2016 American Control Conference (ACC)
Boston Marriott Copley Place
July 6-8, 2016. Boston, MA, USA

# Optimal Control and Coordination of Connected and Automated Vehicles at Urban Traffic Intersections

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*Abstract*— **We address the problem of coordinating online a continuous flow of connected and automated vehicles (CAVs) crossing two adjacent intersections in an urban area. We present a decentralized optimal control framework whose solution yields for each vehicle the optimal acceleration/deceleration at any time in the sense of minimizing fuel consumption. The solution, when it exists, allows the vehicles to cross the intersections without the use of traffic lights, without creating congestion on the connecting road, and under the hard safety constraint of collision avoidance. The effectiveness of the proposed solution is validated through simulation considering two intersections located in downtown Boston, and it is shown that coordination of CAVs can reduce significantly both fuel consumption and travel time.**

## I. INTRODUCTION

Connected and automated vehicles (CAVs) can improve transportation safety and efficiency using traffic lights and vehicle-to-infrastructure communication [1]. There are also significant opportunities to coordinate CAVs for improving both safety and traffic flow using either centralized or decentralized approaches. In this paper, we categorize an approach as centralized if there is at least one task in the system that is globally decided for all vehicles by a single central controller. In a decentralized approach, a "coordinator" may be used to handle or distribute information available in the system without, however, getting involved in any control task.

To date, traffic lights are the prevailing method used to control the traffic flow through an intersection. Recent technological developments which exploit the ability to collect traffic data in real time have made it possible for new methods to be applied to traffic light control [2]. Most of these approaches are computationally inefficient and not immediately amenable to online implementations. More recently, however, data-driven approaches have been developed leading to online adaptive traffic light control as in [3]. Aside from the obvious infrastructure cost and the need for dynamically controlling green/red cycles, traffic light systems also lead to problems such as significantly increasing the number of rear-end collisions at an intersection. These issues have provided the motivation for drastically new approaches capable of providing a smoother traffic flow and more fuel-efficient driving while also improving safety.

The advent of CAVs provides the opportunity for such new approaches. Dresner and Stone [4] proposed a reservation scheme for automated vehicle intersection control whereby a centralized controller coordinates a crossing schedule based on requests and information received from the vehicles located inside some communication range. This scheme has been expanded since then [5]–[7]. Increasing the throughput of an intersection is one desired goal and it can be achieved through the travel time optimization for all vehicles located within a radius from the intersection. There have been several research efforts to address the problem of vehicle coordination at intersections within a decentralized control framework [8]–[11]. One of the main challenges in this case is the possibility of having deadlocks in the solutions as a consequence of the use of local information.

In this paper, we address the problem of optimally controlling online the fuel consumption of a varying number of CAVs subject to congestion and safety constraints as they cross two urban intersections. The contribution of the paper is a decentralized control problem framework whose solution yields for each vehicle the optimal acceleration/deceleration at any time without creating congestion on the connecting road and under the hard constraint of collision avoidance.

The structure of the paper is as follows. In Section II, we extend our work on a single intersection [12] and provide a model for two intersections. In Section III, we formulate the problem of CAV coordination and optimal control for two intersections and provide an analytical solution. In Section IV, we present simulation results in theVISSIM simulation environment considering two intersections located in downtown Boston and offer concluding remarks in Section V.

## II. THE MODEL

We consider two intersections, 1 and 2, located within a distance $D$ (Fig. 1). The region at the center of each intersection, called *merging zone*, is the area of potential lateral collision of the vehicles. Although this is not restrictive, we consider the *merging zones* in both intersections to be squares of equal sides $S$. Each intersection has a *control zone* and a



This research was supported by the Laboratory Directed Research and Development Program of Oak Ridge National Laboratory, managed by UT-Battelle, LLC, for the U. S. Department of Energy. The work of Cassandras and Zhang is supported in part by NSF under grants CNS- 1239021, ECCS-1509084, and IIP-1430145, by AFOSR under grant FA9550-15-1-0471, and by ONR under grant N00014-09-1-1051.

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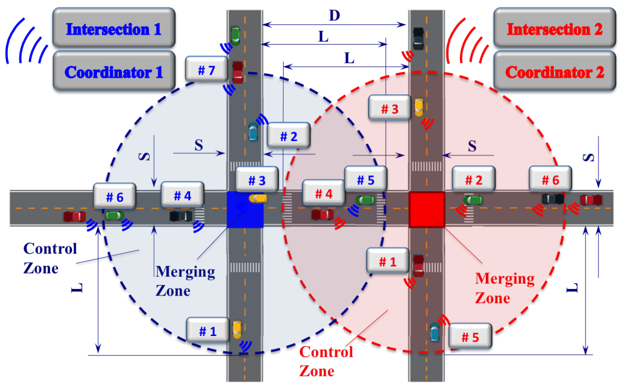


Fig. 1. Two intersections with connected and automated vehicles.

coordinator that can communicate with the vehicles traveling within it. The distance between the entry of the control zone and the entry of the merging zone is $L > S$, and it is assumed to be the same for all entry points to a given control zone.

We consider a time-varying number of CAVs $N_z(t) \in \mathbb{N}$ present at control zone $z = 1, 2$ at time $t \in \mathbb{R}$. When a CAV reaches the control zone of intersection $z$ at some instant $t$, the coordinator assigns a unique identity consisting of a pair $(i, j)$. Here, $i = N_z(t) + 1$ is an integer corresponding to the position of the CAV in a first-in-first-out (FIFO) queue for this control zone. The elements of this queue can belong to any of four subsets (precisely defined in Definition 3.1) depending on the road and lane traveled by each CAV so that $j \in \{1, \dots, 4\}$ is an integer corresponding to the appropriate subset. If two or more vehicles enter the control zone of any intersection at the same time, then the corresponding coordinator selects randomly their position in the queue.

The vehicles in the control zone of intersection $z = 1$ traveling from west to east (see Fig. 1) remain in the queue imposed by coordinator 1 until they exit the corresponding merging zone. In the region between the exit point of merging zone 1 and the entry point of control zone 2, the vehicles cruise with the speed they had when they exited that merging zone and then enter the queue imposed by the coordinator of intersection $z = 2$. A similar process applies to vehicles in control zone 2 traveling from east to west.

The objective of each vehicle is to derive an optimal acceleration/deceleration at any time so as to minimize fuel consumption over the time interval defined from its entry time at a control zone to its exit time from the merging zone while avoiding congestion between the two intersections. We consider an indication of potential congestion the speed reduction of any of the vehicles traveling on this road below a desired minimum value. Accordingly, we specify congestion-avoidance constraints as described in the next section. In addition, we impose hard constraints so to avoid either rear-end collision, or lateral collision inside the merging zone.

Let $\mathcal{N}_z(t) = \{1, \dots, N_z(t)\}$, $z = 1, 2$, be the queue associated with the control zone of intersection $z$. We represent the dynamics of each vehicle $i$, $i \in \mathcal{N}_z(t)$, moving along a specified lane with a state equation

$$\dot{x}_i = f(t, x_i, u_i), \qquad x_i(t_i^0) = x_i^0, \tag{1}$$

where $t \in \mathbb{R}^+$ is the time, $x_i(t)$, $u_i(t)$ are the state of the vehicle and control input, $t_i^0$ is the time that vehicle $i$ enters the control zone, and $x_i^0$ is the value of the initial state. For simplicity, we assume that each vehicle is governed by a second order dynamics

$$\dot{p}_i = v_i(t) \ , \ \dot{v}_i = u_i(t) \tag{2}$$

where $p_i(t) \in \mathcal{P}_i$, $v_i(t) \in \mathcal{V}_i$, and $u_i(t) \in \mathcal{U}_i$ denote the position, speed and acceleration/deceleration (control input) of each vehicle $i$. Let $x_i(t) = \begin{bmatrix} p_i(t) & v_i(t) \end{bmatrix}^T$ denote the state of each vehicle $i$, with initial value $x_i^0 = \begin{bmatrix} 0 & v_i^0 \end{bmatrix}^T$, taking values in the state space $\mathcal{X}_i = \mathcal{P}_i \times \mathcal{V}_i$. The sets $\mathcal{P}_i$, $\mathcal{V}_i$ and $\mathcal{U}_i$, $i \in \mathcal{N}_z(t)$, $z = 1, 2,$, are complete and totally bounded subsets of $\mathbb{R}$. It follows that $\mathcal{P}_i$, $\mathcal{V}_i$, and $\mathcal{U}_i$ are Borel measurable sets. The state space $\mathcal{X}_i$ for each vehicle $i$ is closed with respect to the induced topology on $\mathcal{P}_i \times \mathcal{V}_i$ and thus, it is compact.

We need to ensure that for any initial state $(t_i^0, x_i^0)$ and every admissible control $u(t)$, the system (1) has a unique solution $x(t)$ on some interval $[t_i^0, t_i^f]$, where $t_i^f$ is the time that vehicle $i \in \mathcal{N}_z(t)$ exits the merging zone of intersection $z$. The following observations from (2) satisfy some regularity conditions required both on $f$ and admissible controls $u(t)$ to guarantee local existence and uniqueness of solutions for (1): a) $f$ is continuous in $u$ and continuously differentiable in $x$, b) The first derivative of $f$ in $x$, $f_x$, is continuous in $u$, and c) The admissible control $u(t)$ is continuous in $t$. We impose the following assumption regarding the final conditions when a vehicle exits the merging zone, which is intended to enhance safety awareness:

*Assumption 2.1:* The vehicle speed inside any merging zone is constant.

This assumption is not restrictive and could be modified appropriately. In addition, to ensure that the control input and vehicle speed are within a given admissible range, the following constraints are imposed.

$$\begin{aligned} &u_{min} \le u_i(t) \le u_{max}, \quad \text{and} \\ &\quad 0 \le v_{min} \le v_i(t) \le v_{max}, \quad \forall t \in [t_i^0, t_i^f], \end{aligned} \tag{3}$$

where $u_{min}$, $u_{max}$ are the minimum deceleration and maximum acceleration allowable, and $v_{min}$, $v_{max}$ are the minimum and maximum speed limits respectively.

## III. Vehicle Coordination

### A. Decentralized Control Problem Formulation

When a vehicle enters a control zone $z = 1, 2$, it receives a unique identity $(i, j)$ from the coordinator, as described in the previous section. Since the coordinator is not involved in any decision on the vehicle control, we can formulate $N_1(t)$ and $N_2(t)$ decentralized tractable problems for intersection 1 and 2 respectively that may be solved on line. Before we proceed with the decentralized problem formulation we need to establish some definitions.

Recall that $\mathcal{N}_z(t) = \{1, \ldots, N_z(t)\}$ is the FIFO queue of vehicles in control zone $z = 1, 2$. A vehicle index $i \in \mathcal{N}_z(t)$ also indicates which vehicle is closer to the merging zone, i.e., if $i < k$ then $L - p_i < L - p_k$.

*Definition 3.1:* Each vehicle $i \in \mathcal{N}_z(t)$ belongs to at least one of the following four subsets: 1) $\mathcal{R}_i^z(t)$ contains all vehicles traveling on the same road as vehicle $i$ and towards the same direction but on different lanes, 2) $\mathcal{L}_i^z(t)$ contains all vehicles traveling on the same road and lane as vehicle $i$, 3) $\mathcal{C}_i^z(t)$ contains all vehicles traveling on different roads from $i$ and having destinations that can cause collision at the merging zone, and 4) $\mathcal{O}_i^z(t)$ contains all vehicles traveling on the same road as vehicle $i$ and opposite destinations that cannot, however, cause collision at the merging zone.

To illustrate the definitions of the subsets of $\mathcal{N}_z(t)$, observe that in Fig. 1 vehicles # 4 and # 6 (blue label) belong to $\mathcal{L}_6^1(t)$; vehicles # 4 and # 7 (blue label) belong to $\mathcal{C}_7^1(t)$ while vehicles # 2 and # 3 (red label) belong to $\mathcal{C}_3^2(t)$; vehicles # 4 and # 5 (blue label) belong to $\mathcal{O}_5^1(t)$ while vehicles # 3 and # 5 (red label) belong to $\mathcal{O}_5^2(t)$.

*Definition 3.2:* The *unique identity* that the coordinator assigns to each vehicle $i \in \mathcal{N}_z$, $z = 1, 2$, at time $t$ when the vehicle arrives at control zone $z$, is a pair $(i, j)$ where $i = N_z(t) + 1$ is an integer representing the location of the vehicle in the FIFO queue $\mathcal{N}_z(t)$ and $j \in \{1, \ldots, 4\}$ is an integer based on a one-to-one mapping from $\{\mathcal{R}_i^z(t)$, $\mathcal{L}_i^z(t)$, $\mathcal{C}_i^z(t)$, $\mathcal{O}_i^z(t)\}$ onto $\{1, \ldots, 4\}$.

*Assumption 3.3:* Each vehicle $i$ has proximity sensors and can observe and/or estimate local information that can be shared with other vehicles.

*Definition 3.4:* For each vehicle $i$ when it enters a control zone, we define the *information set* $Y_i(t)$ as

$$Y_i(t) \triangleq \Big\{p_i(t), v_i(t), \mathcal{Q}_j^z, j = 1, \ldots, 4, z = 1, 2, s_i(t), t_i^f\Big\}, \quad \forall t \in [t_i^0, t_i^f], \tag{4}$$

where $p_i(t), v_i(t)$ are the position and speed of vehicle $i$ inside the *control zone* it belongs to, and $\mathcal{Q}_j^z \in \{\mathcal{R}_i^z(t)$, $\mathcal{L}_i^z(t)$, $\mathcal{C}_i^z(t)$, $\mathcal{O}_i^z(t)\}$, $z = 1, 2$, is the subset assigned to vehicle $i$ by the coordinator (see Definition 3.1). The first of the two new elements in $Y_i(t)$ yet to be defined is $s_i(t) = p_k(t) - p_i(t)$; this represents the distance between vehicle $i$ and some vehicle $k$ which is immediately ahead of $i$ in the same lane (the index $k$ is made available to $i$ by the coordinator). The last element above, $t_i^f$, is the time targeted for vehicle $i$ to exit the merging zone, whose evaluation is discussed next. Note that once the vehicle $i$ enters the control zone, then immediately all information in $Y_i(t)$ becomes available to $i$: $p_i(t), v_i(t)$ are read from the sensors; $\mathcal{Q}_j^z$ is assigned by the coordinator, as is the value of $k$ based on which $s_i(t)$ is also evaluated; $t_i^f$ can also be computed at that time, as described next.

The time $t_i^f$ that the vehicle $i$ exits the merging zone is based on imposing constraints aimed at avoiding congestion (in the sense of maintaining vehicle speeds above a certain value). There are three cases to consider, depending on the value of $\mathcal{Q}_j^z$:

1) if the predecessor of vehicle $i$ in queue $\mathcal{N}_z(t)$, i.e., vehicle $i-1$, belongs to either $\mathcal{R}_i^z(t)$ or $\mathcal{O}_i^z(t)$, $z = 1, 2$, then both $i-1$ and $i$ can share the merging zone at the same time; thus, to minimize the distances between vehicles in the queue (hence, not unnecessarily reduce speeds) both $i-1$ and $i$ should be entering and exiting the merging zone at the same time. Therefore, we impose the constraint $t_i^f = t_{i-1}^f$.

2) If vehicle $i-1$ belongs to $\mathcal{L}_i^z(t)$, $z = 1, 2$, then, by the same argument, both $i-1$ and $i$ should have the *minimal safe distance* allowable, denoted by $\delta$, by the time vehicle $i-1$ enters the merging zone, i.e., $t_i^f = t_{i-1}^f + \frac{\delta}{v_i(t_{i-1}^f)}$, where $v_i(t_{i-1}^f) = v_{i-1}(t_{i-1}^f)$.

3) Finally, if vehicle $i-1$ belongs to $\mathcal{C}_i^z(t)$, $z = 1, 2$, we constrain the merging zone to contain only one vehicle so as to avoid a lateral collision. Therefore, vehicle $i$ is allowed to enter the merging zone only when vehicle $i-1$ exits the merging zone, where $t_i^m$ is the time that the vehicle $i$ enters the merging zone), i.e., $t_i^f = t_{i-1}^f + \frac{S}{v_i(t_{i-1}^f)}$, where $v_i(t_{i-1}^f) = \frac{L}{t_{i-1}^f - t_i^0}$.

Note that, in all cases, once vehicle $i$ enters the control zone, vehicle $i-1$ is already present, thus $t_{i-1}^f$, $v_{i-1}(t_{i-1}^f)$, and $\mathcal{Q}_j^z$, $z = 1, 2$, are available through $Y_{i-1}(t)$. Moreover, to ensure the absence of rear-end collision between two consecutive vehicles traveling on the same lane we impose the constraint $s_i(t) \geq \delta$ (obviously, this applies only when $N_z(t) > 1$).

However, $t_i^f$ above may not be feasible due to the speed and acceleration constraint in (3). There are two cases to consider, based on whether vehicle $i$ can reach $v_{max}$ prior to $t_{i-1}^f$:

$(i)$ If vehicle $i$ enters the control zone at $t_i^0$, it accelerates with $u_{max}$ until it reaches $v_{max}$ and then cruises at this speed until it leaves the merging zone at time $t_i^1$. It is straightforward to show (details found in [13]) that

$$t_i^1 = t_i^0 + \frac{L + S}{v_{max}} + \frac{(v_{max} - v_i^0)^2}{2 u_{max} v_{max}}. \tag{5}$$

$(ii)$ Vehicle $i$ reaches the merging zone at $t_i^m$ with speed $v_i(t_i^m) < v_{max}$. It is again straightforward to show that in this case

$$t_i^2 = t_i^0 + \frac{v_i(t_i^m) - v_i^0}{u_{max}} + \frac{S}{v_i(t_i^m)}. \tag{6}$$

where $v_i(t_i^m) = \sqrt{2 L u_{max} + (v_i^0)^2}$. Thus, setting $t_i^c = \max\{t_i^1, t_i^2\}$, the value of $t_i^f$ is computed as described above and summarized as follows, where $z = 1, 2$:

$$t_i^f = \begin{cases} t_1^f, & \text{if } i = 1, \\ \max\ \{t_{i-1}^f, t_i^c\}, & \text{if } i-1 \in \mathcal{R}_i^z \text{ (or } \mathcal{O}_i^z), \\ \max\ \{t_{i-1}^f + \frac{\delta}{v_i(t_{i-1}^f)}, t_i^c\}, & \text{if } i-1 \in \mathcal{L}_i^z, \\ \max\ \{t_{i-1}^f + \frac{S}{v_i(t_{i-1}^f)}, t_i^c\}, & \text{if } i-1 \in \mathcal{C}_i^z. \end{cases} \tag{7}$$

*Definition 3.5:* For each vehicle $i \in \mathcal{N}_z(t), z = 1, 2$, we define the *rear-end control interval* $R_i$ as

$$R_i(t) \triangleq \Big\{u_i(t) \in [u_{min}, u_{max}] \mid s_i(t) \geq \delta, \\ \forall i \in \mathcal{N}_z(t), z=1,2, \forall t \in [t_i^0, t_i^f], |\mathcal{N}_z| > 1\Big\}. \quad (8)$$

*Remark 3.6:* At each time $t$, each vehicle $i \in \mathcal{N}_z, z = 1,2$, communicates with the preceding vehicle $i-1$ in the queue and accesses the values of $t_{i-1}^f$, $v_{i-1}(t_{i-1}^f)$, $\mathcal{Q}_j^z$, $j = 1,\dots,4$, $z = 1,2$ *from its information set* (Definition 3.4). This information is necessary for vehicle $i$ to compute $t_i^f$ appropriately and satisfy (7) and (8).

*Lemma 3.7:* The decentralized communication structure aims for each vehicle $i$ to solve an optimal problem for $t \in [t_i^0, t_i^f]$ the solution of which depends only on the solution of the vehicle $i$-1.

Due to space limitations, all proofs are omitted but may be found in [13].

Consequently the decentralized control problem for each CAV approaching either intersection can be formulated so as to minimize the $L^2$-norm of its control input (acceleration/deceleration). It has been shown [14] that there is monotonic relationship between fuel consumption for each vehicle $i$, $f_i^{fuel}(t)$, and its control input $u_i$. Thus, the problem of minimizing the acceleration/deceleration is equivalent to the problem of minimizing fuel consumption, and it is formulated as follows:

$$\min_{u_i \in R_i} \frac{1}{2}\int_{t_i^0}^{t_i^f} u_i^2 \cdot K_i \; dt \\ \text{Subject to : (2), (7) } \forall i \in \mathcal{N}_z, z=1,2, \quad (9)$$

where $K_i$ is a factor to capture CAV diversity. However, for simplicity in the rest of the paper we set $K_i = 1$. Both rear-end and lateral collision avoidance constraints are satisfied at time $t_i^f$.

### *B. Analytical solution of the decentralized control problem*

For the analytical solution and online implementation of the decentralized problem (9), we apply Hamiltonian analysis by considering that when the CAVs enter the control zone, the constraints are not active. Clearly, this is in general not true. For example, a vehicle may enter the control zone with speed higher than the speed limit. In this case, we need to solve an optimal control problem starting from an infeasible state. To address this situation requires additional analysis which is the subject of ongoing research.

From (9), the state equations (2), the control/state constraints (3), and rear-end collision avoidance constraint for each vehicle $i \in \mathcal{N}_z(t), z = 1,2$, the Hamiltonian function can be formulated as follows

$$H_i\big(t, x(t), u(t)\big) = \frac{1}{2}u_i^2 + \lambda_i^p \cdot v_i + \lambda_i^v \cdot u_i \\ + \mu_i^a \cdot (u_i - u_{max}) + \mu_i^b \cdot (u_{min} - u_i) + \mu_i^c \cdot (v_i - v_{max}) \\ + \mu_i^d \cdot (v_{min} - v_i) + \mu_i^\delta \cdot (p_i - p_k + \delta), i,k \in \mathcal{N}_z, z = 1,2,$$

where $\lambda_i^p$ and $\lambda_i^v$ are the co-state components, and $\mu^T$ is the vector of the Lagrange multipliers. The solution of the problem including the rear-end collision avoidance constraint may become intractable due to the numerous scenarios of activation/deactivation of the constraints. Thus, we will not include it in the analysis below. Note that we can guarantee rear-end collision avoidance at time $t_i^f$, but it remains to show that the constraint does not become active at any time in $(t_i^0, t_i^f]$ assuming it is not active at $t = t_i^0$. The Lagrange multipliers are $\mu_i^a = \mu_i^b = \mu_i^c = \mu_i^d = 0$ if the constraints are not active, and they are greater than zero if the constraints become active. The Euler-Lagrange equations become

$$\dot{\lambda}_i^p = -\frac{\partial H_i}{\partial p_i} = 0, \quad (10)$$

and

$$\dot{\lambda}_i^v = -\frac{\partial H_i}{\partial v_i} = \begin{cases} -\lambda_i^p, & v_i(t) - v_{max} < 0 \text{ and} \\ & v_{min} - v_i(t) > 0, \\ -\lambda_i^p + \mu_i^c, & v_i(t) - v_{max} = 0, \\ -\lambda_i^p - \mu_i^d, & v_{min} - v_i(t) = 0. \end{cases} \quad (11)$$

The necessary condition for optimality is

$$\frac{\partial H_i}{\partial u_i} = u_i + \lambda_i^v + \mu_i^a - \mu_i^b = 0, \quad (12)$$

To address this problem, the constrained and unconstrained arcs will be pieced together to satisfy the Euler-Lagrange equations and necessary condition of optimality.

If the inequality control and state constraints are not active, we have $\mu_i^a = \mu_i^b = \mu_i^c = \mu_i^d = \mu_i^\delta 0$. Applying the necessary condition (12), the optimal control can be given

$$u_i + \lambda_i^v = 0, \quad i \in \mathcal{N}(t). \quad (13)$$

The Euler-Lagrange equations yield $\dot{\lambda}_i^p = -\frac{\partial H_i}{\partial p_i} = 0$ and $\dot{\lambda}_i^v = -\frac{\partial H_i}{\partial v_i} = -\lambda_i^p$; hence $\lambda_i^p = a_i$ and $\lambda_i^v = -(a_i t + b_i)$, where $a_i$ and $b_i$ are constants of integration corresponding to each vehicle $i$. Consequently, the optimal control input (acceleration/deceleration) as a function of time is given by

$$u_i^*(t) = a_i t + b_i. \quad (14)$$

Substituting the last equation into the vehicle dynamics equations (2) we can find the optimal speed and position for each vehicle, namely

$$v_i^*(t) = \frac{1}{2}a_i t^2 + b_i t + c_i \quad (15)$$

$$p_i^*(t) = \frac{1}{6}a_i t^3 + \frac{1}{2}b_i t^2 + c_i t + d_i, \quad (16)$$

where $c_i$ and $d_i$ are constants of integration. These constants can be computed by using the initial and final conditions. Since we seek to derive the optimal control (14) online, we can designate initial values $p_i(t_i^0)$ and $v_i(t_i^0)$, and initial time, $t_i^0$, to be the current values of the states $p_i(t)$ and $v_i(t)$ and time $t$, where $t_i^0 \leq t \leq t_i^f$. Therefore the constants of integration will be functions of time and states, i.e., $a_i(t,p_i,v_i), b_i(t,p_i,v_i), c_i(t,p_i,v_i)$, and $d_i(t,p_i,v_i)$. To derive online the optimal control for each vehicle $i$, we need to update the integration constants at each time $t$. Equations (15) and (16), along with the initial and final conditions defined above, can be used to form a system of four equations of the form $\mathbf{T}_i \mathbf{b}_i = \mathbf{q}_i$, namely

$$\begin{bmatrix} \frac{1}{6}t^3 & \frac{1}{2}t^2 & t & 1 \\ \frac{1}{2}t^2 & t & 1 & 0 \\ \frac{1}{6}(t_i^f)^3 & \frac{1}{2}(t_i^f)^2 & t_i^f & 1 \\ \frac{1}{2}(t_i^f)^2 & t_i^f & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a_i \\ b_i \\ c_i \\ d_i \end{bmatrix} = \begin{bmatrix} p_i(t) \\ v_i(t) \\ p_i(t_i^f) \\ v_i(t_i^f) \end{bmatrix}. \tag{17}$$

Hence we have

$$\mathbf{b}_i(t, p_i(t), v_i(t)) = (\mathbf{T}_i)^{-1}.\mathbf{q}_i(t, p_i(t), v_i(t)), \tag{18}$$

where $\mathbf{b}_i(t, p_i(t), v_i(t))$ contains the four integration constants $a_i(t, p_i, v_i), b_i(t, p_i, v_i), c_i(t, p_i, v_i), d_i(t, p_i, v_i)$. Thus (14) can be written as

$$u_i^*(t, p_i(t), v_i(t)) = a_i(t, p_i(t), v_i(t))t + b_i(t, p_i(t), v_i(t)). \tag{19}$$

Since (17) can be computed on line, the controller can yield the optimal control on line for each vehicle $i$, with feedback indirectly provided through the re-calculation of the vector $\mathbf{b}_i(t, p_i(t), v_i(t))$ in (18).

Similar results are obtained when the constraints become active. Due to space limitations, this analysis is omitted but may be found in [12]. Note that the control for vehicle $i$ remains unchanged until an "event" occurs that affects its behavior. Therefore, the time-driven controller above can be replaced by an event-driven one without affecting its optimality properties under conditions described in [15].

### C. Interdependence of the Intersections

The two intersections are interdependent, i.e., the coordination of vehicles at the merging zone of one intersection affects the behavior of vehicle coordination of the other merging zone, and a potential congestion on the connecting road of length $D$ (Fig. 1) can disturb the traffic flow. As the number of vehicles $N_z(t), z = 1, 2$, inside the control zones increases, the imposed safety constraints may reinforce some of the vehicles to slow down. When the speed of a vehicle $i$, traveling on the road that connects the two intersections, drops below the desired minimum speed, $v_{min}$, we are interested in a control "mechanism" to accelerate the preceding vehicles to create more space on the road for the following vehicles.

*Definition 3.8:* For each vehicle $i$, we define $\tau_i$ the additional minimum time required for the vehicle to reach the merging zone with the desired minimum speed $v_{min}$, i.e., $\tau_i = \frac{L - p_i(t)}{v_{min}} - \frac{L - p_i(t)}{v_i(t)}$.

Therefore if a vehicle $i \in \mathcal{N}_z, z = 1, 2$, travels towards an intersection and the speed drops below the desired minimum speed $v_i(t) < v_{min}$, then the first vehicle in the queue must expedite its time, $t_1^f$, to exit the merging zone by $\tau_i(t)$, which means now we have $t_1^f = t_1^f - \tau_k$. By doing so, it will also change the value of $t_2^f$ of the second vehicle by $\tau_i(t)$ and so on, and thus all vehicles from 1 to $i$-1 will accelerate to create the required space for vehicle $i$ to cruise with at least $v_{min}$.

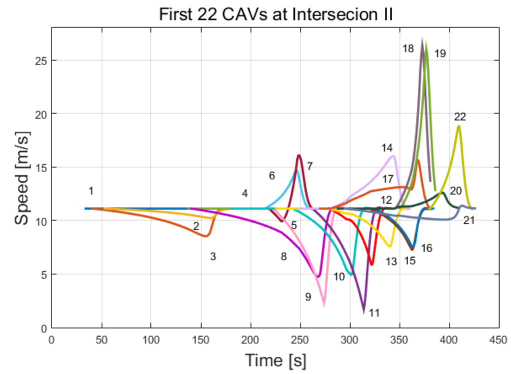


Fig. 2. The speed profile of the first 22 vehicles at intersection II for the unconstrained case.

In this context, the *information set* for each vehicle is expanded to include $\tau_i$, namely

$$Y_i(t) \triangleq \left\{p_i(t), v_i(t), \mathcal{Q}_j^z, j = 1, \ldots, 4, z = 1, 2, s_i(t), t_i^f, \tau_i\right\}, \quad \forall t \in [t_i^0, t_i^f]. \tag{20}$$

## IV. Simulation Results

The effectiveness of the efficiency of the proposed solution is validated through simulation in VISSIM considering two intersections located in downtown Boston. For each direction, only one lane is considered. In both intersections the length of the control and merging zones is $L = 245$ m and $S = 35$ m respectively. The distance between the two intersections is $D_{e-w} = 160$ m and $D_{w-e} = 145$ m, respectively. The safe following distance is $\delta = 10$ m. The arrival rate is given by a Poisson process with $\lambda = 450$ veh/h. The speed of each vehicle entering the control zone is $v_i^0 = 11.11$ m/s. Note that the last two assumptions are only made for simplicity and are by no means constraining. The desired minimum speed inside the control zones is 7 m/s.

We considered two simulations where we: 1) relaxed the upper and lower limits of the speed and control, and 2) included the limits. For the latter case, the upper and lower speed limits are $v_{max} = 13$ m/s and $v_{min} = 0.5$ m/s respectively. The speed of the first 22 vehicles crossing the intersection II for both cases is shown in Fig. 2 and Fig. 3 respectively. The label on each profile corresponds to the number in the queue of the control zone II assigned by the coordinator. The position trajectory of the first 22 vehicles crossing the intersection II is shown in Fig. 4.

Combining Fig. 3 and Fig. 4, we notice that vehicle #4 assigns $t_4^f = t_1^f$ in (7). Vehicle #7 assigns $t_7^f = t_6^f + \frac{\delta}{v_7(t_6^f)}$ in (7), to keep a safe distance $\delta$ from vehicle $\#6 \in \mathcal{L}_7$. Vehicle #8 assigns $t_8^f = t_7^f + \frac{S}{v_8(t_7^f)}$ in (7), which right after vehicle $\#7 \in \mathcal{C}_8$ exits the merging zone, it enters the merging zone. Vehicle #22, assigns $t_{22}^f = t_{21}^f$ in (7), seems to accelerate to catch up with vehicle #21 to arrive at the merging zone at the same time.

To investigate the interdependence between two intersections, we focus on the behavior of three adjacent vehicles

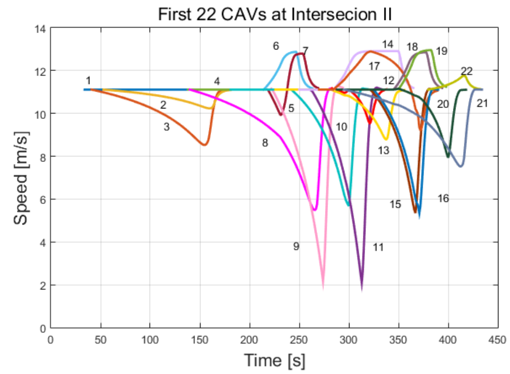


Fig. 3. The speed profile of the first 22 vehicles at intersection II for the constrained case.

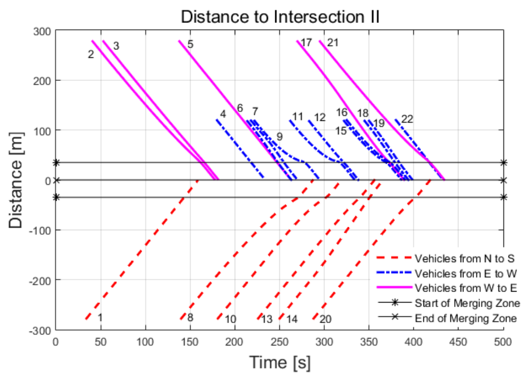


Fig. 4. Distance to the end of merging zone of the first 22 vehicles at Intersection II.

in intersection II vehicles #7, #8 and #9, where vehicles #7 and #9 travel from intersection I to intersection II. When the speed of vehicle #8 becomes lower than the desired minimum speed 7 m/s, the vehicle #7 is "forced" to accelerate, which creates extra space for vehicle #8 to speed up (Fig. 3 and Fig. 4). Similarly, when the speed of vehicle #9 becomes lower than the minimum desired speed, vehicle #8 also accelerates.

To quantify the impact of vehicle coordination on fuel consumption, we used the polynomial metamodel [14] that yields vehicle fuel consumption as a function of the speed and acceleration. In the simulation, we considered 448 CAVs in total crossing the two intersections and we compared our approach to a baseline scenario that includes traffic lights with a traffic light cycle of 30 sec. It was shown that, with coordination of CAVs, fuel consumption is improved by 42.4% while the average travel time is also improved by 37.3% compared to the baseline scenario. The fuel consumption improvement is due to the fact that the vehicles do not come to a full stop, thereby conserving momentum and fuel while also improving travel time. The average travel time for some distance inside the control zone is the same for both cases as the vehicles approach the merging zone; however, in the baseline scenario the vehicles need to come to a full stop resulting in increasing the time to cross the merging zone.

## V. Concluding Remarks

The paper addressed the problem of coordinating online a continuous flow of CAVs crossing two adjacent intersections in an urban area. We presented a decentralized optimal control framework whose solution (when feasible) yields for each vehicle the optimal acceleration/deceleration at any time aimed at minimizing fuel consumption. Ongoing research investigates the feasibility of the solution when at the time the vehicles enter the control zone some of the constraints are active and the computational implications. Future research should consider the diversity in CAV types crossing the intersections and the existence of a potential trade-off between fuel consumption and congestion.

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