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Automating mobility in smart cities☆

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ABSTRACT

Smart Cities are examples of Cyber-Physical Systems whose goals include improvements in transportation, energy distribution, emergency response, and infrastructure maintenance, to name a few. When it comes to mobility, the availability of large amounts of data, ubiquitous wireless connectivity, and the critical need for scalability open the door for new control and optimization methods with the aim of automating all aspects of mobility, from interconnected self-driving vehicles to sharing transportation resources. We address two key questions: can control and optimization methods enable this automation and, if so, how can we quantify its benefits to justify the challenging technological, economic, and social transitions involved? An optimal control framework is presented to show how Connected Automated Vehicles (CAVs) can operate in a dynamic resource contention environment, primarily urban intersections without any traffic lights. We also describe how large amounts of actual traffic data can be harnessed and drive inverse optimization methods to quantify the value of CAVs in terms of eliminating the prevailing Price of Anarchy: the gap between current "selfish" user-centric and optimal "social" system-centric traffic equilibria which are achievable with automated mobility.

1. Introduction

As of 2014, 54% of the earth's population resides in urban areas, a percentage expected to reach 66% by 2050. This increase would amount to 2.5 billion people added to urban populations (World's Population, 2014). At the same time, there are now 28 mega-cities (with 10M people or more) worldwide, accounting for 22% of the world's urban population and projections are for more than 41 mega-cities by 2030. It stands to reason that the management and sustainability of urban areas have become one of the most critical challenges our society faces today. Consequently, cities are looking for ways that ensure a sustainable, comfortable, economically viable future for their citizens by becoming "smart." The emerging prototype for a "Smart City" is one of an urban environment with a new generation of innovative services for transportation, energy distribution, healthcare, environmental monitoring, business, commerce, emergency response, and social activities. The term *Smart City* is broadly used to capture the overall vision outlined above, as well as the intellectual content that supports it. The technological infrastructure of a Smart City is based on a network of sensors and actuators embedded throughout the urban terrain, interacting with wireless mobile devices (e.g., smartphones). The data collected and flowing through such a *Cyber-Physical System* (CPS) may involve traffic conditions, the occupancy of parking spaces, air/water quality information, the structural health of bridges, roads, or buildings, and the location and status of city resources. Enabling such a Smart City setting requires a cyber-physical infrastructure combined with new software platforms and strict requirements for mobility, security, safety, privacy, and the processing of massive amounts of information.

It is worth emphasizing that the ultimate value of a Smart City's infrastructure lies in "closing the loop" that consists of sensing, communicating, decision making, and actuating - rather than simply collecting and sharing data (see Fig. 1). As also discussed in Lamnabhi-Lagarrigue et al. (2017), this requires a balanced understanding of both "physical" and "cyber" components taking into account the important issues of privacy, security, and safety; proper energy management necessitated by the wireless nature of most data collection and actuation mechanisms involved; and the development of new control and optimization methods suitable for this environment.

Among the multitude of functions a Smart City must support, transportation dominates in terms of resource consumption, strain on the environment, and citizen frustration. Based on the 2011 Urban Mobility Report, the cost of commuter delays has risen by 260% over the past 25 years and 28% of U.S. primary energy is now used in transportation Shrank, Lomax, and Eisele (2011). A report

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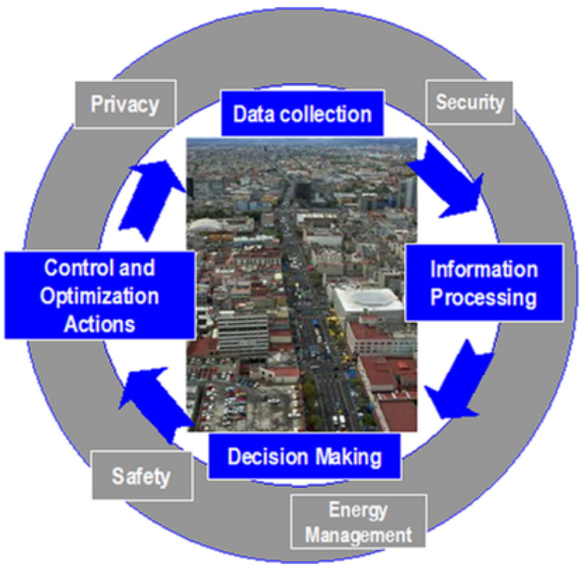

Fig. 1. Cyber-physical infrastructure for a Smart City.

by INRIX (2015) focusing on the U.S., the U.K., France and Germany projects the combined annual cost of gridlock to these countries to reach $293B by 2030 – a 50% increase from 2013. Traffic congestion also leads to an increase in vehicle emissions; in large cities, as much as 90% of CO emissions are due to mobile sources. On the safety side, according to the US National Highway Traffic Safety Administration (NHTSA) in 2012 there were 5,615 million crashes in the U.S. leading to 33,561 deaths

Core disruptive technologies include vehicle connectivity and automation, and the notion of shared personalized transportation infrastructure enabled by mobility-on-demand systems. *Connected Automated* (or *Autonomous*) *Vehicles* (CAVs) provide the most intriguing opportunity for enabling users to better monitor transportation network conditions and make better operating decisions to improve safety and reduce pollution, energy consumption, and travel delays. Intuitively, there are simple arguments for automated vehicles: (*i*) Humans are bad drivers (data indicate that 94% of accidents are due to human error). (*ii*) In contrast to humans, computers can maintain steady cruising speeds which improves fuel efficiency. (*iii*) Computers are also better at processing data whose abundance is now overwhelming humans. (*iv*) Computers are able to make fast and accurate driving adjustments. (*v*) Computers assisting drivers do not get distracted like humans do, they do not blink and they do not sleep. There are of course numerous counter-arguments regarding moral and legal issues, security, privacy, and the question of integrating CAVs with normal vehicles. One could also expect that, while safety would be generally enhanced through the use of CAVs, accidents, though rare, may be very serious. Finally, there are of course many technical challenges which provide the main motivation for the work described in this paper.

In what follows, we address two key questions related to CAVs along with relevant emerging analytical frameworks. First, in Section 2, we consider the question of quantifying the benefits of a transportation system which, at least partially, involves a network of CAVs, so as to justify the challenging technological, economic, and social transitions involved. This quantification is based on the *Price of Anarchy* (PoA) concept often used in game theory to measure the gap between the performance attained by a team consisting of cooperating players ("social" optimality) as opposed to the performance attained by a collection of noncooperating players ("selfish" optimality). We will show that based on actual data from the transportation system in the area around the city of Boston, the PoA can be significant, thus justifying the effort towards the development of CAVs. Next, in Section 3, we present an optimal control framework as a starting point to enable CAV-based automated mobility in urban settings with preliminary experimental results showing substantial improvements in terms of both lower traffic congestion metrics and energy consumption.

2. Estimating the price of anarchy

A transportation network as it functions today is a system with non-cooperative agents (drivers) in which each driver seeks to minimize his/her own cost by choosing the best route to reach a destination without taking into account the overall system performance. In such a non-cooperative setting, one often observes convergence to a Nash equilibrium. However, it is known that the Nash equilibrium is often not the best strategy from the system's point of view and results in a suboptimal behavior compared to the *socially optimal* policy. In today's transportation network, each selfish agent (driver) follows the path derived from a *user optimal* policy. In order to quantify the suboptimality under selfish driving, we use the Price of Anarchy (PoA) mentioned in the previous section as a measure to compare system performance under a user-optimal policy vs. a system-optimal policy.

We begin by reviewing a model introduced in Zhang, Pourazarm, Cassandras, and Paschalidis (2016). We consider a transportation network $(\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ denotes the set of nodes and $\mathcal{A}$ the set of links. The set $\mathcal{W} = \{\mathbf{w}_i : \mathbf{w}_i = (w_{si}, w_{ti}),\ i = 1, \ldots, R\}$ indicates the set of all Origin-Destination (OD) pairs. Assume the graph $(\mathcal{V}, \mathcal{A})$ is strongly connected and let $\mathbf{N} \in \{0, 1, -1\}^{|\mathcal{V}|\times|\mathcal{A}|}$ be its node-link incidence matrix. Denote by $\mathbf{e}_a$ the vector with an entry being 1 corresponding to link a and all the other entries being 0. For any OD pair $\mathbf{w} = (w_s, w_t)$, denote by $d^{\mathbf{w}} \geq 0$ the amount of the flow demand from w_s to w_t. Let $\mathbf{d}^{\mathbf{w}} \in \mathbb{R}^{|\mathcal{V}|}$ be the vector which is all zeros, except for two entries $-d^{\mathbf{w}}$ and $d^{\mathbf{w}}$ corresponding to nodes w_s and w_t respectively.

The basic transportation optimization problem can be formulated as a Variational Inequality (VI) problem (Dafermos & Sparrow, 1969; LeBlanc, Morlok, & Pierskalla, 1975; Patriksson, 2015) seeking to determine $\mathbf{x}^* \in \mathcal{F}$ such that

$$\mathbf{t}(\mathbf{x}^*)'(\mathbf{x} - \mathbf{x}^*) \geq 0, \quad \forall \mathbf{x} \in \mathcal{F}$$

where x_a is the total flow on link $a \in \mathcal{A}$ and $\mathbf{x}$ the vector of these flows. The cost function $\mathbf{t}(\mathbf{x})$ consists of link costs $t_a(\mathbf{x}) : \mathbb{R}_+^{|\mathcal{A}|} \to \mathbb{R}_+$. Given $\epsilon > 0$, an *ϵ-approximate solution* $\hat{\mathbf{x}} \in \mathcal{F}$ (the set of feasible flow vectors) solves the problem

$$\mathbf{t}(\hat{\mathbf{x}})'(\mathbf{x} - \hat{\mathbf{x}}) \geq -\epsilon, \quad \forall \mathbf{x} \in \mathcal{F}.$$

A fundamental difficulty involved with solving this problem is that the cost functions $t_a(\mathbf{x})$ are unknown since they depend on individual driver behavior, i.e., how different drivers make routing decisions which, in turn, affect the link flows. On the other hand, the extensive availability of traffic data provides an opportunity to use these data as the input to an "inverse optimization" problem formulation through which one can obtain cost functions which best fit the data.

2.1. Inverse optimization

Let us consider flow data $(\mathbf{x}_k, \mathcal{F}_k)$, $k = 1, \ldots, K$, $\mathbf{x}_k \in \mathcal{F}_k$. Then, the *inverse* VI problem amounts to seeking a function $\mathbf{t}$ such that $\boldsymbol{\epsilon}_k$ is an ϵ_k-approximate solution to the optimization problem above for each k:

$$\begin{aligned}&\min_{\mathbf{t},\epsilon} \quad \|\epsilon\| \\ &\text{s.t.} \quad \mathbf{t}(\mathbf{x}_k)'(\mathbf{x}-\mathbf{x}_k) \geq -\epsilon_k, \quad \forall \mathbf{x}\in\mathcal{F}_k,\ \forall k=1,\dots,K.\end{aligned} \tag{1}$$

Thus, we seek to estimate the cost functions used by the selfish drivers leading to a Nash equilibrium based on observed traffic data which provide the given flows $\mathbf{x}_k \in \mathcal{F}_k$. We solve this problem using a nonparametric estimation approach which expresses the cost function in a reproducing kernel Hilbert space $\mathcal{H}$. A detailed technical development is provided in Zhang, Pourazarm, Cassandras, and Paschalidis, so we limit ourselves here to an outline of the overall approach. We assume that the cost functions have the form

$$t_a(x_a) = t_a^0 f\left(\frac{x_a}{m_a}\right), \tag{2}$$

where t_a^0 is the free-flow travel time of $a\in\mathcal{A}$, $f(0)=1$, $f(\cdot)$ is strictly increasing and continuously differentiable on $\mathbb{R}_+$, and m_a is the flow capacity of $a\in\mathcal{A}$. Aiming at recovering such a cost function that has both good data reconciling and generalization properties (i.e., $t_a(x_a)$ should fit "old" data well but should not be overfitting; it must also have the ability to predict "new" data), we reformulate the inverse VI problem (1) as follows:

$$\min_{f,\mathbf{y},\boldsymbol{\epsilon}} \quad \|\boldsymbol{\epsilon}\| + \gamma\|f\|_{\mathcal{H}}^2 \tag{3}$$

$$\text{s.t.} \quad \mathbf{e}_a'\mathbf{N}_k'\mathbf{y}^{\mathbf{w}} \leq t_a^0 f\left(\frac{x_a}{m_a}\right), \tag{4}$$

$$\begin{aligned}&\forall \mathbf{w}\in\mathcal{W}^{(k)},\ a\in\mathcal{A}^{(k)},\ k=1,\dots,K,\\ &\sum_{a\in\mathcal{A}^{(k)}} t_a^0 x_a f\left(\frac{x_a}{m_a}\right) - \sum_{\mathbf{w}\in\mathcal{W}^{(k)}} \left(\mathbf{d}^{\mathbf{w}}\right)'\mathbf{y}^{\mathbf{w}} \leq \epsilon_k,\end{aligned} \tag{5}$$

$$\begin{aligned}&\forall k=1,\dots,K,\\ &f\left(\frac{x_a}{m_a}\right) \leq f\left(\frac{x_{\tilde a}}{m_{\tilde a}}\right),\end{aligned} \tag{6}$$

$$\begin{aligned}&\forall a,\ \tilde a\in\bigcup_{k=1}^K\mathcal{A}^{(k)}\ \text{s.t.}\ \frac{x_a}{m_a}\leq\frac{x_{\tilde a}}{m_{\tilde a}},\\ &\boldsymbol{\epsilon}\geq\mathbf{0},\quad f\in\mathcal{H},\\ &f(0)=1\end{aligned} \tag{7}$$

where $\mathbf{y}, \boldsymbol{\epsilon}$ are decision vectors ($\mathbf{y}$ is a dual variable which can be interpreted as the "price" of $\mathbf{d}^{\mathbf{w}}$). Note also that $\gamma > 0$ is a regularization parameter: a smaller γ should result in recovering a "tighter" $f(\cdot)$ in terms of data reconciling, while a larger γ would lead to a "better" $f(\cdot)$ in terms of generalization properties. Moreover, $\|f\|_{\mathcal{H}}^2$ denotes the squared norm of $f(\cdot)$ in $\mathcal{H}$, (4) is needed for dual feasibility, (5) is the suboptimality (primal-dual gap) constraint, (6) enforces $f(\cdot)$ to be non-decreasing, and (7) is a normalization constraint.

It is clear that the above formulation is too abstract to explicitly solve, since it is an optimization problem over functions f. To make it tractable, we specify $\mathcal{H}$ by selecting its *reproducing kernel* (Evgeniou, Pontil, & Poggio, 2000) to be a polynomial

$$\phi(x,y) = (c+xy)^n = \sum_{i=0}^n \binom{n}{i} c^{n-i} x^i y^i,$$

for some choice of $c \geq 0$ and $n\in\mathbb{N}$ (see Zhang, Pourazarm, Cassandras, & Paschalidis, 2017b). We can now rewrite this problem as

$$\begin{aligned}&\min_{\beta,\mathbf{y},\boldsymbol{\epsilon}} \quad \|\boldsymbol{\epsilon}\| + \gamma\sum_{i=0}^n \frac{\beta_i^2}{\binom{n}{i}c^{n-i}}\\ &\text{s.t.} \quad \mathbf{e}_a'\mathbf{N}_k'\mathbf{y}^{\mathbf{w}} \leq t_a^0\sum_{i=0}^n\beta_i\left(\frac{x_a}{m_a}\right)^i,\\ &\forall\mathbf{w}\in\mathcal{W}^{(k)},\ a\in\mathcal{A}^{(k)},\ k=1,\dots,K,\\ &\sum_{a\in\mathcal{A}^{(k)}} t_a^0 x_a\sum_{i=0}^n\beta_i\left(\frac{x_a}{m_a}\right)^i - \sum_{\mathbf{w}\in\mathcal{W}^{(k)}}\left(\mathbf{d}^{\mathbf{w}}\right)'\mathbf{y}^{\mathbf{w}} \leq \epsilon_k,\\ &\forall k=1,\dots,K,\\ &\sum_{i=0}^n\beta_i\left(\frac{x_a}{m_a}\right)^i \leq \sum_{i=0}^n\beta_i\left(\frac{x_{\tilde a}}{m_{\tilde a}}\right)^i,\\ &\forall a,\ \tilde a\in\bigcup_{k=1}^K\mathcal{A}^{(k)}\ \text{s.t.}\ \frac{x_a}{m_a}\leq\frac{x_{\tilde a}}{m_{\tilde a}},\\ &\boldsymbol{\epsilon}\geq\mathbf{0},\quad \beta_0=1\end{aligned}$$

where the function $f(\cdot)$ in (3) is parameterized by $\beta = (\beta_i;\ i=0,1,\dots,n)$. Assuming an optimal $\beta^* = \left(\beta_i^*;\ i=0,1,\dots,n\right)$ is obtained by solving this problem, our estimator for $f(\cdot)$ is

$$\hat f(x) = \sum_{i=0}^n \beta_i^* x^i = 1 + \sum_{i=1}^n \beta_i^* x^i. \tag{8}$$

The input to this problem are the observed traffic data $\mathbf{x}_k$; these are also used in order to estimate an OD demand matrix $\mathcal{W}$ which is an integral part of the problem. This is a separate equally challenging task which we have addressed in Zhang, Pourazarm, Cassandras, and Paschalidis (2017a) and Zhang et al.

2.2. Forward optimization

Given a transportation network $(\mathcal{V},\mathcal{A})$, with O-D demand matrix $\mathcal{W}$, we define its *total travel latency cost* as

$$L(\mathbf{x}) = \sum_{a\in\mathcal{A}} x_a t_a(x_a).$$

The socially optimal flow vector, denoted by $\mathbf{x}^{\text{social}} = (x_a^{\text{social}};\ a\in\mathcal{A})$, is the solution to the following system-centric*forward problem*, which is a Non-Linear Program (NLP) (Patriksson, 2015; Pourazarm, Cassandras, & Wang, 2016):

$$\min_{\mathbf{x}\in\mathcal{F}} \sum_{a\in\mathcal{A}} x_a t_a(x_a)$$

where the functions $t_a(\cdot)$ are the ones obtained through the solution of the inverse optimization problem, i.e., the values from (2) using (8) to estimate f based on the observed traffic data $\mathbf{x}_k$.

We can now explicitly define the PoA as

$$\text{PoA} = \frac{L(\mathbf{x}^{\text{user}})}{L(\mathbf{x}^{\text{social}})} = \frac{\sum_{a\in\mathcal{A}} x_a^{\text{user}} t_a(x_a^{\text{user}})}{\sum_{a\in\mathcal{A}} x_a^{\text{social}} t_a(x_a^{\text{social}})} \geq 1, \tag{9}$$

where $\mathbf{x}^{\text{user}} = (x_a^{\text{user}};\ a\in\mathcal{A})$ is the Nash equilibrium flow vector (also referred to as *Wardrop equilibrium* in transportation systems) which is directly observable or indirectly inferable. By the definition of $\mathbf{x}^{\text{social}}$, we always have PoA ≥ 1 and, clearly, the larger the PoA, the larger the inefficiency induced by selfish drivers. Thus, PoA quantifies the inefficiency that a societal group has to deal with due to non-cooperative behavior of its members.

2.3. Price of Anarchy experimental results

In our experimental study, $\mathbf{x}^{\text{user}}$ is inferred from actual data from the Eastern Massachusetts (EMA) road network (Transportation Networks for Research, 2017; Zhang et al., 2017b). Details of how this inference is made from the raw data are provided in Zhang et al. In particular, we process data from two

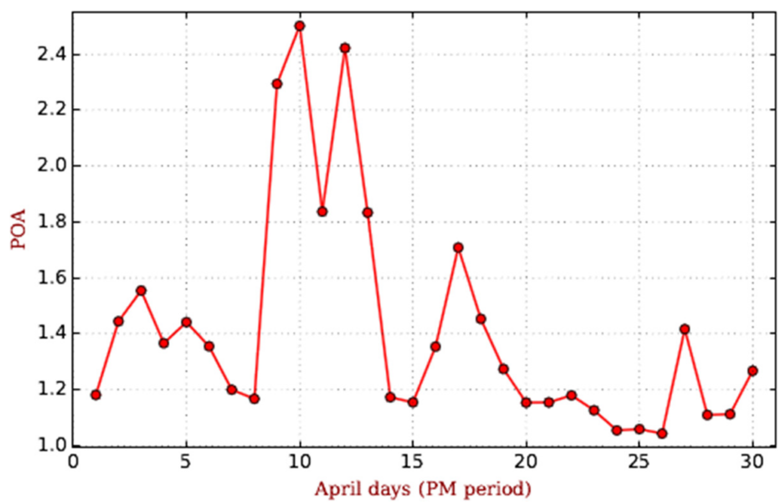

Fig. 2. Price of Anarchy (PoA) for a subset of the Eastern Massachusetts road network in April 2012.

datasets: (*i*) The speed dataset, made available to us by the Boston Region Metropolitan Planning Organization (MPO), includes the spatial average speeds for more than 13,000 road segments (with an average length of 0.7 miles of EMA), providing the average speed for every minute of the year 2012. For each road segment, identified with a unique *tmc* (traffic message channel) code, the dataset provides information such as speed data (instantaneous, average and free-flow speed) in miles per hour, date and time, and traveling time (in minutes) through that segment. (*ii*) The flow capacity (in vehicles per hour) dataset, also provided by the MPO, includes capacity data for more than 100,000 road segments (with an average length of 0.13 miles) in EMA.

The actual values of the PoA may be obtained by network link and time of day. In most cases, we have found the PoA to be significantly higher that 1 and, in many cases, PoA >2. An example is shown in Fig. 2 for a subset of the EMA road network over days (PM period) in April 20,102 (higher PoA values observed certain days can usually be associated with specific circumstances such as bad weather conditions or road construction; in general, they motivate closer scrutiny of the data to provide interpretations).

The implication of results such as those in Fig. 2 is that if drivers could be induced to follow paths in accordance to the social optimal flows $\mathbf{x}^{\text{social}}$, then congestion levels could be decreased by a factor of 2 or more. This is precisely what a CAV-based transportation system can accomplish, since routing decisions would be automatically made and implemented; at the very least, human drivers would be advised to follow a set of recommended routes evaluated to be socially optimal.

This PoA-based analysis results in a quantifiable measure of the benefits of automated urban mobility based on actual data. Obtaining results such as PoA >2 is a clear motivating force for further exploring how CAVs can be deployed in a transportation system.

3. Reducing the price of anarchy through automation

Automating mobility through the use of CAVs has the potential of achieving a social optimum by exploiting the ability to directy control the movement of vehicles and enforcing routing decisions when a CAV is assigned a particular origin-destination pair. Recall, however, that the PoA is based on a congestion metric. Going a step further, we propose an optimal control framework which combines *energy* and *congestion* as performance criteria, while also guaranteeing safety in the form of avoiding lateral and rear-end collisions. The most challenging parts of an urban setting where such a framework needs to operate flawlessly are intersections. In the intersections envisioned by this framework there are no traffic lights and there is a perpetual flow of vehicles, thus eliminating the most time-consuming and energy-intensive aspects of a traffic network, i.e., the need for vehicles to stop and restart.

One of the very early efforts in using some form of autonomous connected vehicles was proposed in Athans (1969) and Levine and Athans (1966) where the merging problem was formulated as a linear optimal regulator to control a single string of vehicles. The key features of an automated intelligent vehicle-highway system (IVHS) were extensively discussed in Varaiya (1993) where a related control system architecture is also proposed. In what follows, we focus on the problem of optimally controlling CAVs crossing an urban intersection without any explicit traffic signaling so as to minimize energy consumption subject to a throughput maximization requirement and to hard safety constraints. A recent study Tachet et al. (2016) has in fact suggested that transitioning from intersections with traffic lights to autonomous ones has the potential of doubling capacity and reducing delays, which is consistent with our own early findings. Several related research efforts have been reported in the literature proposing either *centralized* (if there is at least one task in the system that is globally decided for all vehicles by a single central controller) or *decentralized* approaches for coordinating CAVs at intersections. A centralized reservation scheme is proposed in Dresner and Stone (2004) to control a single intersection of two roads with no turns allowed. Since then, numerous centralized approaches have been reported in the literature (e.g., de La Fortelle, 2010; Dresner & Stone, 2008; Huang, Sadek, & Zhao, 2012), to achieve safe and efficient control of traffic through intersections. Some approaches have focused on coordinating vehicles to improve the travel time (e.g., Yan, Dridi, and El Moudni, 2009; Zhu and Ukkusuri, 2015; Zohdy, Kamalanathsharma, and Rakha, 2012). Others, such as Lee, Park, Malakorn, and So (2013), have considered minimizing the overlap in the position of vehicles inside the intersection rather than arrival time. In Kim and Kumar (2014), an approach is proposed based on Model Predictive Control (MPC) that allows each vehicle to optimize its movement locally with respect to any objective of interest. Queuing theory is used in Miculescu and Karaman (2014) to model the problem as a polling system that determines the sequence of times assigned to the vehicles on each road. In decentralized approaches, each vehicle determines its own control policy based on the information received from other vehicles on the road or from a coordinator. Two conflict resolution schemes are proposed in Alonso et al. (2011) in which an autonomous vehicle can make a decision about the appropriate order of crossing the intersection to avoid collision with

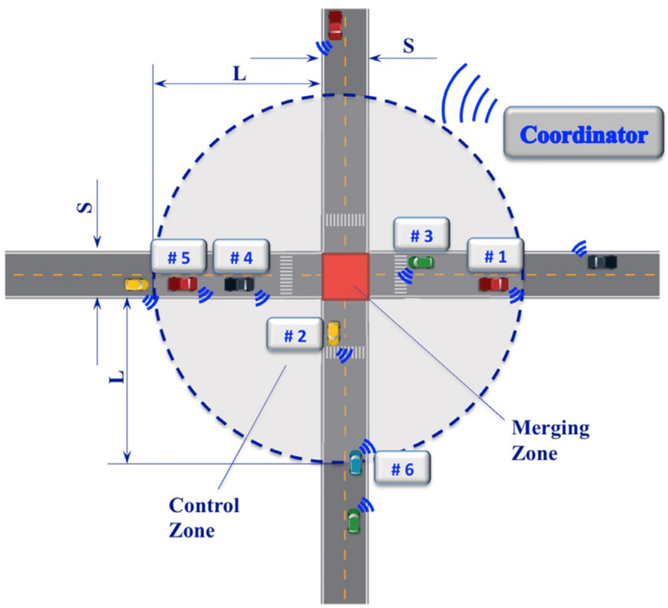

Fig. 3. Model of an intersection with CAVs.

other manually driven vehicles, whereas in Colombo and Del Vecchio (2014) the invariant set for the control inputs that ensure lateral collision avoidance is constructed. A detailed discussion of research efforts in this area can be found in Rios-Torres and Malikopoulos (2016).

3.1. Intersection model

Our analysis is based on a model introduced in Zhang, Malikopoulos, and Cassandras (2016) and is provided in full detail in Malikopoulos, Cassandras, and Zhang (2017). An intersection consists of a region at its center called *Merging Zone* (MZ) where potential lateral collision of vehicles may occur and a *Control Zone* (CZ) of length L within which all CAVs can communicate and coordinate their motion (see Fig. 3). Let $N(t) \in \mathbb{N}$ be the number of CAVs inside the CZ at time $t \in \mathbb{R}^+$ and $\mathcal{N}(t) = \{1, \ldots, N(t)\}$ be a queue which designates the order in which these vehicles will be entering the MZ. Thus, letting t_i^m be the assigned time for vehicle i to enter the MZ, we require that $t_i^m \geq t_{i-1}^m$ for all $i \in \mathcal{N}(t)$, $i > 1$. The policy through which the order (schedule) is specified may be the result of a higher level optimization problem as long as the condition

$$t_i^m \geq t_{i-1}^m, \ \forall i \in \mathcal{N}(t), \ i > 1 \tag{10}$$

is preserved in between CAV arrival events at the CZ.

The dynamics of each CAV $i \in \mathcal{N}(t)$ moving along a specified lane are assumed to satisfy

$$\begin{aligned} \dot{p}_i &= v_i(t), \quad p_i(t_i^0) = 0 \\ \dot{v}_i &= u_i(t), \quad v_i(t_i^0) \text{ given} \end{aligned} \tag{11}$$

where t_i^0 is the time when CAV i enters the CZ, and $p_i(t) \in \mathcal{P}_i$, $v_i(t) \in \mathcal{V}_i$, $u_i(t) \in \mathcal{U}_i$ denote the position, speed and acceleration/deceleration (control input) of each CAV i inside the CZ with $p_i(t_i^0) = 0$ (upon entering the CZ at time t_i^0) and a given $v_i(t_i^0) = v_i^0$. To ensure that the control input and vehicle speed are always within a given admissible range, the following constraints are imposed:

$$\begin{aligned} &u_{i,\min} \leqslant u_i(t) \leqslant u_{i,\max}, \quad \text{and} \\ &\quad 0 \leqslant v_{\min} \leqslant v_i(t) \leqslant v_{\max}, \quad \forall t \in [t_i^0, t_i^m], \end{aligned} \tag{12}$$

where $u_{i,\min}$, $u_{i,\max}$ are the minimum and maximum control inputs (maximum deceleration/acceleration) for each vehicle $i \in \mathcal{N}(t)$, and $v_{\min}$, $v_{\max}$ are the minimum and maximum speed limits respectively. For simplicity, in the sequel we set $u_{i,\min} = u_{\min}$ and $u_{i,\max} = u_{\max}$.

Depending on its physical location inside the CZ, CAV $i-1 \in \mathcal{N}(t)$ belongs to only one of the following four subsets of $\mathcal{N}(t)$ with respect to CAV i: (*i*) $\mathcal{R}_i(t)$ contains all CAVs traveling on the same road as i and towards the same direction but on different lanes, (*ii*) $\mathcal{L}_i(t)$ contains all CAVs traveling on the same road and lane as vehicle i, (*iii*) $\mathcal{C}_i(t)$ contains all CAVs traveling on different roads from i and having destinations that can cause collision at the MZ, (e.g., $\mathcal{C}_6(t)$ contains CAVs 3,4,5 in Fig. 3), and (*iv*) $\mathcal{O}_i(t)$ contains all CAVs traveling on the same road as i and opposite destinations that cannot, however, cause collision at the MZ.

Based on this definition, it is clear that a rear-end collision can only arise if CAV $k \in \mathcal{L}_i(t)$ is directly ahead of i. Thus, to ensure the absence of any rear-end collision, we assume a predefined safe distance $\delta < S$ and impose the rear-end safety constraint

$$s_i(t) = p_k(t) - p_i(t) \geqslant \delta, \ \ \forall t \in [t_i^0, t_i^f], \ \ k \in \mathcal{L}_i(t) \tag{13}$$

where t_i^f is the time that CAV $i \in \mathcal{N}(t)$ exits the MZ. Henceforth, we reserve the symbol k to denote the CAV which is physically immediately ahead of i in the same lane. On the other hand, a lateral collision involving CAV i may occur only if some CAV $j \neq i$ belongs to $\mathcal{C}_i(t)$. This leads to the following definition for each CAV $i \in \mathcal{N}(t)$:

$$\Gamma_i \triangleq \left\{ t \mid t \in [t_i^m, t_i^f] \right\}.$$

Consequently, to avoid a lateral collision for any two vehicles $i, j \in \mathcal{N}(t)$ on different roads, the following constraint should hold

$$\Gamma_i \cap \Gamma_j = \varnothing, \qquad \forall t \in [t_i^m, t_i^f], \ \ j \in \mathcal{C}_i(t). \tag{14}$$

This constraint implies that no two CAVs from different roads which may lead to a lateral collision are allowed to be in the MZ at the same time. If the length of the MZ is too large, making this constraint overly conservative, then it can be modified appropriately.

In this modeling framework, we assume that each CAV i has proximity sensors and can measure local information without errors or delay and that none of the constraints (12) and (13) is active at t_i^0. We also assume that the speed of the CAVs inside the MZ is constant, i.e., $v_i(t) = v_i(t_i^m) = v_i(t_i^f)$, $\ \forall t \in [t_i^m, t_i^f]$. This implies that

$$t_i^f = t_i^m + \frac{S}{v_i(t_i^m)}. \tag{15}$$

For simplicity of notation in the remainder of the paper, we will write $v_i(t_i^0) \equiv v_i^0$, $v_i(t_i^m) \equiv v_i^m$ and $v_i(t_i^f) \equiv v_i^f$.

3.2. Energy minimization and throughput maximization problems

We begin by considering the controllable acceleration/deceleration $u_i(t)$ of each CAV i which minimizes the following cost functional:

$$J_i(u_i(t), t_i^m, v_i^m) = \int_{t_i^0}^{t_i^m} C_i(u_i(t))dt, \tag{16}$$

$$\text{subject to} : (11), (12), (13), (14), \ \ p_i(t_i^0) = 0, \ p_i(t_i^m) = L,$$

$$\text{and given } t_i^0, \ v_i^0, \ t_i^m$$

where $C_i(\cdot)$ is a monotonically increasing function of its argument. We view $C_i(u_i(t))$ as a measure of the energy, which is a monotonically increasing function of the control input (acceleration/deceleration) consumed by CAV i in traveling between

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$p_i(t_i^0)=0$ and $p_i(t_i^m)=L$ (a special case arises when $C_i(u_i(t))=\frac{1}{2}u_i^2(t)$). Thus, we minimize transient engine operation leading to direct benefits in fuel consumption and emissions. In this problem, t_i^0, v_i^0 are known upon arrival of CAV i at the CZ and t_i^m is also specified. Clearly, not all t_i^m can satisfy the safety constraints (13) and (14). Moreover, in general, a value of t_i^m that satisfies (13) and (14) may depend on other CAVs $j \neq i$; therefore, it may not be possible for CAV i to solve (16) in a *decentralized* manner, i.e., based only on local information. We address the question of specifying appropriate t_i^m for each instance of (16) in what follows.

Before proceeding, we note that the obvious unconstrained solution to (16) is $u_i^*(t)=0$ for all $t \in [t_i^0, t_i^m]$ since $C_i(u_i(t))$ is a monotonically increasing function with respect to $u_i(t)$. This applies to $i=1$ in which case (13) and (14) are inactive, since CAV 1 is not constrained by an any prior CAV in the queue. This also implies that $v_1^*(t)=v_i^0$ for all $t \in [t_i^0, t_i^m]$ and $t_1^m = L/v_i^0$.

We now turn our attention to the problem of maximizing the traffic throughput at the intersection, in terms of minimizing the gaps between the vehicles in a given queue $\mathcal{N}(t)$, under the hard safety constraints (13) and (14). Thus, setting $\mathbf{t}_{(2:N(t))} = [t_2^m \dots t_{N(t)}^m]$, we define the following optimization problem:

$$\min_{\mathbf{t}_{(2:N(t))}} \sum_{i=2}^{N(t)} \left(t_i^m - t_{i-1}^m\right) = \min_{\mathbf{t}_{N(t)}} \left(t_{N(t)}^m - t_1^m\right),$$
$$\text{subject to}: (10), (12), (13), (14) \tag{17}$$

where t_1^m is not included since it is obtained from the solution of (16) when $i=1$, i.e., $t_1^m = L/v_i^0$. The equivalence between the two expressions in (17) (due to the cancellation of all terms in the sum except the first and last) reflects the equivalence between minimizing the total time to process all CAVs in the queue and the average interarrival time of CAVs at the MZ.

As stated in (17), the problem does not incorporate constraints on t_i^m, $i=2,\dots,N(t)$, that are imposed by the CAV dynamics. In other words, we should write $t_i^m = t_i^m(\mathbf{u}_{(1:i)}(t))$ where $\mathbf{u}_{(1:i)}(t) = [u_1(t;t_1^m)\dots u_i(t;t_i^m)]$ denotes the controls applied to all CAVs $i=1,\dots,N(t)$ over $[t_i^0, t_i^m]$ for any given t_i^0, t_i^m. Let $\mathscr{A}_i$ denote a set of feasible controls:

$$\mathscr{A}_i \triangleq \Big\{u_i(t;t_i^m) \in \mathcal{U}_i \text{ subject to:}$$
$$(10), (11), (12), (13), (14),\ \ p_i(t_i^0)=0,\ p_i(t_i^m)=L,$$
$$\text{and given } t_i^0,\ v_i^0,\ t_i^m\Big\}. \tag{18}$$

Then, we rewrite (17) as

$$\min_{\mathbf{t}_{(2:N(t))}} \sum_{i=2}^{N(t)} \left(t_i^m(\mathbf{u}_{(1:i)}(t)) - t_{i-1}^m(\mathbf{u}_{(1:i-1)}(t))\right) \tag{19}$$
$$= \min_{\mathbf{t}_{N(t)}} \left(t_{N(t)}^m(\mathbf{u}_{(1:N(t))}(t)) - t_1^m(\mathbf{u}_{(1)}(t))\right),$$
$$\text{subject to}: u_i(t;t_i^m) \in \mathscr{A}_i,\ \forall i \in \mathcal{N}(t),\ (10),\ (12),\ (13),\ (14).$$

The solution of (19) provides a sequence $\{t_2^{m*},\dots,t_{N(t)}^{m*}\}$ which designates the MZ arrival times of all CAVs in the current queue so as to minimize the total time needed for them to clear the intersection, hence maximizing the throughput over the current $N(t)$ CAVs. This solution may then be used in (16) to specify the terminal time of each energy minimization problem. As formally shown in Malikopoulos et al. (2017), it turns out that this solution has a simple iterative structure and depends only on the hard safety constraints (13) and (14), as well as the state and control constraints (12):

$$t_i^{m*} = \begin{cases} t_1^{m*} & \text{if } i=1 \\ \max\left\{t_{i-1}^{m*}, t_k^{m*} + \dfrac{\delta}{v_k^m}, t_i^c\right\} & \text{if } i-1 \in \mathcal{R}_i(t) \cup \mathcal{O}_i(t) \\ \max\left\{t_{i-1}^{m*} + \dfrac{\delta}{v_{i-1}^m}, t_i^c\right\} & \text{if } i-1 \in \mathcal{L}_i \\ \max\left\{t_{i-1}^{m*} + \dfrac{S}{v_{i-1}^m}, t_i^c\right\} & \text{if } i-1 \in \mathcal{C}_i \end{cases} \tag{20}$$

where $t_i^c = t_i^1 \mathbb{1}_{v_i^m = v_{\max}} + t_i^2(1-\mathbb{1}_{v_i^m = v_{\max}})$ and

$$t_i^1 = t_i^0 + \frac{L}{v_{\max}} + \frac{(v_{\max} - v_i^0)^2}{2u_{i,\max} v_{\max}} \tag{21}$$

$$t_i^2 = t_i^0 + \frac{[2Lu_{i,\max} + (v_i^0)^2]^{1/2} - v_i^0}{u_{i,\max}} \tag{22}$$

It follows that t_i^{m*} is always recursively determined from t_{i-1}^{m*} and v_{i-1}^m and possibly t_k^{m*}, v_k^m where v_{i-1}^m and v_k^m depend on the specific controls used when solving problem (19). However, note that there is no guarantee that there exist feasible controls satisfying all constraints in (18) over all $t \in [t_i^0, t_i^m]$. In fact, it is easy to see that the safety constraint (13) may not hold depending on the initial conditions (t_i^0, v_i^0) for CAV i. It is shown in Malikopoulos et al. (2017), however, that there exists a nonempty feasible region $\mathcal{F}_i \subset \mathbb{R}^2$ of initial conditions (t_i^0, v_i^0) such that $s_i(t) \geq \delta$ for all $t \in (t_i^0, t_i^m)$ so that all safety constraints are guaranteed to hold throughout $[t_i^0, t_i^m]$.

3.3. Decentralized optimal control framework

We are now in a position to return to the energy minimization problem (16) with the value of t_i^m for any $i=1,\dots,N(t)$ specified through (20) in a recursive manner. This allows us to solve these problems in a decentralized manner with the optimal control problem for each CAV i formulated as follows:

$$\min_{u_i(t)} \frac{1}{2} \int_{t_i^0}^{t_i^m} u_i^2(t)\ dt \tag{23}$$

subject to the vehicle dynamics described earlier, the speed and acceleration constraints, the lateral and rear-end safety constraints, and $p_i(t_i^0)=0$, $p_i(t_i^m)=L$ with given t_i^0, v_i^0. This quadratic cost functional captures the energy consumption over $[t_i^0, t_i^m]$ so that (23) is an energy minimization problem for each CAV with the value of t_i^m selected through (20) so as to maximize the throughput of the intersection over a given queue $\mathcal{N}(t)$ subject to the requirement $t_i^m \geq t_{i-1}^m$. Thus, a solution of (23), if it exists, combines energy minimization with throughput maximization while guaranteeing all safety constraints. Alternatively, a terminal cost of the form $\frac{1}{2}[v_i^2(t_i^m) - \bar{v}_i^2]$ may be added, where $\bar{v}_i$ is a desired terminal speed selected as a target vehicle throughput rate.

There are several questions to address regarding the optimal control problem (23), starting with the existence of feasible solutions and including the ability for each CAV to solve its own problem in a decentralized fashion and within a manageable computation time for real-time operation.

3.3.1. Feasibility

As mentioned above, it is shown in Malikopoulos et al. (2017) that there always exist initial conditions (t_i^0, v_i^0) which guarantee a solution of (23) satisfying all safety constraints. This defines a feasibility region $\mathcal{F}_i \subset \mathbb{R}^2$ for solving the optimal control problem (23). Enforcing such feasible initial conditions requires a "pre-control zone" within which a CAV is subject to a controller aiming to adjust its (t_i^0, v_i^0) so that $(t_i^0, v_i^0) \in \mathcal{F}_i$. In practice, if this becomes difficult to enforce, optimal control may be foregone for any CAV that fails to satisfy $(t_i^0, v_i^0) \in \mathcal{F}_i$. This is possible due to the decentralized nature of the problems in (23).

3.3.2. Computational complexity

The complete solution of the optimal control problem (23) can be analytically derived (see Malikopoulos et al. (2017)) and leads to $u_i^*(t) = a_i t + b_i$, $v_i^*(t) = \frac{1}{2}a_i t^2 + b_i t + c_i$, and $p_i^*(t) = \frac{1}{6}a_i t^3 + \frac{1}{2}b_i t^2 + c_i t + d_i$ as long as none of the problems constraints are active. The four coefficients a_i, b_i, c_i, d_i can also be explicitly evaluated as the solution of four linear algebraic equations. When the constraints are active, it is still possible to obtain exact expressions for the optimal control as detailed in Malikopoulos et al. (2017). This relatively simple solution structure requires minimal computational effort. Moreover, a new solution is required only whenever an event, such as a new CAV arrival at the CZ, takes place.

Returning to the PoA estimated as described earlier based on actual traffic data, the ability to automate the movement of vehicles allows a Smart City to at least reduce the PoA by controlling routing decisions when a CAV is assigned a particular origin-destination pair. The optimal control framework based on (23) goes beyond a congestion-based PoA metric by combining energy and throughput as performance criteria, while also guaranteeing the enforcement of safety requirements at the most vulnerable component of an urban transportation network, i.e., intersections. Explicit numerical results obtained to date and included in Malikopoulos et al. (2017) show that, compared to an intersection operating under traffic light control, it is possible to reduce the average energy consumption of vehicles by about 40% while also reducing the average travel time by about 40%.

The model in Fig. 3 can be extended to two or more intersections (see Zhang et al., 2016) and may also include left and right turns as described in Zhang, Malikopoulos, and Cassandras (2017). Clearly, one important aspect not included in this model is the presence of pedestrians. This needs to be accounted for as an additional traffic flow which is combined with that of vehicles and presents several additional challenges for integrating CAVs into current transportation systems.

4. Towards shared automated mobility using CAVs

We have addressed two key questions related to the path towards automated mobility in Smart Cities. First, we have quantified at least some of the potential benefits of introducing CAVs into a transportation system with the objective of justifying the technological, economic, and social transitions involved. This was accomplished through an estimation of the Price of Anarchy (PoA) measuring the gap between the performance attained by a system-centric approach with cooperating drivers ("social" optimality) as opposed to the performance attained by the current noncooperative transportation environment. Explicit results based on traffic data from the area around the city of Boston indicate that the PoA can be significant, thus justifying the effort towards the development of CAVs. Second, with this motivation in mind, we have presented an optimal control framework developed so as to enable CAV-based automated mobility in urban settings combining both energy and congestion as the performance metrics of interest.

The proposed optimal control framework is merely a first step which has established the feasibility of a mobility automation approach while paving the way for a multitude of related questions and open research directions. Examples include: How do CAVs coexist with regular vehicles? What is the minimal fraction of CAVs within a transportation system which justifies the benefits of automated mobility? How are pedestrians accommodated if traffic lights no longer exist? What is the role of Electric Vehicles (EVs) in an automated mobility setting?

Interestingly, it has been argued that if "self-driving cars" are effective, then users who are now shying away from car ownership and who opt for public transportation will eventually gravitate back to them, ironically causing additional congestion, fuel consumption and undesirable emissions to the environment. The ultimate solution, therefore, may be *automated shared on-demand mobility*: a CAV provided to a user (or group of users) where and when he/she needs it. This encompasses a public transportation system where vehicles operate on a dynamic on demand basis rather than inefficient static predetermined schedules. The ultimate goal of such an approach is to maintain a sufficiently low total number of vehicles in the system and reap the joint sustainable benefits of automation in terms of lower congestion, lower energy consumption with less wasted fuel, and a cleaner environment.

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