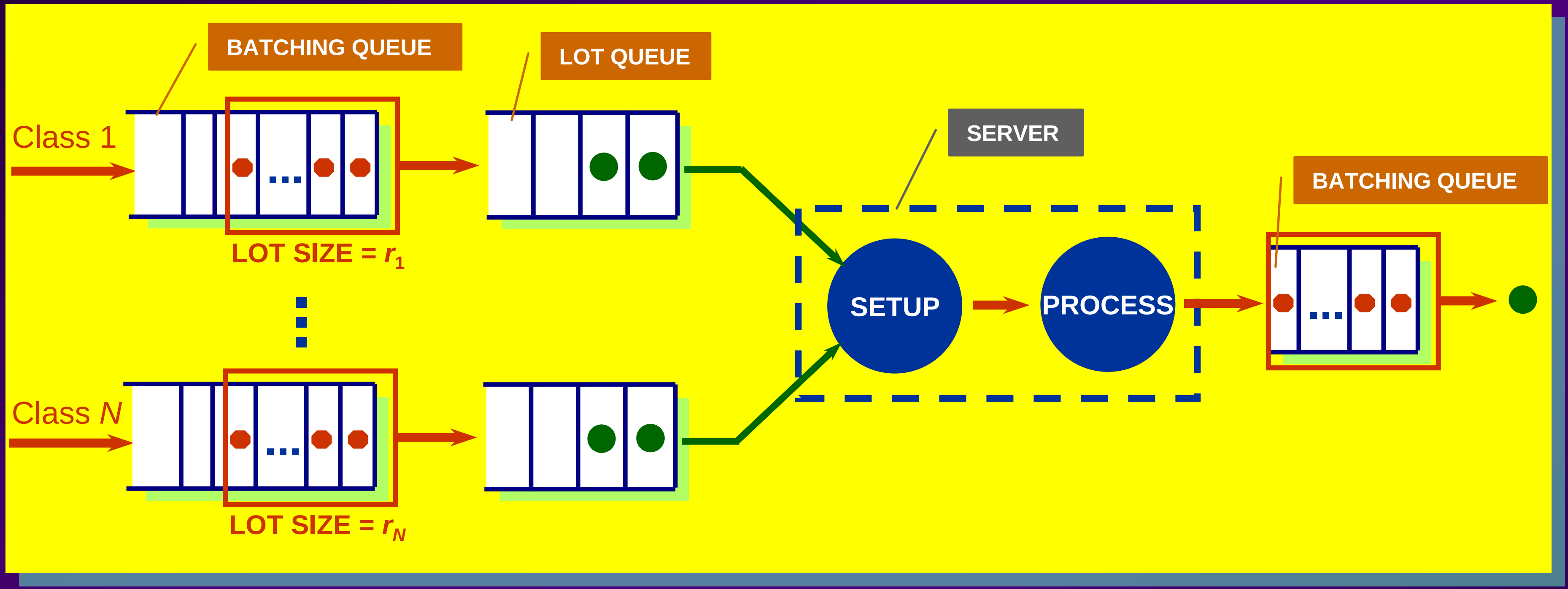


Convergence in ~ 20 iterations vs. 118,755

20 iterations ≈ 630,000 events

- $\lambda = 5.0$, $\mu_i = 1.0$, $p_i = 1/6$ for all $i = 1,\dots,6$
- Optimal = [4,4,4,4,4,4]

LOT SIZING IN MANUFACTURING

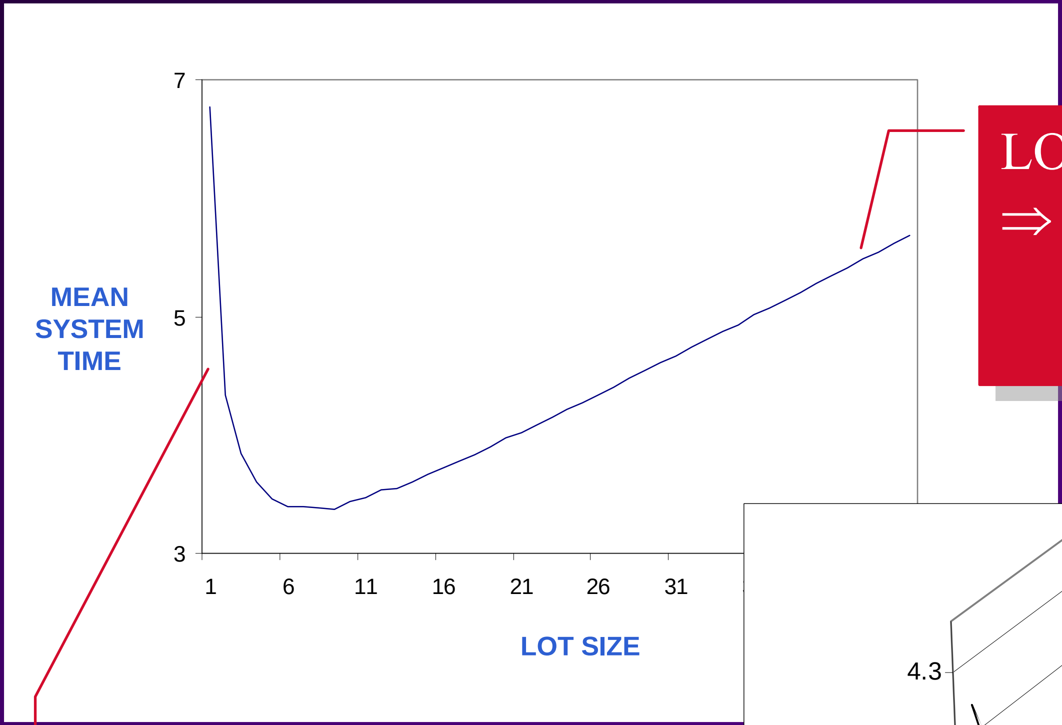


LOT SIZING: Determine Lot Sizes to minimize Average Lead Time

COMPLEXITY: For 3 product classes and lot sizes in [1,100] :

10^6 possible allocations

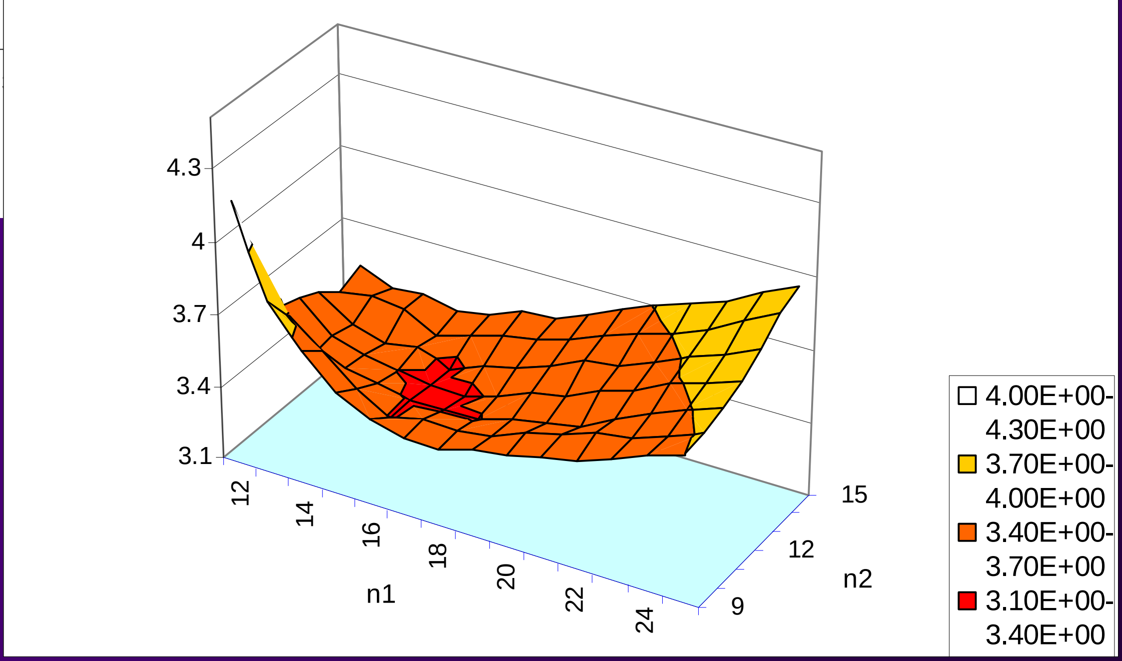
LOT SIZING IN MANUFACTURING



LOT SIZE r_i too ***large***

$\Rightarrow$ All other classes delayed

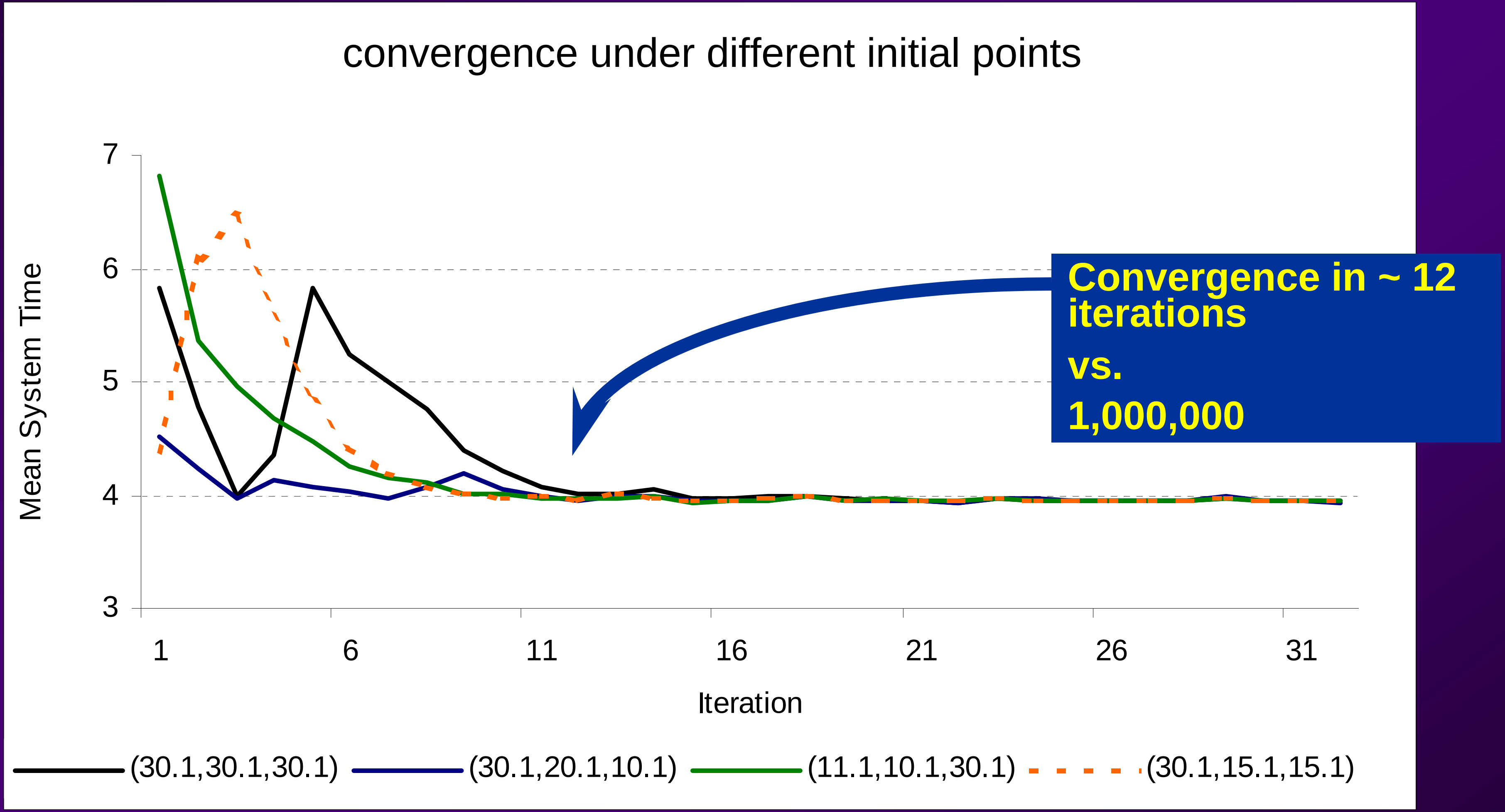
+ class i parts delayed waiting for lot at output



LOT SIZE r_i too ***small***

$\Rightarrow$ Too many setups, instability possible

CONCURRENT SIMULATION + OPTIMIZATION



ABSTRACTION (AGGREGATION) OF DES

THREE FUNDAMENTAL *COMPLEXITY LIMITS*

$1/T^{1/2}$ LIMIT

NP-HARD LIMIT

O.

CE

C

(e.

Tradeoff between
GENERALITY and **EFFICIENCY**
of an algorithm

[*Wolpert and Macready, IEEE TEC, 1997*]

NO-FREE-LUNCH LIMIT

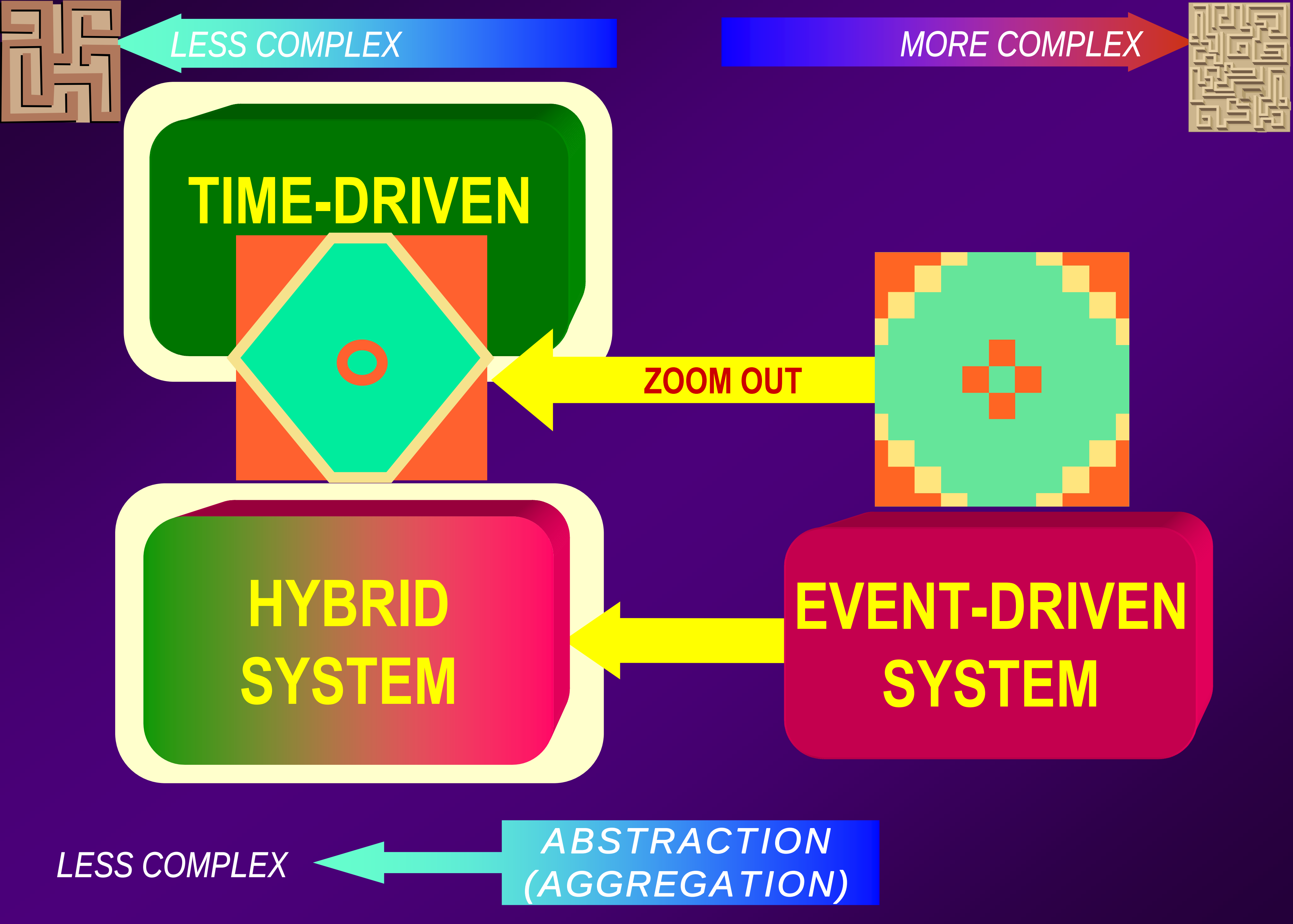
THREE FUNDAMENTAL *COMPLEXITY LIMITS*

$1/T^{1/2}$ LIMIT

NP-HARD LIMIT

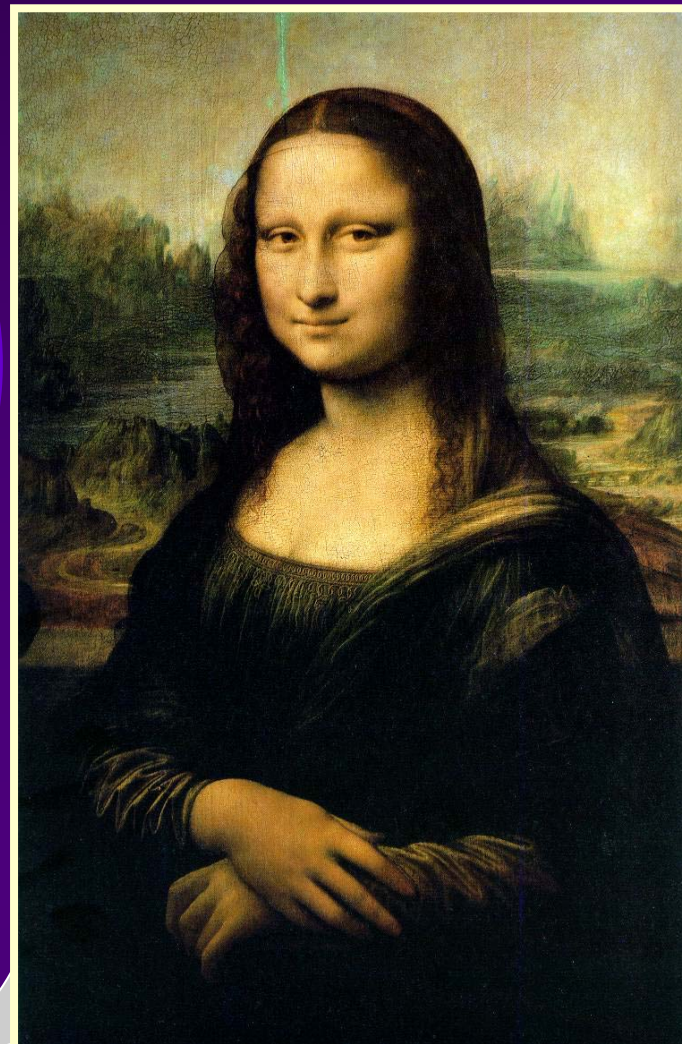
Effect is MULTIPLICATIVE!

NO-FREE-LUNCH LIMIT

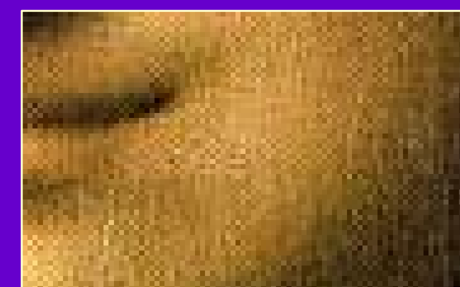

LESS COMPLEX
MORE COMPLEX
TIME-DRIVEN
ZOOM OUT
HYBRID SYSTEM
EVENT-DRIVEN SYSTEM
LESS COMPLEX
ABSTRACTION (AGGREGATION)

WHAT IS THE RIGHT ABSTRACTION LEVEL ?

TOO FAR…
model not detailed enough



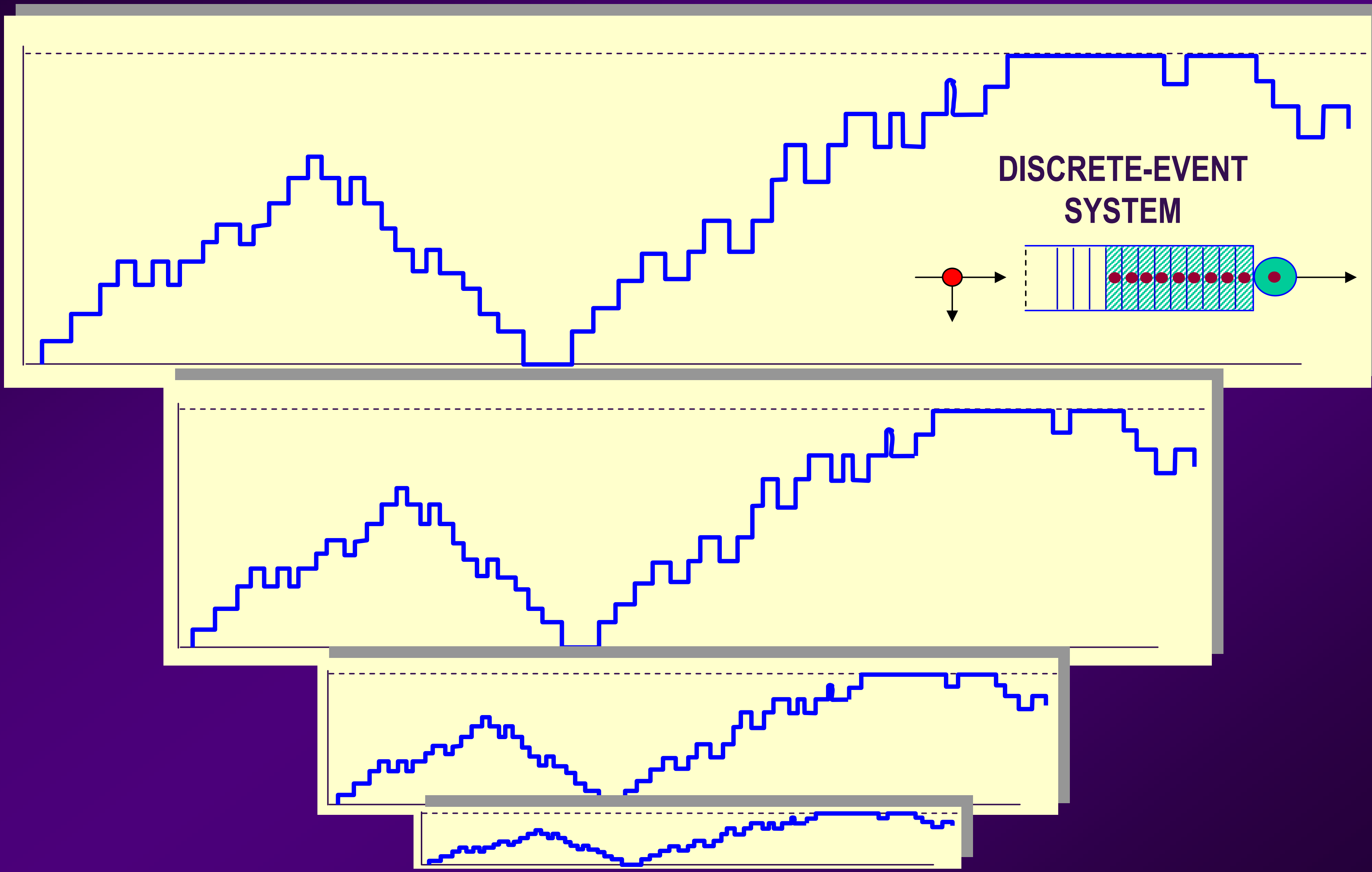
JUST RIGHT…
good model



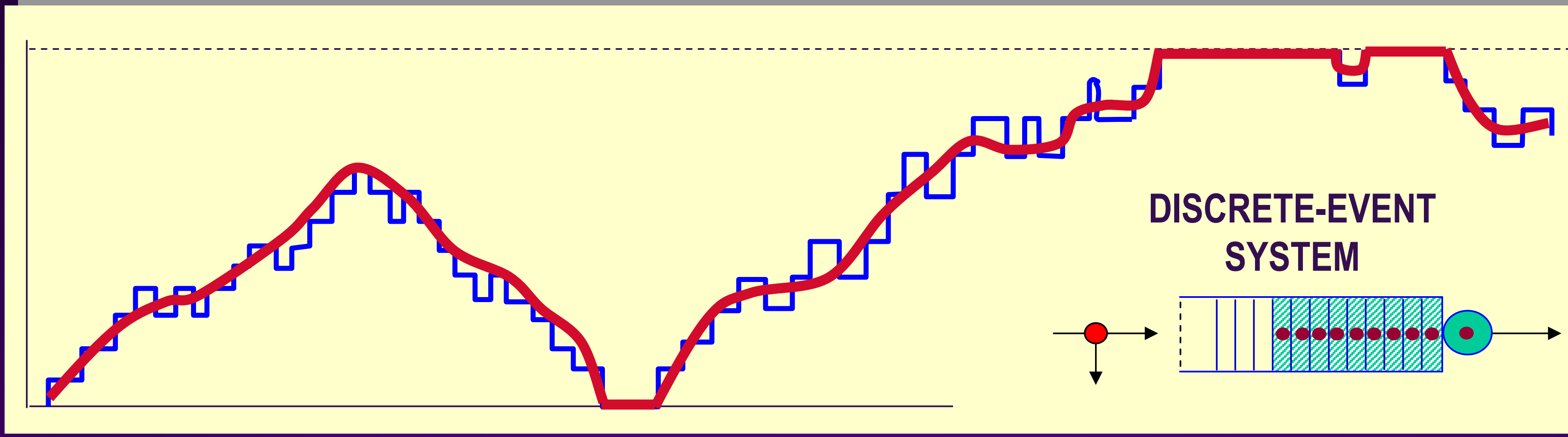
TOO CLOSE…
too much undesirable detail

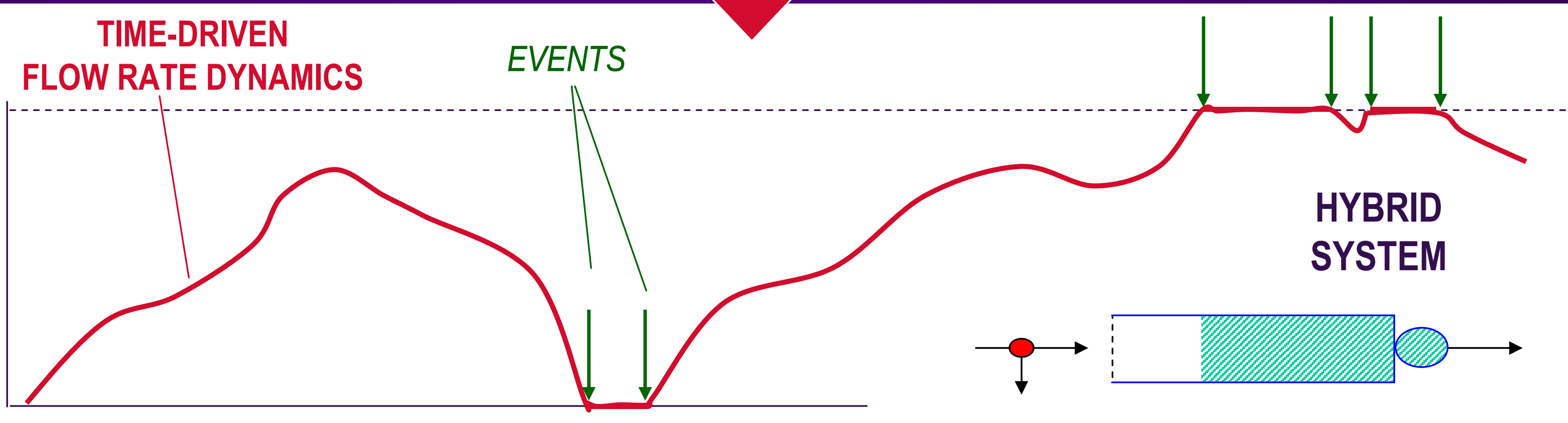
CREDIT: W.B. Gong

ABSTRACTION OF A DISCRETE-EVENT SYSTEM

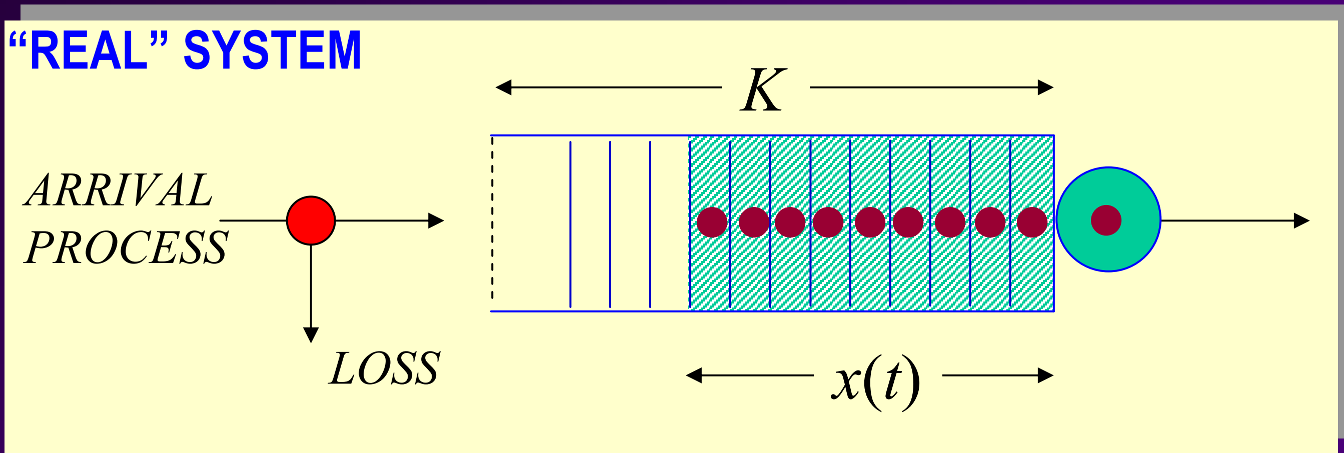


ABSTRACTION OF A DISCRETE-EVENT SYSTEM





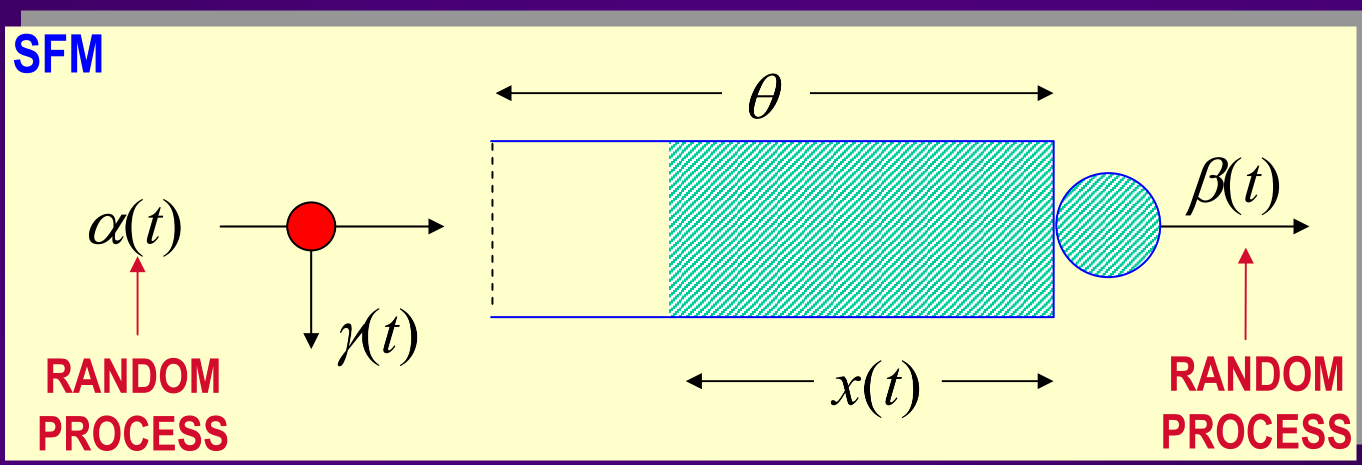
THRESHOLD BASED BUFFER CONTROL



$L(K)$: Loss Rate

$Q(K)$: Mean Queue Lenth

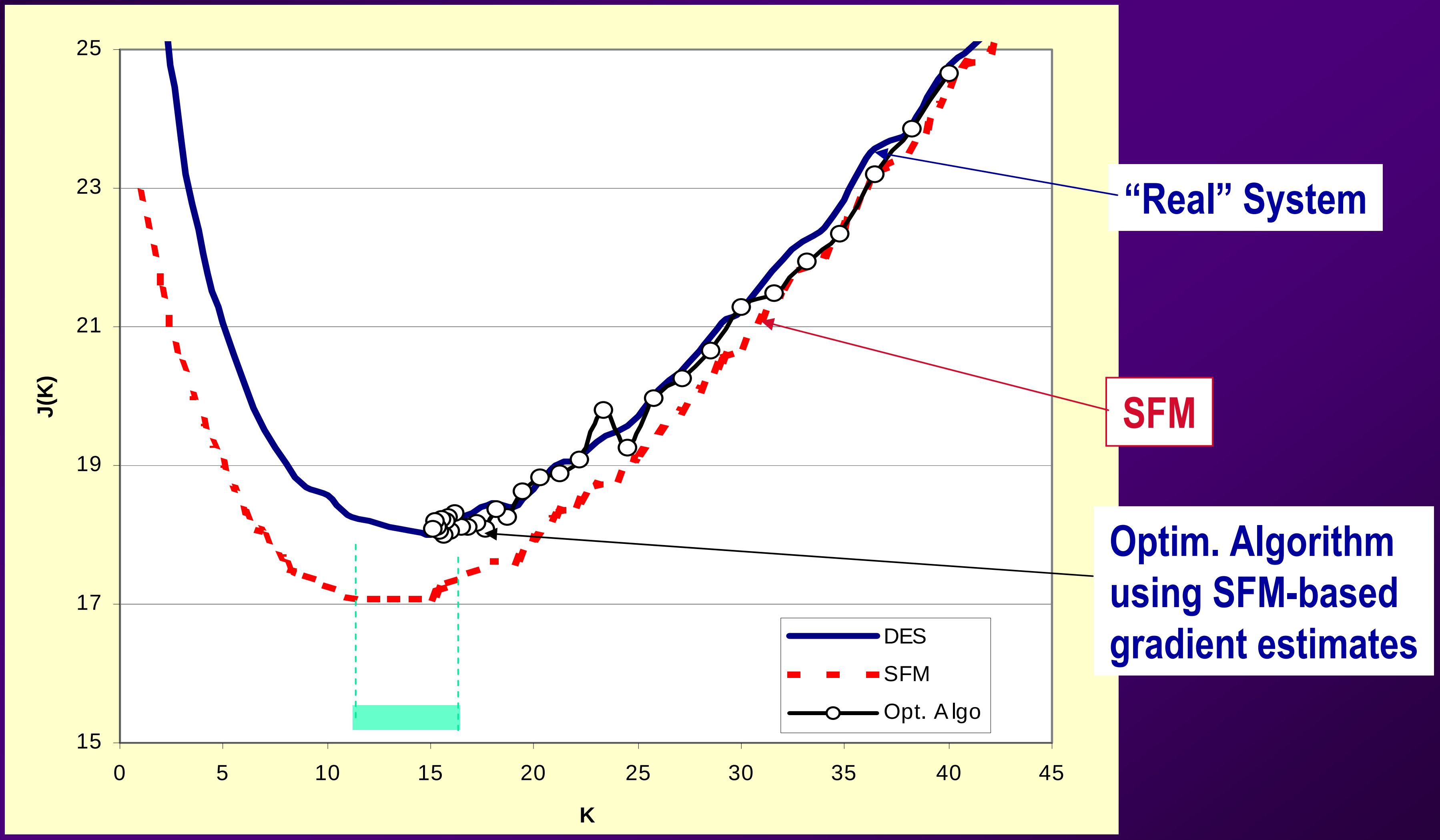
PROBLEM: Determine K to minimize $[Q(K) + R \cdot L(K)]$



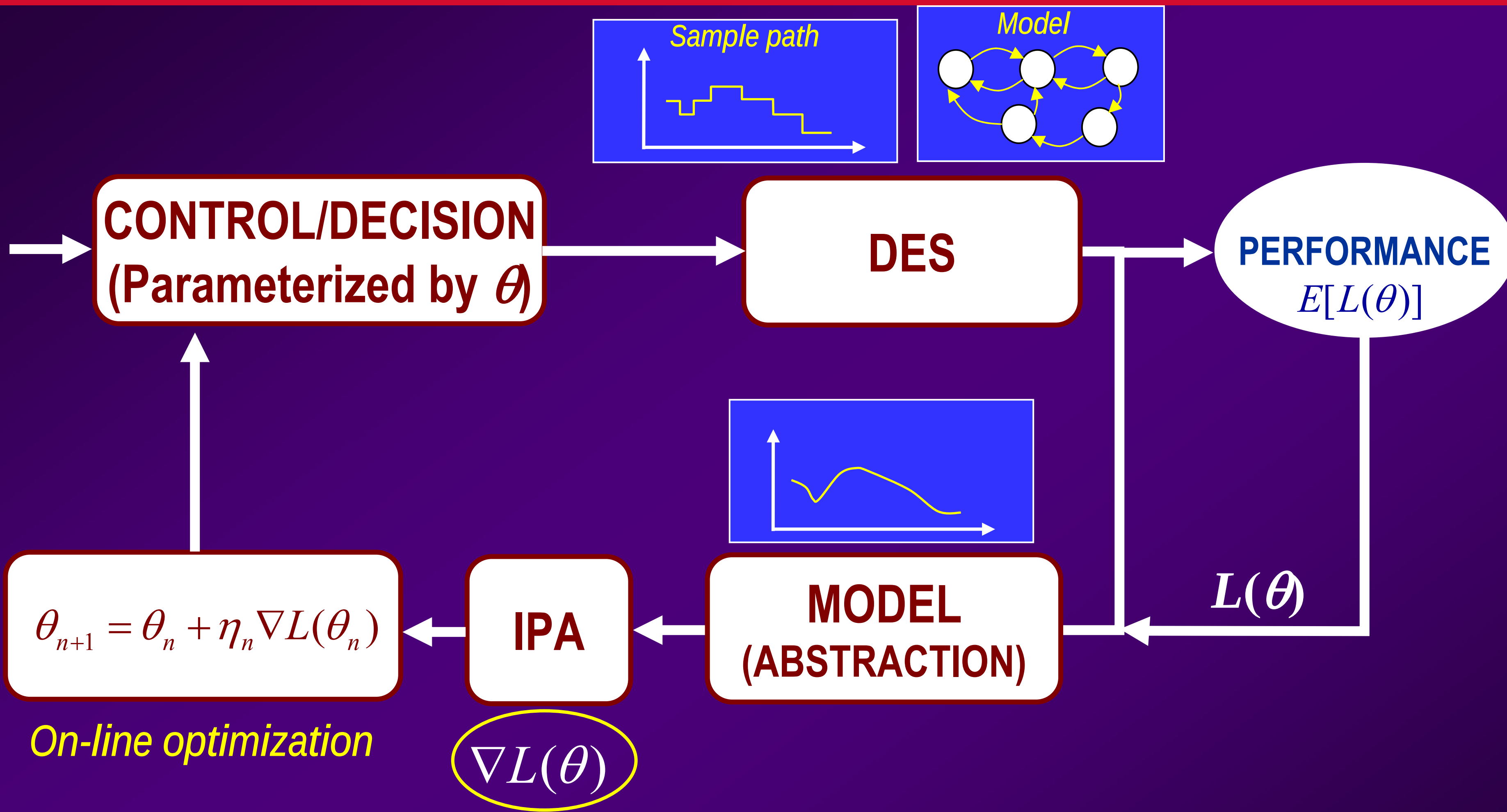
SURROGATE PROBLEM: Determine θ to minimize $[Q^{SFM}(\theta) + R \cdot L^{SFM}(\theta)]$

THRESHOLD BASED BUFFER CONTROL

CONTINUED



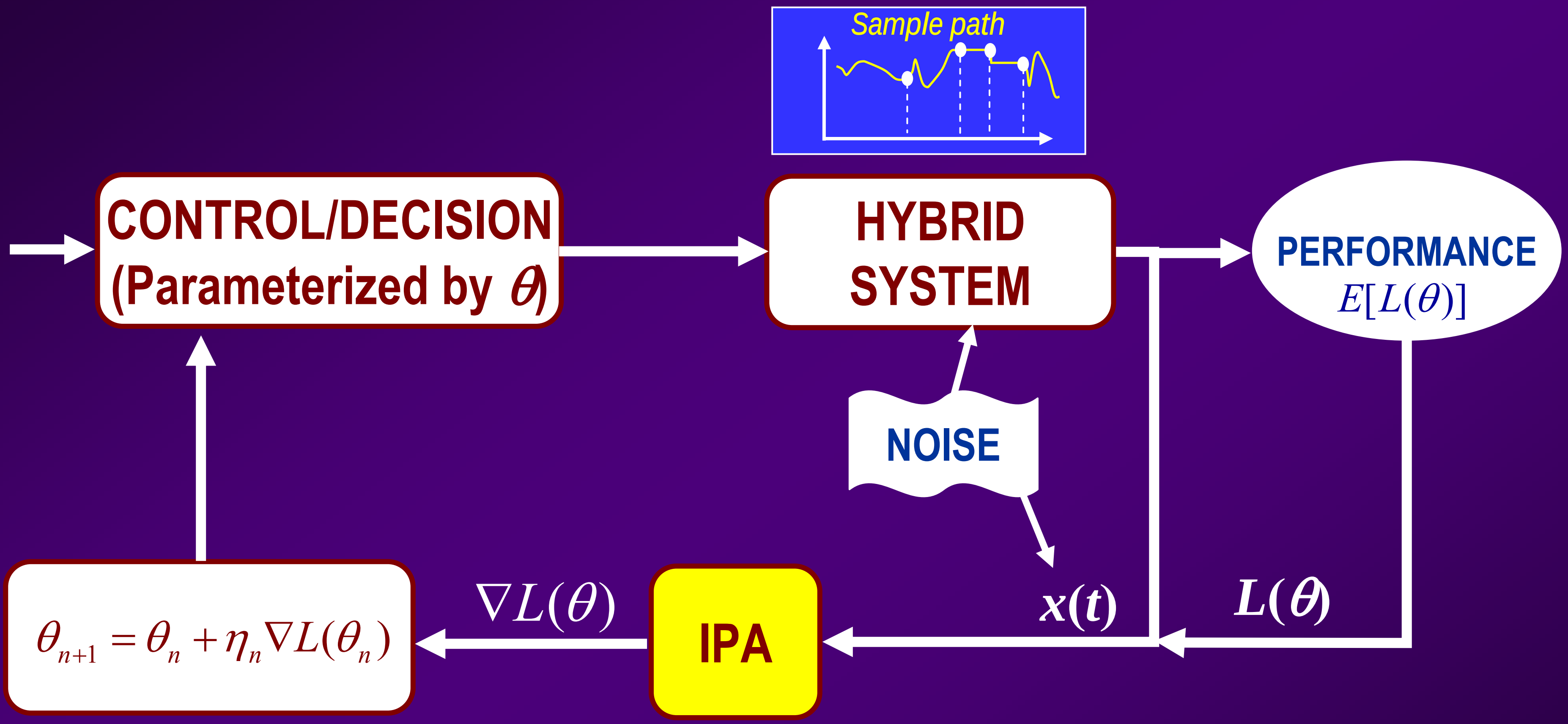
REAL-TIME STOCHASTIC OPTIMIZATION



- **Unbiased estimators**
- **General distributions**
- **Simple on-line implementation**

Not always true in DES !

REAL-TIME STOCHASTIC OPTIMIZATION: *HYBRID SYSTEMS*



A general framework for an IPA theory in Hybrid Systems?

PERFORMANCE OPTIMIZATION AND IPA

Performance metric (objective function):

$$J\left(\theta; x(\theta,0), T\right) = E\left[L\left(\theta; x(\theta,0), T\right)\right]$$

$$\Downarrow$$

$$L\left(\theta\right) = \sum_{k=0}^{N} \int_{\tau_k}^{\tau_{k+1}} L_k(x, \theta, t)\,dt$$

IPA goal:

- Obtain unbiased estimates of $\dfrac{dJ\left(\theta; x(\theta,0), T\right)}{d\theta}$, normally $\dfrac{dL\left(\theta\right)}{d\theta}$

- Then: $\theta_{n+1} = \theta_n + \eta_n \dfrac{dL(\theta_n)}{d\theta}$

NOTATION: $x'\left(t\right) = \dfrac{\partial x\left(\theta, t\right)}{\partial\theta}, \quad \tau'_k = \dfrac{\partial \tau_k\left(\theta\right)}{\partial\theta}$

RECALL: HYBRID AUTOMATA

$$G_h = (Q, X, E, U, f, \phi, Inv, guard, \rho, q_0, \mathbf{x}_0)$$

Q: set of discrete states (modes)

X: set of continuous states (normally R^n)

E: set of events

U: set of admissible controls

f: vector field, $f : Q \times X \times U \to X$

ϕ: discrete state transition function, $\phi : Q \times X \times E \to Q$

Inv: set defining an invariant condition (domain), $Inv \subseteq Q \times X$

$guard$: set defining a guard condition, $guard \subseteq Q \times Q \times X$

ρ: reset function, $\rho : Q \times Q \times X \times E \to X$

q_0: initial discrete state

$\mathbf{x}_0$: initial continuous state

THE IPA CALCULUS

IPA: *THREE FUNDAMENTAL EQUATIONS*

System dynamics over $(\tau_k(\theta),\ \tau_{k+1}(\theta)]$: $\quad \dot{x} = f_k(x,\theta,t)$

NOTATION: $\quad x'(t) = \dfrac{\partial x(\theta,t)}{\partial \theta}, \quad \tau_k' = \dfrac{\partial \tau_k(\theta)}{\partial \theta}$

---

1. **Continuity at events:** $x(\tau_k^+) = x(\tau_k^-)$

Take $d/d\theta$:

$$x'(\tau_k^+) = x'(\tau_k^-) + [f_{k-1}(\tau_k^-) - f_k(\tau_k^+)]\tau'_k$$

If no continuity, use reset condition $\Rightarrow$

$$x'(\tau_k^+) = \frac{d\rho(q,q',x,\upsilon,\delta)}{d\theta}$$

IPA: *THREE FUNDAMENTAL EQUATIONS*

2. Take $d/d\theta$ of system dynamics $\dot{x} = f_k(x,\theta,t)$ over $(\tau_k(\theta),\ \tau_{k+1}(\theta)]$:

$$\frac{dx'(t)}{dt} = \frac{\partial f_k(t)}{\partial x} x'(t) + \frac{\partial f_k(t)}{\partial \theta}$$

Solve $\dfrac{dx'(t)}{dt} = \dfrac{\partial f_k(t)}{\partial x} x'(t) + \dfrac{\partial f_k(t)}{\partial \theta}$ **over** $(\tau_k(\theta),\ \tau_{k+1}(\theta)]$:

$$x'(t) = e^{\int_{\tau_k}^{t} \frac{\partial f_k(u)}{\partial x} du} \left[\int_{\tau_k}^{t} \frac{\partial f_k(v)}{\partial \theta} e^{-\int_{\tau_k}^{v} \frac{\partial f_k(u)}{\partial x} du} dv + x'(\tau_k^+) \right]$$

initial condition from 1 above

NOTE: If there are no events (pure time-driven system), IPA reduces to this equation

IPA: *THREE FUNDAMENTAL EQUATIONS*

3. Get τ_k' depending on the event type:

- **Exogenous** event: By definition, $\boxed{\tau_k' = 0}$

- **Endogenous** event: occurs when $g_k(x(\theta,\tau_k),\theta) = 0$

$$\boxed{\tau_k' = -\left[\frac{\partial g}{\partial x} f_k(\tau_k^-)\right]^{-1}\left(\frac{\partial g}{\partial \theta} + \frac{\partial g}{\partial x} x'(\tau_k^-)\right)}$$

- **Induced** events:

$$\boxed{\tau_k' = -\left[\frac{\partial y_k(\tau_k)}{\partial t}\right]^{-1} y_k'(\tau_k^+)}$$

IPA: *THREE FUNDAMENTAL EQUATIONS*

Ignoring resets and induced events:

1. $x'(\tau_k^+) = x'(\tau_k^-) + [f_{k-1}(\tau_k^-) - f_k(\tau_k^+)] \cdot \tau'_k$

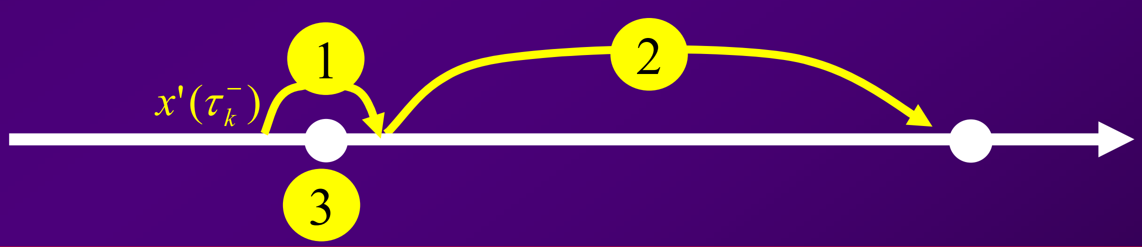
2. $$x'(t) = e^{\int_{\tau_k}^{t} \frac{\partial f_k(u)}{\partial x} du} \left[\int_{\tau_k}^{t} \frac{\partial f_k(v)}{\partial \theta} e^{-\int_{\tau_k}^{v} \frac{\partial f_k(u)}{\partial x} du} dv + x'(\tau_k^+) \right]$$

3. $\tau'_k = 0$ **or** $$\tau'_k = -\left[\frac{\partial g}{\partial x} f_k(\tau_k^-) \right]^{-1} \left(\frac{\partial g}{\partial \theta} + \frac{\partial g}{\partial x} x'(\tau_k^-) \right)$$

Recall:

$$x'(t) = \frac{\partial x(\theta, t)}{\partial \theta}$$

$$\tau'_k = \frac{\partial \tau_k(\theta)}{\partial \theta}$$



Cassandras et al, Europ. J. Control, 2010

IPA PROPERTIES

Back to performance metric: $$L(\theta) = \sum_{k=0}^{N} \int_{\tau_k}^{\tau_{k+1}} L_k(x,\theta,t)\,dt$$

NOTATION: $$L_k'(x,\theta,t) = \frac{\partial L_k(x,\theta,t)}{\partial \theta}$$

Then: $$\frac{dL(\theta)}{d\theta} = \sum_{k=0}^{N} \Bigg[\underbrace{\tau_{k+1}' \cdot L_k(\tau_{k+1}) - \tau_k' \cdot L_k(\tau_k)}_{\text{What happens at event times}} + \underbrace{\int_{\tau_k}^{\tau_{k+1}} L_k'(x,\theta,t)\,dt}_{\text{What happens between event times}} \Bigg]$$

IPA PROPERTIES

THEOREM 1: If either 1,2 holds, then $dL(\theta)/d\theta$ depends only on information available at event times τ_k:

1. $L(x,\theta,t)$ **is independent of** t **over** $[\tau_k(\theta),\ \tau_{k+1}(\theta)]$ **for all** k

2. $L(x,\theta,t)$ **is only a function of** x **and for all** t **over** $[\tau_k(\theta),\ \tau_{k+1}(\theta)]$**:**

$$\frac{d}{dt}\frac{\partial L_k}{\partial x} = \frac{d}{dt}\frac{\partial f_k}{\partial x} = \frac{d}{dt}\frac{\partial f_k}{\partial \theta} = 0$$

[Yao and Cassandras, 2010]

$$\frac{dL(\theta)}{d\theta} = \sum_{k=0}^{N}\left[\tau'_{k+1}\cdot L_k(\tau_{k+1}) - \tau'_k \cdot L_k(\tau_k) + \cancel{\int_{\tau_k}^{\tau_{k+1}} L'_k(x,\theta,t)dt}\right]$$

IMPLICATION: - Performance sensitivities can be obtained from information limited to event times, which is easily observed

- *No need to track system in between events !*

IPA PROPERTIES

EXAMPLE WHERE THEOREM 1 APPLIES (simple tracking problem):

$$\min_{\theta,\phi} E\left[\int_0^T [x(t) - g(\phi)]dt\right] \quad \Rightarrow \frac{\partial L}{\partial x} = 1$$

$$\text{s.t.} \quad \dot{x}_k = a_k x_k(t) + u_k(\theta_k) + w_k(t) \quad \Rightarrow \frac{\partial f_k}{\partial x_k} = a_k, \quad \frac{\partial f_k}{\partial \theta_k} = \frac{du_k}{d\theta_k}$$

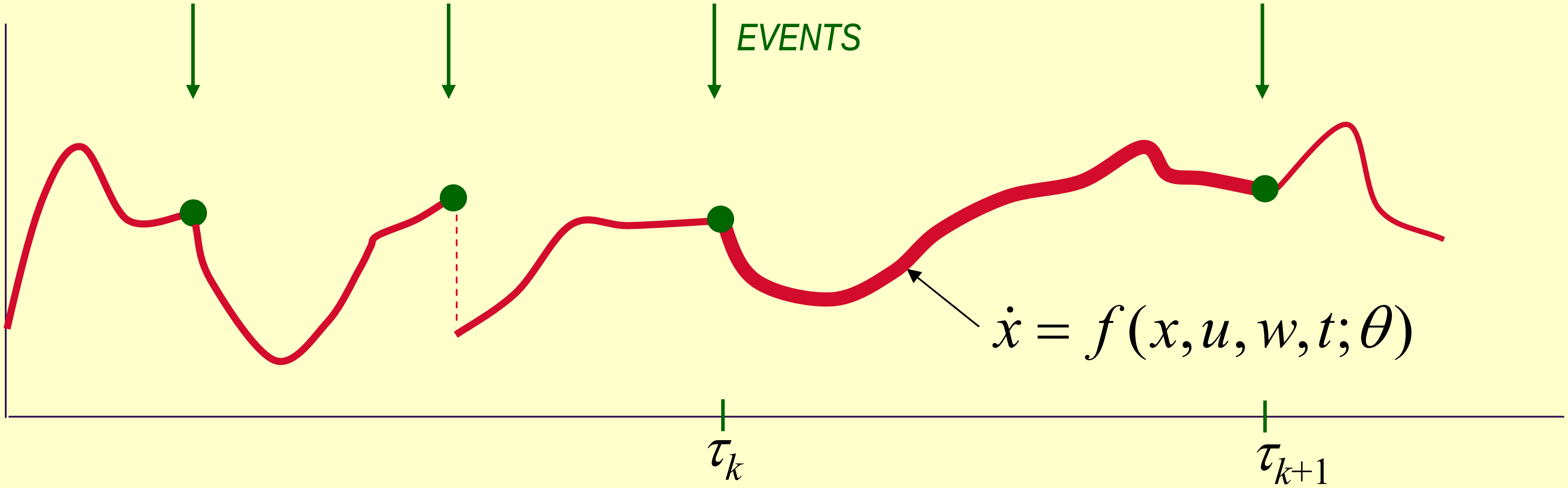
$$k = 1, \ldots, N$$

NOTE: THEOREM 1 provides *sufficient* conditions only.
IPA still depends on info. limited to event times if

$$\dot{x}_k = a_k x_k(t) + u_k(\theta_k, t) + w_k(t)$$
$$k = 1, \ldots, N$$

for "nice" functions $u_k(\theta_k,t)$, e.g., $b_k\theta t$

IPA PROPERTIES



Evaluating $x(t;\theta)$ requires full knowledge of w and f values (obvious)

However, $\dfrac{dx(t;\theta)}{d\theta}$ may be *independent* of w and f values (*NOT* obvious)

It often depends only on:
- event times τ_k
- possibly $f(\tau_{k+1}^-)$

IPA PROPERTIES

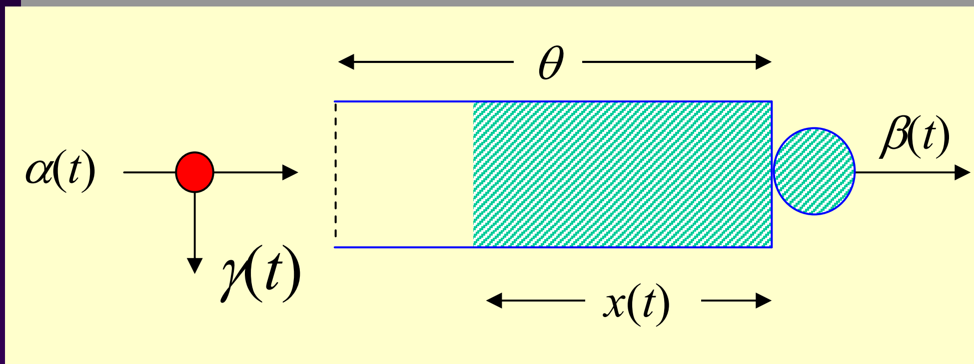
In many cases:

- *No need for a detailed model* (captured by f_k) to describe state behavior in between events

- This explains why *simple abstractions of a complex stochastic system* can be adequate to perform sensitivity analysis and optimization, as long as event times are accurately observed and local system behavior at these event times can also be measured.

- This is true in *abstractions of DES as HS* since:
  Common performance metrics (e.g., workload) satisfy THEOREM 1

THRESHOLD-BASED ADMISSION CONTROL



$$\frac{dx(t)}{dt} = \begin{cases} 0 & x(t)=0, \alpha(t) \le \beta(t) \\ 0 & x(t)=\theta, \alpha(t) \ge \beta(t) \\ \alpha(t)-\beta(t) & \text{otherwise} \end{cases}$$

$$J_T(\theta) = Q_T(\theta) + RL_T(\theta)$$

WORKLOAD

$$Q_T(\theta) = \int_0^T x(t,\theta)dt = \sum_{k\in\Omega} \int_{\tau_k}^{\tau_{k+1}} x(t,\theta)dt$$

$$\Omega = \{k : x_i(t) > 0 \text{ for all } t \in [\tau_k, \tau_{k+1}]\}$$

LOSS

$$L_T(\theta) = \sum_{k\in\Psi} \int_{\tau_k}^{\tau_{k+1}} [\alpha(t) - \beta(t)]dt$$

$$\Psi = \{k : x_i(t) = \theta_i \text{ for all } t \in [\tau_k, \tau_{k+1}]\}$$

Assume: $\{\alpha(t)\}$, $\{\beta(t)\}$ **piecewise continuously differentiable, independent of** θ

IPA FOR LOSS WITH RESPECT TO θ

$$L_T(\theta) = \sum_{k\in\Psi} \int_{\tau_k}^{\tau_{k+1}} \left[\alpha(t) - \beta(t)\right] dt$$

$$\frac{\partial L_T(\theta)}{\partial\theta} = \sum_{k\in\Psi} \left[\alpha(\tau_{k+1}^-) - \beta(\tau_{k+1}^-)\right]\tau'_{k+1} - \sum_{k\in\Psi} \left[\alpha(\tau_k^+) - \beta(\tau_k^+)\right]\tau'_k$$

Need to obtain EVENT TIME derivatives

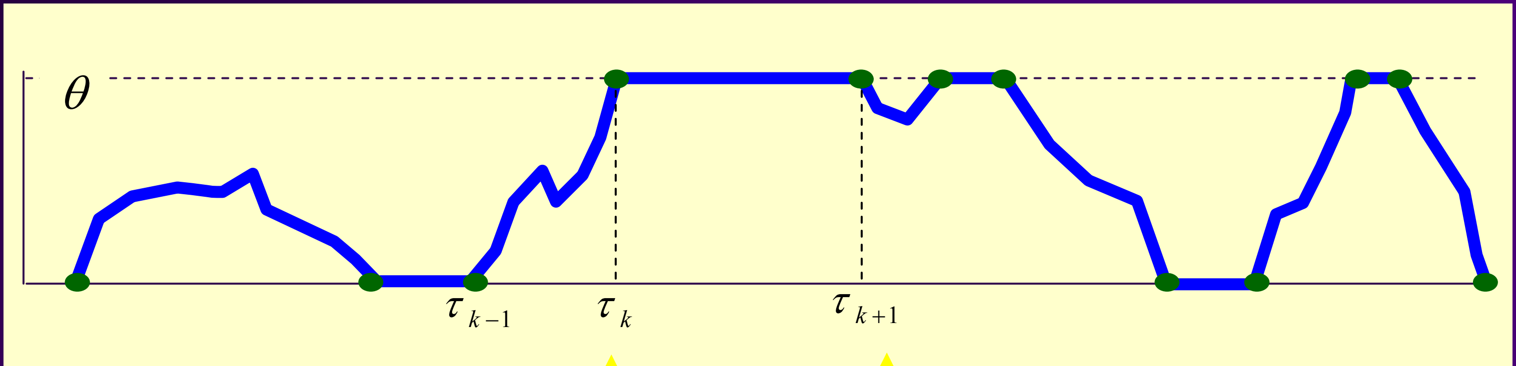
Apply:

1. $x'(\tau_k^+) = x'(\tau_k^-) + [f_{k-1}(\tau_k^-) - f_k(\tau_k^+)] \cdot \tau'_k$

2. $x'(t) = e^{\int_{\tau_k}^{t} \frac{\partial f_k(u)}{\partial x} du} \left[\int_{\tau_k}^{t} \frac{\partial f_k(v)}{\partial\theta} e^{-\int_{\tau_k}^{v} \frac{\partial f_k(u)}{\partial x} du} dv + x(\tau_k^+) \right]$

3. $\tau'_k = 0$ **or** $\tau'_k = -\left[\frac{\partial g}{\partial x} f_k(\tau_k^-)\right]^{-1} \left(\frac{\partial g}{\partial\theta} + \frac{\partial g}{\partial x} x'(\tau_k^-)\right)$

IPA WITH RESPECT TO θ



Endogenous event:

$$\tau_k' = -\left[\frac{\partial g}{\partial x} f_k(\tau_k^-)\right]^{-1}\left(\frac{\partial g}{\partial \theta} + \frac{\partial g}{\partial x} x'(\tau_k^-)\right)$$

Exogenous event:

$$\tau_{k+1}' = 0$$

Here:

$$g_k = x(\tau_k) - \theta \Rightarrow \tau_k' = \frac{1 - x'(\tau_k^-)}{\alpha(\tau_k) - \beta(\tau_k)}$$

$$x'(\tau_k^-) = e^{\int_{\tau_{k-1}}^{\tau_k} 0\,du}\left[\int_{\tau_{k-1}}^{\tau_k} 0 \cdot e^{-\int_{\tau_{k-1}}^{v} 0\,du} dv + 0\right] = 0$$

$$\frac{\partial L_T(\theta)}{\partial \theta} = -\sum_{k \in \Psi}\left[\alpha(\tau_k) - \beta(\tau_k)\right]\tau_k'$$

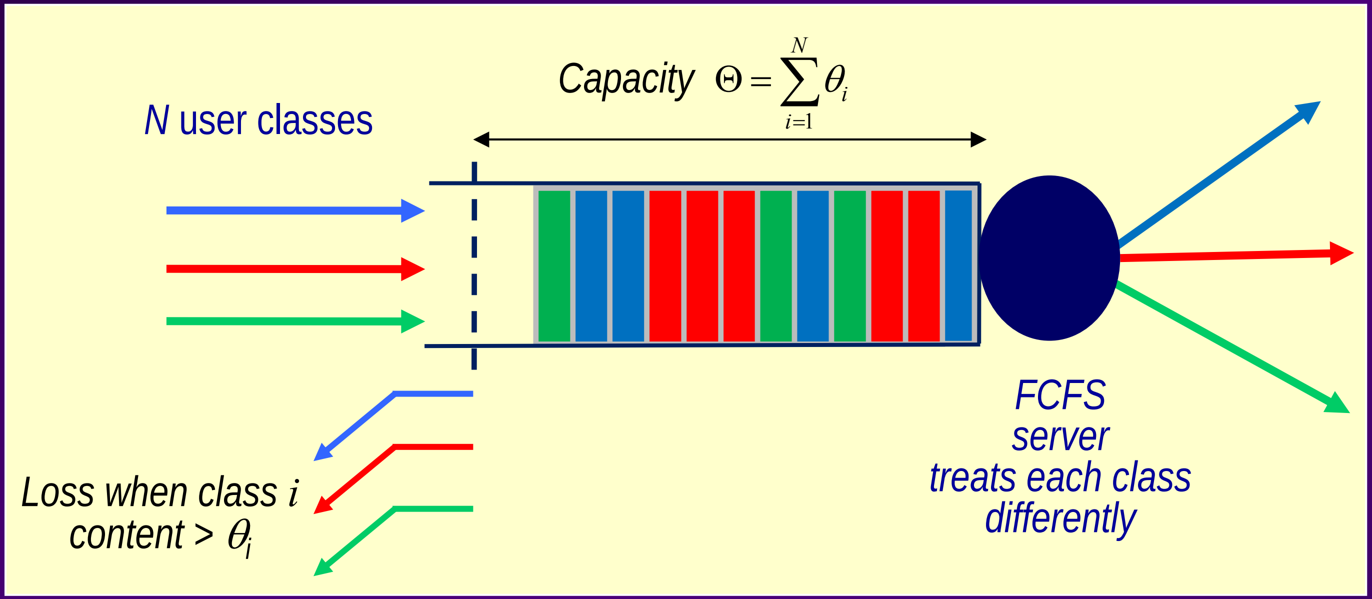
$$\boxed{\frac{\partial L_T(\theta)}{\partial \theta} = -|\Psi|}$$

Just count Overflow intervals

RESOURCE CONTENTION GAMES

MULTIPLE USERS COMPETE FOR RESOURCE

PROBLEM: Determine $(\theta_1,\ldots,\theta_N)$ to optimize system performance



SYSTEM-CENTRIC OPTIMIZATION:

System optimizes $J(\theta_1,\ldots,\theta_N)$ by controlling $(\theta_1,\ldots,\theta_N)$

vs

USER-CENTRIC OPTIMIZATION:

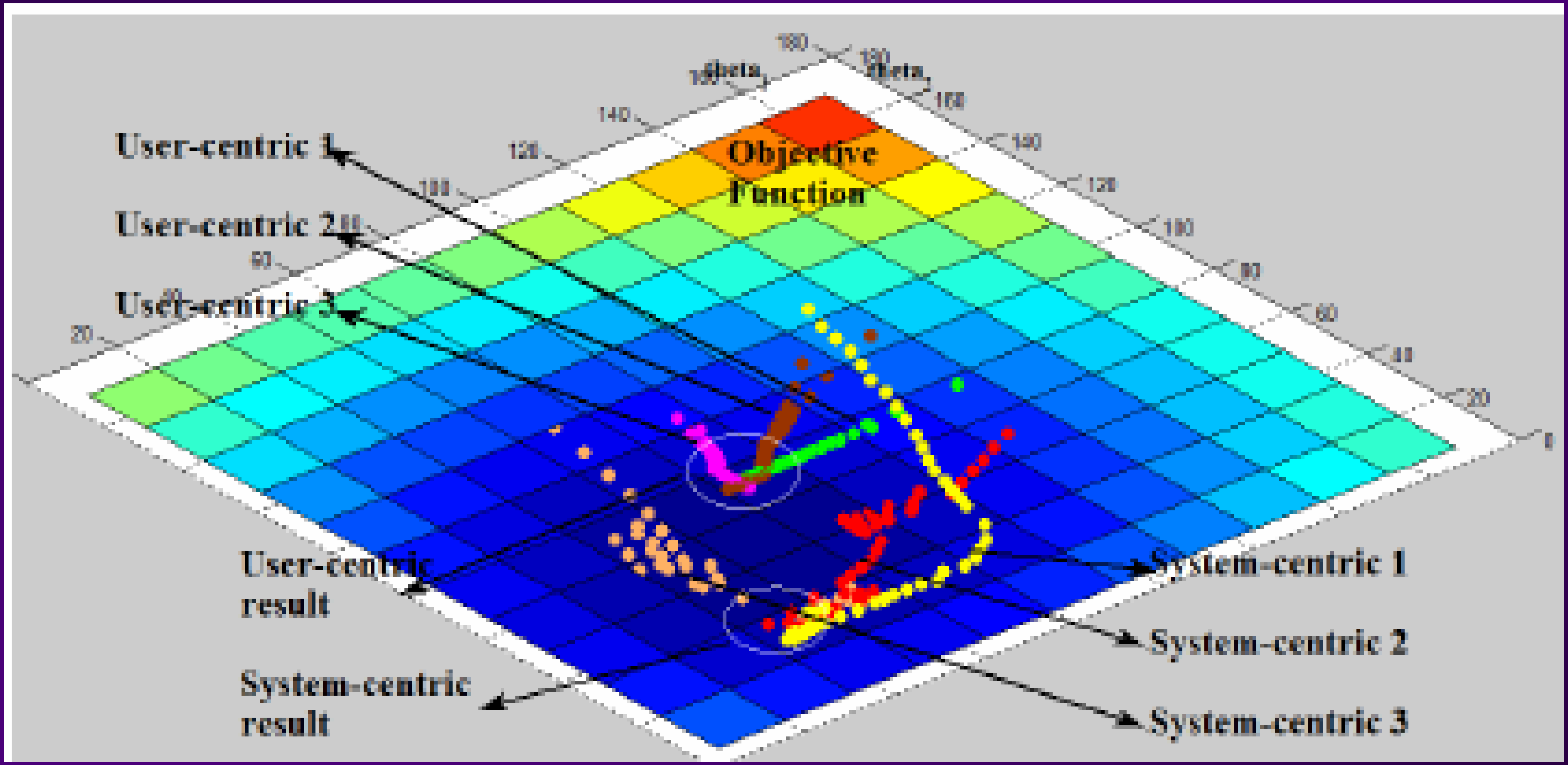
Each user optimizes $J_i(\theta_1,\ldots,\theta_N)$ by controlling *only* θ_i

$\Rightarrow$ ***Resource Contention Game***

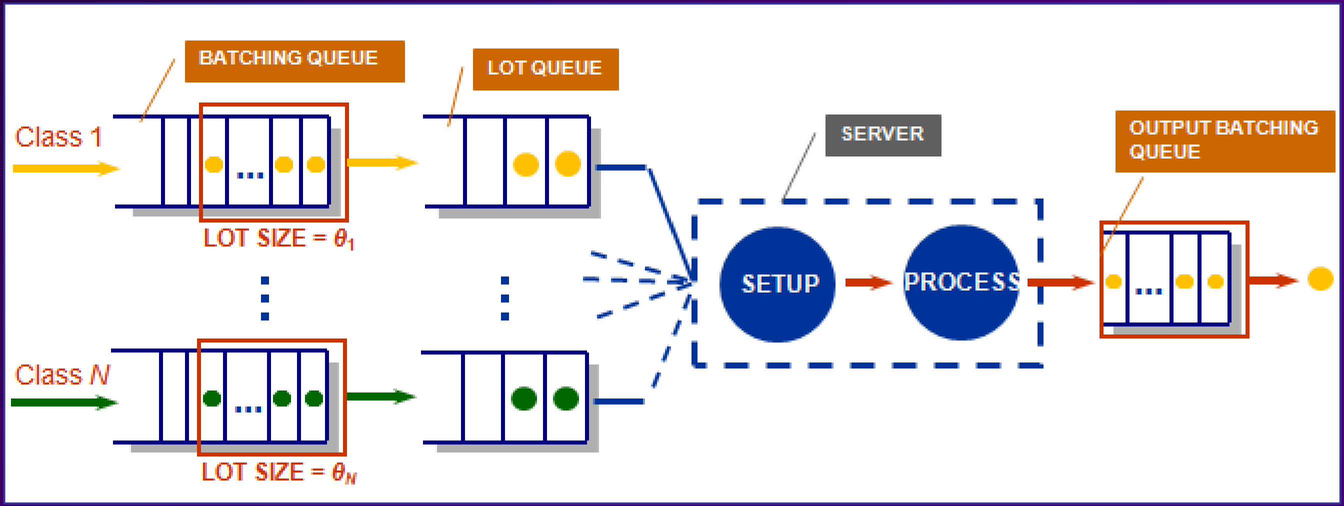
SYSTEM-CENTRIC v GAME SOLUTIONS

Generally, GAME solution is worse than SYSTEM-CENTRIC solution

→ “*the price of anarchy*”



THE OPTIMAL LOT-SIZING PROBLEM

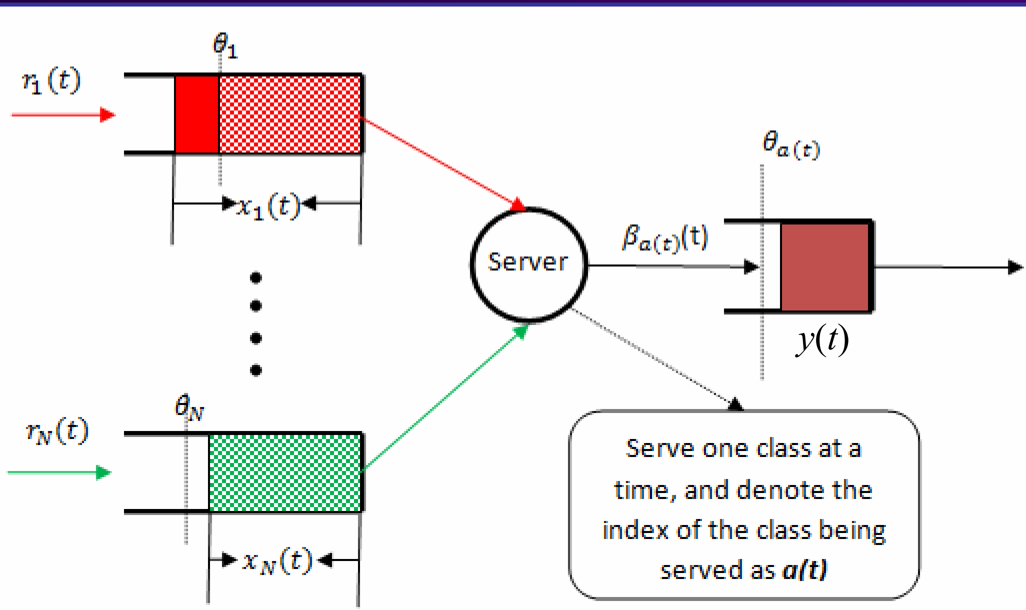


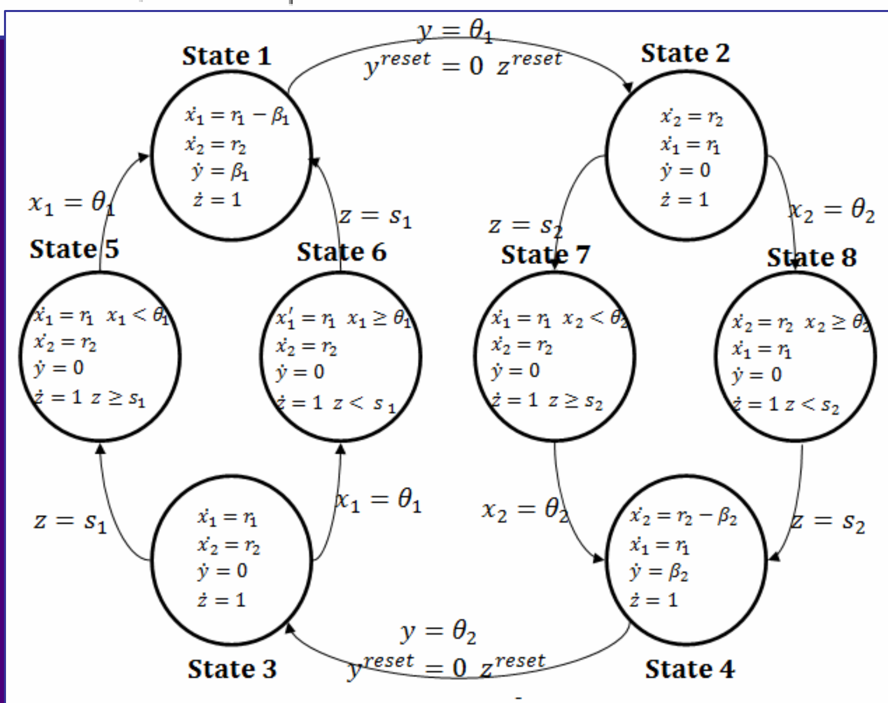
SYSTEM-CENTRIC OPTIMIZATION: Determine N lot size parameters to minimize overall MEAN DELAY

USER-CENTRIC OPTIMIZATION: Determine *ith* lot size parameter to minimize *ith* user MEAN DELAY

K. Baker, P.S. Dixon, M.J. Magazine, and E.A. Silver. *Management Science*, 1978.
P. Afentakis, B. Gavish, and U. Karmarkar. *Management Science*, 1984.
U.S. Karmarkar. *Management Science*, 1987.
J. Maes and L.V. Wassenhove. *Journal of Operations Research*, 1988.
G. Belvaux and L.A. Wolsey. *Management Science*, 2000.
N. Absi and S. Kedad-Sidhoum. *Operations Research*, 2007.

THE OPTIMAL LOT-SIZING PROBLEM





$$\frac{dx_i(t)}{dt} = \begin{cases} r_i(t) - \beta_i(t) & a(t) = i \text{ and } z(t) \geq s_{a(t)} \\ & \text{and } x_i(t) + y(t) \geq \theta_i(t) \\ r_i(t) & \text{otherwise} \end{cases}$$

$$\frac{dy(t)}{dt} = \begin{cases} \beta_{a(t)}(t) & y(t) < \theta_{a(t)} \text{ and } z(t) \geq s_{a(t)} \\ & \text{and } x_{a(t)}(t) + y(t) \geq \theta_{a(t)} \\ r_i(t) & \text{otherwise} \end{cases}$$

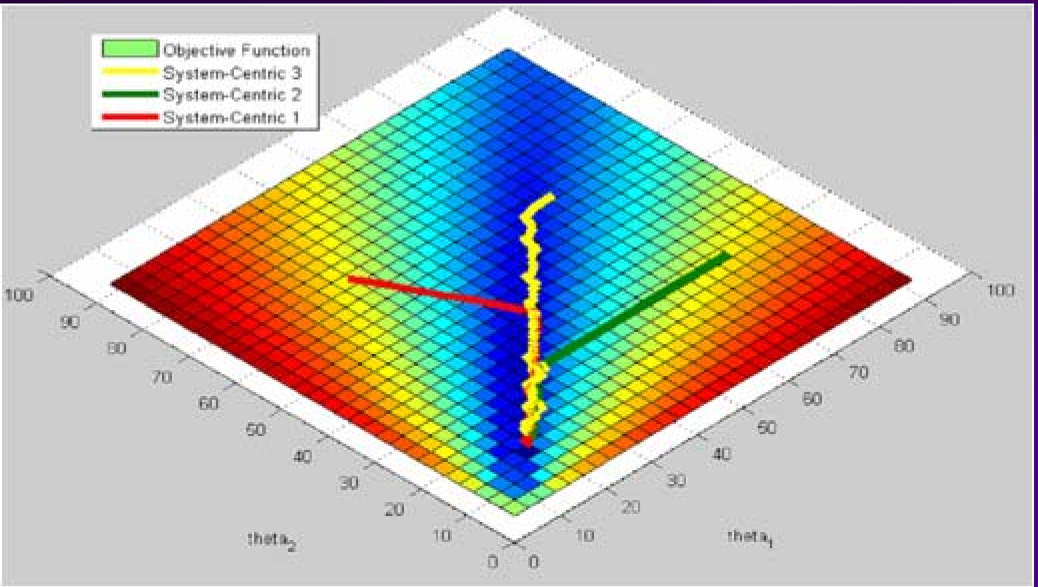
$$y(t) = 0 \text{ if } y(t^-) = \theta_{a(t^-)}$$

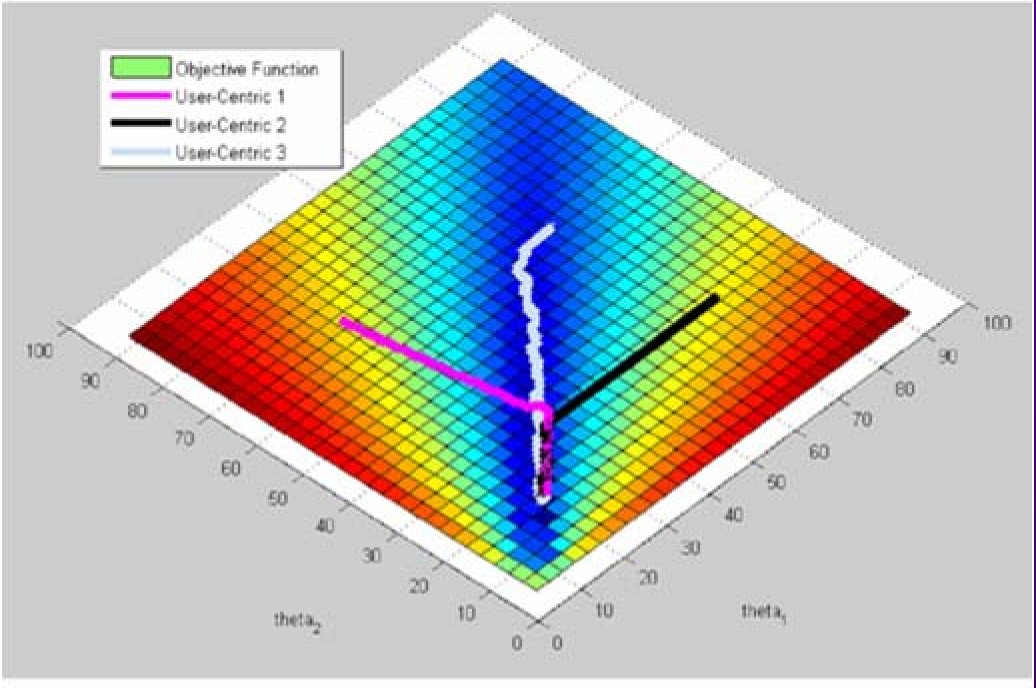
$$\frac{dz(t)}{dt} = 1 \text{ if } y(t) < \theta_{a(t)}$$

$$z(t) = 0 \text{ if } y(t^-) = \theta_{a(t^-)}$$

[*Yao and Cassandras, IEEE CDC, 2010; IEEE TASE, 2012*]

SYSTEM-CENTRIC v GAME SOLUTIONS





SYSTEM-CENTRIC solution…

…coincides with

GAME (USER-CENTRIC) solution!

→ *ZERO "price of anarchy"*

Proof obtained for DETERMINISTIC version

CYBER-PHYSICAL SYSTEMS: *THE NEXT FRONTIER*