# Chapter 5 – Water in the Atmosphere

## Section 1 – The Hydrologic Cycle

In this chapter, we focus on water in the air, both as vapor and as liquid and solid water. To start we will briefly review the three states of water and the conversion of one state to another.

Water can exist in three states—as a solid (ice), as a liquid (water), or as an invisible gas (water vapor). If we want to change the state of water from solid to liquid, liquid to gas, or solid to gas, we must put in energy. This energy, which is drawn in from the surroundings and stored within the water molecules, is called latent heat; in essence, it is the energy needed to break the molecular bonds that maintain the phase state. For instance when solid water (ice) melts, the water molecules absorb enough energy to break molecular bonds that maintain the ice crystals; the same process happens as water changes state





from liquid to gas. In contrast, when water freezes, the water molecules release energy to the surroundings, lowering their internal energy; in turn, they cannot overcome the molecular forces that tend to bind the molecules together and they 'settle' into a crystalline structure. The relation between energy absorption/release and change in phase can be seen in Figure 1.

We are all familiar with melting, freezing, evaporation, and condensation. Sublimation is the direct transition from solid to vapor. Perhaps you have noticed that old ice cubes left in the freezer shrink away from the sides of the ice cube tray and get smaller. They shrink through sublimation—never melting, but losing mass directly as vapor. In this book, we use the term deposition to describe the reverse process, when water vapor crystallizes directly as ice. Frost forming on a cold winter night is a common example of deposition.

Now we want to look at how water in all of its forms is distributed around the globe. The realm of water among ocean, land, and atmosphere is known as the hydrosphere, shown in Figure 2. About 97.5 percent of the hydrosphere consists of ocean salt water. The remaining 2.5 percent is fresh water. The next largest reservoir is fresh water stored as ice in the world's ice sheets and mountain glaciers, which accounts for 1.7 percent of total global water.

Fresh liquid water is found above and below the Earth's land surfaces. Subsurface water lurks in openings in soil and rock. Most of it is held in deep storage as ground water, where plant roots cannot reach. Ground water makes up 0.75 percent of the hydrosphere.

The small remaining proportion of the Earth's water includes the water available for plants, animals, and human use. Plant roots can access soil water. Surface water is held in streams, lakes, marshes, and swamps. Most of this surface water is about evenly divided between

freshwater lakes and saline (salty) lakes. An extremely small proportion makes up the streams and rivers that flow toward the sea or inland lakes.

Only a very small quantity of water is held as vapor and cloud water droplets in the atmosphere—just 0.001 percent of the hydrosphere. However, this small reservoir of water is enormously important. Through precipitation, it supplies water and ice to replenish all freshwater stocks on land. In addition, this water, and its conversion from one form to another in the atmosphere, is an essential part of weather events across the globe. Finally, the flow of water vapor



Figure 2: Partitioning of water throughout the earth system

from warm tropical oceans to cooler regions provides a global flow of heat from low to high latitudes.

The hydrologic cycle represents the flow of water among ocean, land, and atmosphere, shown in Figure 3. It moves water from land and ocean to the atmosphere. Water from the oceans and from land surfaces evaporates, changing state from liquid to vapor and entering the atmosphere. Total evaporation is about six times greater over oceans than land, because oceans cover most of the planet and because land surfaces are not always wet enough to yield much water.

Once in the atmosphere, water vapor can condense or deposit to form precipitation, which falls to the Earth as rain, snow, sleet, or hail. There is nearly four times as much precipitation over oceans than precipitation over land.

When precipitation hits land, it has one of three fates. First, it can evaporate and return to the atmosphere as water vapor. Second, it can sink into the soil and then into the surface rock layers below. This subsurface water emerges from below to feed rivers, lakes, and even ocean margins. Third, precipitation can run off the land, concentrating in streams and rivers that eventually carry it to the ocean or to a lake in a closed inland basin. This flow of water is known as runoff.

Because our planet contains only a fixed amount of water, a global balance must be maintained among flows of water to and from the lands, oceans, and atmosphere. For the ocean, evaporation leaving the ocean is approximately 420 cubic km per year, while the amount entering the ocean via precipitation is 380 cubic km per year. There is an imbalance between the amount of water lost to evaporation and the amount gained through precipitation. This imbalance is made up by the 40 cubic km per year that flows from the land back to the ocean.

Similarly, for the land surfaces of the world, there is a balance. Of the 110 cubic km per year of water that falls on the land surfaces, 70 cubic km per year is re-evaporated back into the atmosphere. The remaining 40 cubic km per year stays in the form of liquid water and eventually

flows back into the ocean.

Of all these pathways, we will be most concerned with one aspect of the hydrologic cycle—the flow of water from the atmosphere to the surface in the form of precipitation. To understand this process, we first need to examine how water vapor in the atmosphere is converted into clouds and subsequently into precipitation.

# Section 2 – Humidity

Previously we defined the actual quantity of water vapor contained within a



Entering ocean	Leaving ocean	Entering land	Leaving land
380 ×1000 km <sup>3</sup> /yr	420 ×1000 km <sup>3</sup> /yr	110 ×1000 km <sup>3</sup> /yr	$70 \times 1000 \mathrm{km^3/yr}$
40 ×1000 km <sup>3</sup> /yr			$40 \times 1000 \mathrm{km^3/yr}$
Total: 420 ×1000 km <sup>3</sup> /yr	Total: 420 ×1000 km <sup>3</sup> /yr	Total: 110 ×1000 km <sup>3</sup> /yr	Total: $110 \times 1000  \text{km}^3/\text{yr}$
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Figure 3: Cycling of water through the earth system

parcel of air as <u>specific humidity</u> and expressed it as kilograms of water vapor per kilogram of air (kg/kg). By definition, specific humidity can only change be adding or removing water vapor molecules, either through evaporation and sublimation (which increases the specific humidity) or condensation and deposition (which decreases the specific humidity).

In fact, at any one time, individual water molecules are continuously changing state—some are evaporating from the land or ocean surface into the atmosphere, while others are condensing. The net transfer determines whether the atmospheric humidity is increasing or decreasing.

However, at some point the atmosphere above a given surface can reach an equilibrium—a balance in which the sum of water molecules changing state in one direction (evaporating say) are balanced by the sum of water molecules changing state in the other direction (condensing). At that point, the air parcel is <u>saturated</u>; in addition, its vapor pressure is called the <u>saturation</u> <u>vapor pressure</u>,  $e_w$ . This condition is shown in Figure 4, in which there are an equal number of air molecules (three) evaporating from, and condensing onto, the surface of the liquid water. One way to think of the saturation vapor pressure is the pressure that must be exerted in order for water molecules to begin to re-condense. In this sense, as the saturation vapor pressure increases, more water vapor molecules must be added before this pressure is reached. At the same time, we know from the equation of state that pressure, in this case vapor pressure, is also related to temperature—as temperatures increase we expect the vapor pressure to increase. This suggests that there is a relation between the saturation vapor pressure of a given air parcel and its temperature.

In fact, it is possible to solve for the change in saturation vapor pressure  $(e_w)$  as a function of temperature:

$$\frac{de_s}{dT} = \frac{L_v}{T(V_v - V_w)}$$
$$L_v = 2.5x10^6 \frac{J}{kg}$$
- Latent heat of vaporization

This is called the <u>Clausius Clapeyron</u> equation and describes the dependence of the saturation vapor pressure upon the temperature. We can further simplify this equation by recognizing that the volume of water vapor  $(V_v)$  is much greater than that for liquid water  $(V_w)$ . This allows us to substitute the equation of state for water vapor into the equation and arrive at:

$$\frac{de_s}{e_s} = \frac{L_v dT}{R_v T(T)}$$

Finally, from the definition of specific humidity we can substitute for the vapor pressure:

$$\frac{dq_s}{q_s} = \frac{L_v dT}{R_v T(T)}$$



Figure 4: Water vapor and liquid water in equilibrium condition, in which evaporation rate is the same as the condensation rate

Integrating, we arrive at an equation that relates the <u>saturation specific humidity</u>,  $q_s$ , to temperature

$$q_s = q_s^0 e^{\left(\frac{L}{R_v}\right)\left(\frac{1}{T_0} - \frac{1}{T}\right)}$$

T(K) - Temperature of fluid

 $T_0(K)$  - Reference temperature of fluid (constant)

 $q_s(kg/kg)$  - Saturation specific of humidity

 $q_s^0(kg/kg)$  - Reference saturation specific humidity (constant)

 $L_{\rm v} = 2.5 \times 10^6$  - Latent heat of vaporization

Here we see that as temperature (T) increases, the difference term increases and therefore the saturation specific humidity increases. Typically saturation specific humidity increases 20% for every 3K change in temperature. More quantitatively, Figure 5 shows that changes in temperature produce exponential changes in the saturation specific humidity. It is the saturation specific humidity, in turn, which dictates how much water can be in the vapor phase within a given air parcel. Hence, we find that warmer air tends to have a higher specific humidity (i.e. water vapor content) than cold air. This temperature effect upon saturation specific humidity will become important as we move forward and look at expected changes associated with climate variability.

At this point, we have defined specific humidity and saturation specific humidity. We now want to define <u>relative humidity</u> (which is what one typically refered to when one talks about 'humidity') as the ratio of the specific humidity to the saturation specific humidity:

$$RH = q/q_{.}$$

To see how these three types of humidity relate to one another, we can consider the formation of dew. During the day, we expect evaporation to occur. Evaporation effectively raises the specific humidity of the air because it increases the amount of water vapor in the atmosphere. At night, however, the air begins to cool. As it cools, the saturation specific humidity decreases, indicating that the air cannot contain as much water vapor as during the day. At the





same time, if the specific humidity stays the same while the saturation specific humidity decreases, we expect the relative humidity to increase. At some point, if the air continues to cool, the saturation specific humidity will be the same as the specific humidity, indicating that the relative humidity is 100% and the air is saturated. If the parcel continues to cool further, the saturation specific humidity will continue to drop. Since the air cannot contain more water vapor than the saturation amount, the actual specific humidity must drop as well. The only way to do that is to have some of the water vapor re-condense as liquid water. Near the ground, this re-condensation occurs in the form of dew. It turns out that in some places of the world, such as coastal forests of Oregon and Washington, dew is the primary moisture source for plant growth in the region.

Ex. 3 If the specific humidity is 0.01 and the saturation specific humidity is 0.015, what is the relative humidity? What is the relative humidity if the temperature goes from 293K to 288K?

Answer:

$$RH = \frac{q}{q_s} = \frac{0.01}{0.015} = 0.667$$

$$q_s(288K) = q_s(293K)e^{\left(\frac{L}{R_v}\right)\left(\frac{1}{293} - \frac{1}{288}\right)} = 0.015 \cdot e^{\left(\frac{2.5 \times 10^6}{461.5}\right)\left(\frac{1}{293} - \frac{1}{288}\right)} = 0.0109$$

$$RH = \frac{q}{q_s} = \frac{0.01}{0.0109} = 0.92$$

### Section 3 - Stability

Now we want to look at how the land surface transfers moisture (and energy and momentum) to the atmosphere. These transfer processes occur within the <u>Boundary layer</u>, a shallow region of the atmosphere which responds rapidly to changes in the surface conditions. This boundary layer can be from 20 meters up to 5km thick, although we typically assume it is around 1 km thick. The relationship between the different regions of the atmosphere can be seen in Figure 6. At the bottom is the boundary layer or mixed layer, showing a



Figure 6: Schematic showing the typical levels of the atmosphere above the earth's surface. Also shown is a typical profile of temperature through these three levels.

fairly homogenous temperature profile. At the interface between the boundary layer and free atmosphere, there is usually an entrainment zone in which there is a sharp discontinuity in temperature (as well as humidity), above which is the free atmosphere.

One important characteristic of the boundary layer is that it responds much more quickly to changing surface conditions than the <u>free atmosphere</u> above it. In fact, there are typically two types of boundary layers. One is referred to as the turbulent or convective boundary layer; the other is termed the stable or neutral boundary layer. For the convective boundary layers, mixing is due to buoyant accelerations associated with rapidly ascending or descending parcels of air within the atmosphere. This process produces a well-mixed profile in temperature, humidity, and wind speed with a very dramatic discontinuity in the entrainment layer. These types of boundary layers are common over land as well as the tropical ocean. They are also common over the midlatitude ocean where cooling at the top of the atmosphere leads to plunging air from above. Neutral boundary layers, on the other hand, are ones in which mixing is due to mechanical accelerations; this mechanical mixing is associated with conversion of mean winds to turbulent motions produced by friction at the surface. In these cases, there tends to be a much shallower boundary layer, again with a strong discontinuity across the entrainment zone.

In this section, we investigate processes leading to convection; in the next section, we look in more detail at the neutral or stable boundary layer.

Previously, we examined the state of the atmosphere at rest. Now we want to examine what might cause air parcels to move; here we want to consider first just the movement of air parcels in the vertical direction. To determine source of this movement, we have to determine how the properties of an air parcel in motion change as the parcel is displaced from its original location. A key assumption for this process is that there is no heat added or lost from the parcel during its movement. This implies the parcel has to be displaced fast relative to heat conduction. The assumption itself is called the <u>Adiabatic assumption</u>. It turns out that for many atmospheric processes, the adiabatic assumption is fairly good. This assumption holds for two main reasons. First, air is a poor conductor of heat so that, for an air parcel of reasonable size (larger than a building), heating or cooling at the edges does not affect the internal properties of the parcel. Second, many motions in the atmosphere occur relatively quickly - during convection, air parcels

rise and sink over the course of 20 minutes. Horizontally, air aloft flows at about 20-50m/s. As such, an air parcel does not get much time to interact with changing conditions around it, again precluding any heating or cooling from the surrounding environment.

From before, we know that the First law of Thermodynamics states:

dE = dQ + dW = dQ - PdV

Now, if we allow for the adiabatic assumption, we essentially are dictating that dQ=0. We can then write:

$$dE = -PdV$$

From before, we also know that,  $dE = c_v dT$ . In addition, if we use a variant on the equation of

state, PV=RT (recognizing that density is inversely related to volume), and differentiate by parts, we find:

PdV + VdP = RdTPlugging in for PdV and combining terms gives: VdP = (R + c)dT

Next, we use the equation of state to substitute for V and then use the hydrostatic equation  $(dP = -\rho g dz)$  to substitute for dP. We then arrive at an equation for the change in temperature with height:

$$\frac{dT}{dz} = -g/(R+C_v) = -g/C_P = -\Gamma_d$$

 $\Gamma_{\mu}$  - Dry adiabatic lapse rate

Note that all of the terms on the right hand side are constant. When we plug in for these constants we find that  $\Gamma = 0.01 K/m = 10 K/km$ . From this equation we find that the temperature of an air parcel decreases with height. In addition, if there are no processes which can heat or cool it (including those related to release of latent heat) we find that we can solve for this rate of temperature change exactly. In fact, this rate simply defines how much a parcel's temperature will change with height, based solely upon changes in the surrounding pressure (i.e. due to work done by the parcel on its surrounding environment). Figure 7 demonstrates how these processes

are related. As a parcel rises, the pressure around it decreases (as shown previously). Hence the parcel expands. This represents work done by the parcel upon the environment. However, because the parcel is doing work, it is losing energy. Without any outside source of energy (i.e. no heating or cooling as dictated by the adiabatic assumption), this energy must come from the internal energy of the parcel itself. Since internal energy is decreasing, the temperature must be decreasing. In contrast, as a



Figure 7: Change in pressure and temperature for an air parcel moving adiabatically

parcel sinks, it is compressed, work is done on the parcel, adding to its energy, thereby increasing its temperature.

Ex.5 What is the saturation vapor pressure for the parcel in Ex.3 if it is lifted 5km? Answer:  $T(5km) = T(0km) + \frac{\partial T}{\partial z} = 293 - 10 \left(\frac{K}{km}\right) \cdot 5km = 243K$  $q_s(243K) = q_s(293K)e^{\left(\frac{L}{R_v}\right)\left(\frac{1}{293} - \frac{1}{243}\right)} = 0.015 \cdot e^{\left(\frac{2.5 \times 10^6}{461.5}\right)\left(\frac{1}{293} - \frac{1}{243}\right)} = 0.00033$ 

At this point, we can use the dry adiabatic lapse rate to determine whether the atmosphere is <u>stable</u> or not. To determine this, we want to calculate the <u>Buoyant</u> force on a moving parcel of air. For a parcel of air that is moving in the vertical, the balance of forces is:

$$-(1/\rho)\frac{\partial P}{\partial z} - g = \frac{dw}{dt}$$

Where w is the vertical velocity and the negative signs indicate that, absent a balancing pressure gradient, gravity makes objects accelerate towards the earth. Re-arranging gives:

$$\frac{\partial P}{\partial z} + \rho g = -\rho \frac{dw}{dt}$$

If we take a parcel and lift it, the surrounding pressure will be determined by the ambient (nonmoving) air, for which we can use the equation for air at rest (i.e. the hydrostatic equation):

$$\frac{\partial P}{\partial z} = -\rho^* g$$

We can combine these two equations, recognizing that the pressure is the same (because the parcel and the ambient air are at the same level) but the densities of the parcel and the ambient air may be different:

$$\frac{dw}{dt} = \left(\frac{\rho^* - \rho}{\rho}\right)g$$

From this equation we can see that if a parcel rises and is less dense that the ambient air around it (e.g.  $\rho < \rho^*$ ), it will continue to rise (dw/dt > 0); however, if it is more dense, then the parcel will sink. A priori it is difficult to tell whether a parcel's density will be greater or less than that of its surroundings. However, we can substitute the equations of state for both the parcel and ambient air:

$$P = \rho RT$$
  $P = \rho^* RT^*$ 

This gives:

$$\frac{dw}{dt} = \left(\frac{T - T^*}{T^*}\right)g$$

From before, we can define the change in the parcel temperature, *T*, by:

$$\frac{dT}{dz} = -\Gamma_d \Longrightarrow T(z) = T_0 - \Gamma_d \partial z$$

We can also measure the change in the ambient (or environmental) temperature,  $T^*$ , which is given by the Environmental Lapse Rate,  $\gamma$ :

$$\frac{dT^*}{dz} \equiv -\gamma$$

By the same token, this equation lets us determine the ambient temperature at a given height:

$$T^*(z) = T_0 - \gamma \partial z$$

Putting these into the equation for motion, we get:

$$\frac{dw}{dt} = \left[\frac{\left(\gamma - \Gamma_d\right)\partial z}{T_0 - \gamma \partial z}\right]g$$

Here, it is important to remember that g is always positive, as is the denominator (which just represents the ambient temperature of the atmosphere at a given level). Hence, the sign of the vertical velocity change is given by the numerator. For a parcel that is lifted, dz>0. Hence, if the environmental lapse rate is greater than the adiabatic lapse rate, the parcel will continue to rise (i.e. dw/dt>0). This represents an <u>unstable</u> situation in which an initial rise in parcel height will be followed by subsequent rising. In contrast, if the environmental lapse rate is less than the adiabatic lapse rate, the parcel will sink, representing a <u>stable</u> condition. What does this indicate in a physical sense? First it is important to remember that both lapse rate is greater than the adiabatic lapse rate, then the environmental temperature decreases with height faster than the adiabatic lapse rate, then the environmental temperature decreases with height faster than the surrounding atmosphere and will therefore continue to rise. On the other hand, if the environmental lapse rate is less than the adiabatic lapse rate is less than the surrounding atmosphere and will therefore continue to rise. On the other hand, if the environmental lapse rate is less than the adiabatic lapse rate, then the environmental temperature decreases less quickly with height; hence an air parcel that rises will be cooler and more dense than the surrounding atmosphere and will therefore sink.

Up until now, we have been considering only the stability of a dry parcel, i.e. one that is unsaturated. However, in almost all processes, moisture content also plays an important role in determining stability. For a *saturated* parcel, it can be shown that the <u>Moist adiabatic lapse rate</u> is:

$$\Gamma_{s} = \Gamma_{d} \left[ \frac{1 + (Lw_{s})/(RT)}{1 + 0.622 \cdot \left(\frac{L}{(R - c_{y})T}\right) \cdot (Lw_{s})/(RT)} \right]$$

$$w_{s} = 0.622e/P$$

Here, the important thing to note is that the size of the moist adiabatic lapse rate compared with the dry adiabatic lapse rate depends upon the ratio of  $0.622L/((R-c_1)T)$ . For the atmosphere, this is usually greater than one, making the denominator of the above equation larger than the numerator. This in turn means that the moist adiabatic lapse rate is less than the dry adiabatic lapse rate, i.e.for a saturated parcel the rate of temperature change with height is less than for an unsaturated parcel. Qualitatively, this is due of course to the fact that as the parcel rises and cools, additional water vapor condenses, releasing latent heat which warms the parcel, offsetting and hence reducing the decrease in temperature associated with the adiabatic process alone.

For determining the stability of a saturated parcel in a given environment, we can use the same equation we used before, but substituting  $\Gamma_s$  for  $\Gamma_d$ . Importantly, this creates another classification for stability. Namely, if the environmental lapse rate is less than the dry adiabatic lapse rate but greater than the moist adiabatic lapse rate, the environment is conditionally stable because the



### Unstable

If the environmental lapse rate is greater than both the dry and moist adiabatic lapse rates, the environment is unstable. As an air parcel moves up through the atmosphere, it is always warmer and less dense than the surrounding environment, causing it to continue to rise.

### **Conditionally stable**

If the environmental lapse rate is less than the dry adiabatic lapse rate, but greater than the moist adiabatic lapse rate, the environment is conditionally stable. An unsaturated air parcel moving up in this environment becomes colder than the surrounding air and sinks back down. However, a saturated air parcel moving up in this same environment is warmer than the surrounding air and continues to rise.

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Figure 8: Vertical profiles of the dry adiabatic lapse rate and environmental lapse rates for an unstable atmosphere; a stable atmosphere; and a conditionally stable atmosphere

stability is conditioned on whether the parcel itself is saturated or not. The other two designations refer to an environmental lapse rate which is greater than both the dry and moist adiabatic lapse rates (absolutely unstable) or one which is less than both the dry and moist adiabatic lapse rates (absolutely stable). These three conditions are shown in Figure 8.

At this point we now know how vertical motions can occur. Namely, air rises (or more precisely, 'continues to move in the direction of its initial motion') because it is unstable compared with the environmental air surrounding it. For a rising air parcel, this means that as it rises it is warmer and less dense than the environmental air around it and hence it will continue to rise against the force of gravity. Importantly, for a sinking air parcel that is unstable, it means that as the air parcel sinks it becomes colder and more dense than the environmental air around it; hence it will continue to sink. At the same time, it must be remembered that the temperature of the rising air parcel is not only determined by adiabatic processes (i.e. those related to work done by the parcel on its surroundings) but also by its moisture content and the release of latent heat to the parcel as water vapor condenses.

On a practical level, we typically associate rising air with warm conditions at the surface. How does this translate into the dynamics we just described? One way to think of this process is to realize that by warming the surface air (and not the air higher in the atmosphere), the surface temperature values (for instance in Figure 8) are shifted to the right. This essentially produces a more rapid decrease of temperature with height, i.e. a larger environmental lapse rate. At some point the environmental lapse rate will become greater than the dry (or moist) adiabatic lapse rate and the environment becomes unstable. At that point, warm air that starts at the surface and is initially lifted (by whatever process) will be cooler than when it was at the surface but warmer than the air around it, and hence will continue to rise, producing what we call <u>convection</u>. Note that convection can also be caused by a cooling of the air in the upper portion of the atmosphere, which again will tend to increase the environmental lapse rate.

Ex.6 For a particular location, the environmental lapse rate at night is 4K/km. If the temperature at 2km does not change during the day, how much does the surface temperature have to warm before convection begins?

Answer:

$$\begin{split} \gamma_{night} &= 5 \frac{K}{km} = \frac{T(0) - T(2km)}{2km} \Rightarrow T_{night}(2km) = T_{night}(0) - 2 \cdot 5 \\ \gamma_{day} &= \Gamma_d = 10 \frac{K}{km} \Rightarrow T_{day}(0) = T_{night}(2km) + 2 \cdot 10 = \left(T_{night}(0) - 10\right) + 20 \\ \Delta T &= T_{day} - T_{night} = 20 - 10 = 10K \end{split}$$

As air is forced upward through convection, we know that it is chilled by the adiabatic process. We also know that as the temperature decreases, the saturation specific humidity of the air parcel decreases. Hence if the air parcel is lifted high enough, and cools enough, the parcel can become saturated simply due to the change in its temperature. At this point, continued lifting leads to condensation of water vapor, cloud formation and, eventually, to precipitation (hence, the level at which saturation and initial condensation occurs is called the <u>lifting condensation level</u>). If the

convection is strong enough, thunderstorms can subsequently form. A thunderstorm is any storm that produces thunder and lightning. At the same time, thunderstorms can also produce high winds, hail, and tornadoes.

Thunderstorms can range from fairly isolated, short-lived storms, sometimes called air-mass thunderstorms, to massive, well-organized complexes of storms, called mesoscale convective systems. Next, we describe the different types of thunderstorms and the environmental conditions necessary for their formation.

Air-mass thunderstorms are isolated thunderstorms generated by daytime heating of the land surface. They occur when surface heating makes the environmental lapse rate—the vertical change in temperature of the surrounding air—unstable with respect to the dry and moist adiabatic lapse rates. At that point, isolated air masses can begin to rise through the air column. As they do so, they cool adiabatically. If the air masses rise high enough to reach the lifting condensation level (also called the convective condensation level), they form cumulus clouds. Continued lifting can subsequently result in enough condensation that precipitation begins to fall.

The typical life cycle of an air-mass thunderstorm involves three stages of development (Figure 9). The first of these is the cumulus stage. In this stage, initial air parcels near the surface are heated and begin to rise. The first air parcels may reach the lifting condensation level and form isolated cumulus clouds. However, these mix with the surrounding dry environmental air, and the cloud water droplets evaporate. This process cools the temperature of the air parcel and prevents it from rising further. However, this mixing adds water vapor to the surrounding environmental air. Then, as the next air parcel rises through the moister environment, its cloud droplets evaporate more slowly. Instead, condensation continues to occur, warming the air parcel and allowing the cloud to rise even higher. As the air parcels continue to rise, condensation continues until water drops become large enough to fall against the force of the updrafts, resulting in precipitation.

The cumulus stage is dominated by the presence of updrafts throughout the air column. It is the rising motion of the heated surface air that produces the updraft. At the same time, however, downdrafts also begin to form during this stage, marking the transition to the mature stage. Downdrafts can be initiated by drag exerted of the falling precipitation, the mixing (or entrainment) of cold, dry environmental air into the cloud, and evaporation of cloud water drops.

During this mature stage, the thunderstorm has formed into an organized convection cell and is at its most active. In one part of the cell, warm, moist air rises through the cooler, drier environmental air. As it does so, significant condensation occurs, releasing latent heat that allows the air parcel to continue to ascend. If enough latent heat is released, these air parcels can ascend 10–15 km to the tropopause. At that level, their ascent is limited by the strong temperature inversion of the tropopause. The cloud top then spreads laterally, forming an anvil cloud that extends downwind from the cumulus cloud. In the other part of the cell, there are significant downdrafts. These downdrafts—¬initiated by the drag of falling precipitation as well as the evaporation of the precipitation as it falls through the cold, dry environmental air—can be as strong as the updrafts.



Vertical motions are limited by mixing with cool, dry environmental air. Continuous mixing moistens the environment, allowing subsequent convection to ascend higher.

In this most intense stage, there are well-organized updrafts and downdrafts. Updrafts can reach the tropopause and then spread out as an anvil cloud. Downdrafts that reach the ground can reinforce convective lifting by producing cold surface outflow that forces warm surface air aloft.

#### Dissipation occurs when cool, dry environmental air is entrained by the rising air parcels. Subsequent sensible and latent cooling, combined with frictional drag imparted by falling rain, result in significant downdrafts that inhibit vertical circulations necessary to sustain convection.

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Figure 9: Stages in the development of an air-mass thunderstorm

The dissipating stage occurs when the stabilizing effects of entrainment overcome the destabilizing effects of convection. As continued mixing between the warm, moist rising air mass and the cool, dry surrounding environment occurs, widespread downdrafts form throughout the air column. These downdrafts inhibit the upward motion associated with convection. Without convection—and the associated condensation and latent heat release—the thunderstorm quickly dies out.

In all, air-mass thunderstorms can develop, mature, and dissipate over the course of an hour or so. Once the mature stage is reached—with its active regions of updrafts and downdrafts—the necessary environmental conditions that would allow them to overcome the stabilizing effect of entrainment are missing, and the storms quickly die out.

In contrast, other thunderstorms can persist for much longer and are called severe thunderstorms. By definition, severe



Figure 10: Anatomy of a severe thunderstorm

thunderstorms must have winds greater than 26 m/s (58 mi/hr). Alternatively, they must either have hail of a certain size (19 mm or 0.75 in. in diameter) or produce a tornado. Weather of this strength usually only accompanies a mature thunderstorm that has been active for many hours.

Severe thunderstorms persist longer than others because they develop an organized convection cell that allows for constant intake of warm, moist air, as seen in Figure 10. The only way to accomplish this development is to have air from outside the region begin to enter the thunderstorm. In that scenario, the downdrafts become so large that they spread out past the radius of the air column itself. As they spread, they force warm, moist air from surrounding regions to lift up and over the colder, denser air. This warm, moist air then flows back toward the thunderstorm and becomes part of the updraft. As long as there is the presence of warm, moist air that can be incorporated into the thunderstorm, it can continue to grow.

At the same time, the strong downdrafts must not inhibit the vertical motions associated with this convection. The interference can be prevented if wind shear—the change in winds with height—is significant. In that case, cool, dry air is entrained only on the upwind side of the convection cell, while warm, moist convecting air is positioned toward the downwind side by the strong winds aloft. This orientation allows the downdrafts to reach the surface without subsequently shutting down convection.

The most severe of these thunderstorms are called supercell thunderstorms. These are massive thunderstorms with a single circulation cell comprising very strong updrafts and downdrafts. Because of their vertical extent, which can be up to 25 km (16 mi), they are affected differently by winds at the surface and winds aloft. If the background wind shear not only involves a change in wind speed with height but also a change in direction—typically in a counterclockwise direction—a rotation of the storm can occur, which is a precursor to the formation of tornadoes.

### Section 3 - Stable Boundary Layer

As mentioned, there are two types of boundary layers—convective boundary layers and neutral (or stable) boundary layers. For neutral boundary layers mixing depends upon mechanical turbulence in the atmosphere, shown in Figure 11. This turbulence, and the thickness of the planetary boundary layer, are dependent upon factors like the surface roughness, wind speed,

topography, surface heating and even advection of heat and moisture by the mean winds.

We can quantify the susceptibility of the atmosphere to this type of mixing by looking at how turbulent motions are produced. For a neutral boundary layer, the source for turbulence is the kinetic energy of the mean wind in the free atmosphere. This kinetic energy is converted to turbulent energy because of the presence of vertical gradients in the velocity (wind shear), as shown in



8 2001 BrooksaCole PublishingTTP (a) Stable air (b) Unstable air Figure 11: Wind-speed can produce turbulence and mixing even in a neutral or stable boundary layer

Figure 12, which cause undulations that can then deform into eddies. The shear itself is produced by frictional forces applied to the atmosphere by the underlying surface. In addition to producing turbulence, this frictional force produces a strong momentum flux to the surface (called the <u>shear stress</u>), which effectively slows down the wind speed in the boundary layer, as seen in Figure 13. This figure shows the utility of planting trees around agricultural



Figure 12: Schematic showing how vertical wind-shear between two levels can lead to turbulence, even in a stable environment. ©Brooks/Cole

areas as a means of damping the surface winds in these regions. By reducing wind speeds in agricultural areas, shelterbelts limit soil erosion, particularly on exposed sandy or dry soils, wind chill factors, and sandblasting, while also enhancing recruitment during seeding.

It turns out that we can write the wind stress between two layers of the atmosphere is:

$$\tau = -\rho \upsilon \frac{\partial u}{\partial z}$$

This equation states that the wind stress is proportional to the vertical gradient in velocity, the density of the fluid, and the <u>kinematic viscosity</u> of air, v, which is a molecular property of a fluid that measures the internal resistance to deformation. This viscosity arises due to the fact that every fluid has an internal resistance to shearing, even air. In addition, every fluid also has a tendency to adhere to solid surfaces which are not moving.

It is important to emphasize again that the shear stress can also be considered a vertical flux of momentum, usually one in which momentum is removed from the winds and transferred to the surface. This alternative definition will arise again throughout this chapter.

Although very simple to write, this equation is actually very difficult to use - it presumes we know the wind shear as well as the kinematic viscosity at various levels and locations throughout the atmosphere. In addition, this equation determines the behavior of individual molecules; in

contrast we are interested in the largerscale behavior of air parcels. However, we can start by assuming that mechanical turbulence in the boundary behaves somewhat like molecular turbulence. Hence, we will continue to estimate the stress, or momentum flux in the atmosphere, based upon the gradient in wind:

Figure 13: Schematic showing how wind speed changes due to the effect of frictionally-induced turbulence in the boundary layer. ©Brooks/Cole

$$\tau_{bulk} = -\rho K_m \frac{\partial u}{\partial z}$$

Here,  $K_m$  is the <u>eddy viscosity</u> for momentum and determines how effectively variations in the wind field are re-distributed. In vector notation, it can be written as:

$$\vec{\tau}_{bulk} = -\vec{i}\,\rho K_m \frac{\partial u}{\partial z} - \vec{j}\rho K_m \frac{\partial v}{\partial z}$$

For this assumption to hold, we assume that air parcels move randomly and interact with other parcels, similar to the random motions of molecules which leads to conduction. Given these assumptions, it is possible to define a mixing length, *l*, which is the average distance a parcel travels before interacting with another parcel. The mixing length itself depends upon the size of the eddies, which in turn can be a function of all the variables we mentioned before. In addition, the mixing length is also related to the depth of the boundary layer although they are not exactly the same. Importantly, it can be shown that the eddy flux of momentum (and typically temperature as well as moisture) is related to the mixing length and the wind shear:

$$\tau = \rho l^2 \left| \frac{\partial u}{\partial z} \right| \frac{\partial u}{\partial z}$$

Hence, the stronger the wind shear and the larger the eddies (or boundary layer), the larger the flux of momentum and energy between the atmosphere and the surface

In addition, from this can equation it can be shown that the wind profile is:

$$|u(z)| = \left(\frac{1}{K}\right)\sqrt{(\tau/\rho)}\ln\left(\frac{z}{z_0}\right)$$

K- von Karmans Constant~0.4

z - Roughness length

where the momentum flux is just given by the shear stress described earlier. In addition, the roughness length is a parameter that characterizes the "roughness" of the underlying surface and can range from 1mm for smooth water to 1m for cities and forests. Note however that roughness length is *not* the same as mixing length and, although it qualitatively characterizes the roughness of the surface, it does not provide a quantitative measure of the height of the boundary layer or the scale of the obstacles on the surface.

Based upon the assumptions we have made, it can be shown that the shear stress is then:

$$\tau = \rho \left( \frac{\mathrm{K}}{\ln(z/z_0)} \right)^2 u(z)^2$$

This can also be written as:

$$\tau = \rho C_D u(z)^2$$

Here the drag coefficient,  $C_D$ , is related to the eddy viscosity and describes how efficiently momentum is transferred from the surface to the atmosphere and vice versa. Typically, we choose a reference height at which to make our measurements (for the atmosphere this is usually

2m or 10m), at which point we can define the drag coefficient as:

$$C_D = \left(\frac{\mathrm{K}}{\ln(z_{ref}/z_0)}\right)^2$$

By fixing this reference height, we assume the drag coefficient remains constant, allowing us to estimate the vertical fluxes of sensible heat, latent heat and momentum using only observations of the mean wind taken at a single height. In general the drag coefficient is also a function of stability. Over the oceans,  $C_d = 1.2x10^3$ . In comparison, over crops,  $C_d = 7.5x10^3$ , which is about 6 times as large as over oceans.

Ex. 3 What is the shear stress applied to the surface by a 10m/s wind blowing over the ocean? Over crops? Notice that the end result has units of pressure.

Answer: For the oceans

$$\tau = \rho C_D u(z)^2 = 1.2 \cdot (1.2 \times 10^{-3}) \cdot 10 = 0.14 \left(\frac{kg}{ms^2}\right)$$

For crops -

 $\tau = \rho C_D u(z)^2 = 1.2 \cdot \left(7.5 \times 10^{-3}\right) \cdot 10 = 0.9 \left(\frac{kg}{ms^2}\right)$ 

So far, we have shown that the momentum flux between the atmosphere and the surface is due to the fact that the moving air wants to pull the underlying surface with it. This in turn produces shear in the atmosphere, which then results in instability or turbulence. This dynamic instability, however, can be reduced by static *stability* associated with the temperature profile of the atmosphere. We can write the generation of turbulent kinetic energy (TKE) as:

$$\frac{\partial (TKE)}{\partial t} = MP + BP + TR - \varepsilon$$

MP - Mechanical production

BP - Buoyant production

TR - Redistribution

ε- Dissipation

Mechanical Production can be written as:

$$MP = -K_m \left| \frac{\partial u}{\partial z} \right| \frac{\partial u}{\partial z}$$

Buoyant production can be written as:

$$BP = -K_{H} \left(\frac{g}{T}\right) \left(\frac{\partial T}{\partial z}\Big|_{adiab} - \frac{\partial T}{\partial z}\Big|_{environ}\right)$$

Assuming the eddy coefficients are the same for momentum and heat, we can then define the

Richardson number as:

$$Ri = -\frac{BP}{MP} = \frac{g}{T} \frac{\left(\Gamma_d - \gamma_e\right)}{\left(\frac{\partial u}{\partial z}\right)^2}$$

Technically, this compares the rate of destruction of turbulent kinetic energy by buoyant forces to the rate of production of turbulent kinetic energy by mechanical forces. To maintain turbulence, the rate of production by shear has to be greater than the rate of destruction by buoyancy. We can now look at how these numbers compare. For Ri < 0, atmospheric turbulence is due to buoyant production (i.e. convection). For 0 < Ri < 0.25, there is also atmospheric turbulence, however it is due to mechanical production (i.e.

wind shear in a stable boundary layer). For Ri>1, atmospheric flow is laminar (smooth) with any mechanical turbulence being dampened by buoyant stability. Finally, for the condition 0.25<Ri<1, the atmosphere is laminar or turbulent depending on the prior state. As shown in Figure 14, during the day when the atmosphere is less stable there are larger eddies and more mechanical mixing; at night when the atmosphere is more stable, the buoyant dampening limits the size of the eddies and hence the depth of the mixed layer.



Figure 14: Vertical profile of wind-speed in the (a) nighttime, stable boundary layer and (b) daytime, stable boundary layer

Ex. 4 For an environmental lapse rate of 8K/km and an average temperature of 295K, would the atmosphere be stable for a wind shear of (2m/s)/m? (20m/s)/m?

Answer: For (2m/s)/m  

$$Ri = -\frac{BP}{MP} = \frac{g}{T} \frac{(\Gamma_d - \gamma_e)}{(\partial u/\partial z)^2} = \frac{9.8}{295} \frac{(10 - 8) \cdot (1000m/km)}{(2)^2} = 16.6$$
For (20m/s)/m-  

$$Ri = -\frac{BP}{MP} = \frac{g}{T} \frac{(\Gamma_d - \gamma_e)}{(\partial u/\partial z)^2} = \frac{9.8}{295} \frac{(10 - 8) \cdot (1000m/km)}{(20)^2} = 0.166$$

## Section 4 - Transfer of Sensible and Latent Heat in the Boundary Layer

In addition to producing a transfer of momentum from the surface to the atmosphere, the mechanical mixing associated with fluctuations in the winds also affects energy and humidity fields. BHL Hind Speed (nph)

<sup>65:00</sup> <sup>12:00</sup> <sup>12:00</sup> <sup>10:00</sup> <sup>67/12</sup> <sup>67/13</sup> <sup>67/13</sup> <sup>67/13</sup> <sup>67/13</sup> <sup>67/13</sup> <sup>67/14</sup> <sup>67/14</sup>

In general terms, the re-distribution—or vertical flux—of momentum, energy, or

moisture, is just the average vertical motion of energy, momentum or moisture over some time period. For momentum, this means we can re-write the wind stress as:

### $\tau = \rho \overline{wu}$

where w and u are the vertical and zonal wind speeds respectively, and the overbar indicates an average over some time.

Because vertical velocities and the scalar fields are constantly fluctuating, we have to average over a fairly long time in order to get a net transport. Typically, we assume that both the vertical velocity and scalar fields can be represented as a mean component and a fluctuating component, shown schematically in Figure 15:

u = u + u'

Importantly, if we take the time-mean of the deviation by itself, the result is identically zero:

### $\overline{u'} = 0$

However, this does not indicate that the mean of two deviations multiplied together necessarily has to be zero, as is shown in Figure 16. In this schematic of the redistribution of heat (as represented by temperature) associated with turbulent eddies in the boundary layer, we see that upward motion is associated with the movement of air parcels with higher than average temperatures. Multiplying these two positive numbers gives us a positive flux. Conversely, we find that downward motion is associated with the movement of colder than average air parcels. Multiplying these two negative numbers again gives a positive flux. Hence, for the situation shown here, there is a net flux of energy from the surface to the atmosphere all along the region of convection. Mathematically, we can write the average flux as:  $\overline{wT} = \overline{w} \cdot \overline{T} + \overline{w'T}$ 

We can do the same thing with the flux of momentum where the scalar field is now either the zonal wind field (u) or meridional wind field (v). Hence, the momentum flux, as represented by the wind stress, becomes:

$$\tau = \rho \left( \overline{w} \overline{u} + \overline{w' u'} \right) \quad \tau_{eddy} = \rho \overline{w' u'}$$

This is referred to as the <u>eddy correlation method</u> for determining fluxes based upon a fluctuating field. Typically, these fluctuations occur on the order of seconds to minutes, which is too

frequent for us to worry about when dealing with climate.

Instead we estimate turbulent fluxes by assuming an eddy diffusion process as we did with momentum. For example, from before we estimated the momentum flux as:

$$\tau = \rho C_D u(z)^2 = \rho C_D |u| u$$

Here, the last term on the right-hand side, u, actually comes from the term  $(u-u_{sfc})$ ,



Figure 16: Diagram showing how fluctuations of vertical velocities and temperatures within the boundary layer can produce a net flux of temperature from warmer layers (red) to cooler layers (blue)

however since the wind speed right at the surface is by definition zero we get the above equation. By extension, this equation tells us that the flux of momentum from the surface to the atmosphere is dependent

upon a drag coefficient, the overall wind speed, and the change in wind speed between the atmosphere and the surface. By analogy, we can approximate the change in sensible heat and latent heat in much the same manner:

$$SH = -\rho c_p C_H |u| (T(z) - T_{sfc})$$
$$LH = -\rho L C_q |u| (q(z) - q_{sfc})$$

In these equations, the transfer of sensible or latent heat is dependent upon the wind speed, as it was for the transfer of momentum. In addition, they are both related to the vertical gradient of the respective state variables (namely temperature and humidity). For these equations though, the flux of temperature is multiplied by the specific heat of air,  $c_p$ , to arrive at a value for the sensible heat exchange; likewise the flux of moisture is multiplied by the latent heat of vaporization, L, to arrive at latent heat exchange. As before, the equations also contain aerodynamic transfer <u>coefficients</u>,  $C_{H}$  and  $C_{a}$ , which are the equivalent of the drag coefficient for momentum. Finally, the transfer of sensible and latent heat is related to the density of fluid (air in this case) - the more dense the fluid, the more efficient it is at transferring energy from the surface to the atmosphere and vice versa. Overall, these two equations are called the Bulk Aerodynamic Formulae for estimating energy and moisture fluxes and represent the predominant way we estimate these fluxes on climatological time-scales. To arrive at them, we made two principal assumptions, namely that the horizontal wind-speed is proportional to the vertical flux of momentum via mechanical mixing (we assumed this before). In addition, we assumed that eddy diffusion is driven by gradients in temperature and moisture (for momentum, this is not assumed but is part of the definition of shear stress). As mentioned,  $C_{a}$ ,  $C_{b}$  are the aerodynamic transfer coefficients, which are analogous to the drag coefficient used for momentum. As with the drag coefficient,

these depend upon the surface roughness and the Richardson number. In addition, we typically assume that all three coefficients are the same, although this does not necessarily need to be the case.

Ex. 5 What is the transfer of sensible heat over crops if the average wind speed is 10m/s and the temperature difference between land and atmosphere is 1K? What is the transfer of latent heat if the humidity difference is 1g/kg? Use the transfer coefficient from Ex.3

Answer: For sensible heat

$$SH = -\rho c_p C_H |u| (T(z) - T_{sfc}) = 1.2 \cdot 1004 \cdot (7.5 \times 10^{-3}) \cdot 10 \cdot (-1)$$
$$SH = 90 \left(\frac{J}{m^2 s}\right)$$

For the latent heat-

$$LH = -\rho LC_q |u| (q(z) - q_{sfc}) = 1.2 \cdot (2.5 \times 10^6) \cdot (7.5 \times 10^{-3}) \cdot 10 \cdot (.001)$$
$$LH = 225 \left(\frac{J}{m^2 s}\right)$$

Now that we have ways to estimate the sensible and latent heating terms, we can start to investigate how excess energy (due to net radiation) is partitioned between the two. To measure this, we define the Bowen ratio as the ratio between the sensible heat fluxes and the latent heat fluxes:

B = SH/LH

We can write the sensible and latent heat fluxes as:

$$SH = -\rho c_p C_H |u| (T(z) - T_{sfc})$$
$$LH = -\rho L C_q |u| (q(z) - q_{sfc})$$

Assuming that the eddy coefficients are the same, the Bowen ratio becomes:

$$B = \frac{c_P}{L} \left( \frac{T(z) - T_{sfc}}{q(z) - q_{sfc}} \right)$$

If we then introduce the equation for specific humidity as a function of vapor pressure and assume a *saturated* underlying surface, we eventually find an equation for *E*, the actual evaporation:

$$E = \left(\frac{\Delta}{\Delta + \gamma}\right) \left(\frac{-F_{rad}^{net}}{L}\right) + \left(\frac{\gamma}{\Delta + \gamma}\right) E_a$$
  
$$\gamma = \frac{c_p}{L} \frac{P}{0.622} - \text{psychrometer constant}$$
  
$$\Delta = \frac{e_s(0) - e_s(z)}{T(0) - T(z)}$$

Here,  $e_s$  is the saturation vapor pressure for the temperature at the given height. In addition,  $E_a$  is the <u>drying power</u> of the atmosphere and represents the evaporation driven by a moisture deficit in the atmosphere:

 $E_a = \rho C_q |u| (q_s(z) - q(z))$ 

Ignoring the math for right now, the first term on the right-hand side of the evaporation equation represents evaporation due to excess radiative heating while the second term represents the evaporation driven by a moisture deficit in the atmosphere. From this equation we find that even if there is no excess heating, we can still have evaporative cooling of the surface due to a moisture deficit in the overlying atmosphere. In addition, if there is excess radiative heating, this will also drive evaporation (unless the overlying atmosphere has the same specific humidity as the air at the surface, which rarely occurs).

What happens if the underlying surface is not saturated? In this case, evaporation will tend to be limited by the amount of available soil moisture. However, we can still solve for the *potential evaporation*,  $E_P$ , which defines the evaporation that would occur if soil moisture was unlimited. By definition, this is simply the evaporation term we solved for above:

$$E_{P} = \left(\frac{\Delta}{\Delta + \gamma}\right) \left(\frac{-F_{rad}^{net}}{L}\right) + \left(\frac{\gamma}{\Delta + \gamma}\right) E_{a}$$

In this scenario  $E_P$ , becomes an indicator of available heat. When there is a limited quantity of water available for evapotranspiration the actual evaporation,  $E_A$ , is some fraction of potential evaporation:

$$E_{A} = \beta E_{P}$$

We can now define a critical soil moisture below which evaporation is limited

$$\beta = \begin{pmatrix} 1 \text{ for } S > S_c \\ S/S_c \text{ for } S < S_c \end{pmatrix}$$

Hence, for wet surfaces  $E_A = E_P = E_{P0}$ . For dry surfaces  $E_A - E_P = q(S)$  where q(s) is the potential evaporation that is not realized. Because this evaporation is not realized, energy that would have gone into latent heat goes instead into sensible heat. This increase in sensible heat increases the temperature

and hence potential evaporation (since an increase in temperature results in a higher saturation specific humidity). Hence, we find that increases in q(s)increases  $E_p$ . Therefore:  $E_p = E_{P0} + q(S)$ Adding the two equations together gives:  $E_A + E_p = 2E_{P0} = const$ . This produces the complimentary hypothesis, shown in Figure 17.



Figure 17: Hypothetical plot of actual evaporation ( $E_A$ ) and potential evaporation ( $E_P$ ) as a function of soil moisture

Essentially, the equation states that the amount of energy within the system is fixed. This energy is partitioned between actual evaporation and potential evaporation. As the available water becomes greater, the actual evaporation tends towards potential evaporation; however, given limited water, actual evaporation decreases and the energy goes into increasing potential evaporation, which in fact represents an increase in temperatures (since it is an increase in temperature which drives the potential evaporation). Importantly, this equation allows us to estimate the actual evaporation just by knowing the evaporation when the soil is saturated (which gives  $E_{P0}$ ) and the temperature (which gives  $E_p$ ). This method tends to be much easier than measuring the actual evaporation directly.

Ex. 6 If the evaporation rate for saturated soils is  $0.05g/(m^2s)$  at a given location, how much warmer will a 1000m-deep boundary layer be after a day if the evaporation is only  $0.02g/(m^2s)$ ?

Answer: For the potential evaporation that is not realized

$$q(S) = E_{P0} - E_A = 0.03 \left(\frac{g}{m^2 s}\right)$$

For the Latent heat associated with this missing evaporation -

$$LH = L_v E_P = 2.5 \times 10^6 \cdot 0.00003 = 75 \left(\frac{J}{m^2 s}\right) = SH$$

For the temperature change -

$$SH = 75 = \rho c_p dT = 1.2 \cdot 1004 \cdot dT$$
$$dT = 0.1660 \left(\frac{K \cdot m}{s}\right) \cdot \left[\frac{24 hr/d \cdot 3600 s/hr}{1000m}\right] = 5.4K$$