EC 718: Problem Set 1

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1. When we did Anscombe–Aumann, I asserted that the fact that $U(f)$ satisfied $U(\lambda f + (1-\lambda)g) = \lambda U(f) + (1-\lambda)U(g)$ implied that we could write U as

$$U(f) = \sum_{s \in S} \sum_{z \in Z} f(s)(z) u_s(z).$$

Prove this assertion.

2. Consider a Choquet expected utility maximizer whose nonadditive probability measure is $v(A) = 0$ for all $A \neq S$ and $v(S) = 1$. Show that such a person evaluates every act by the worst possible outcome it could yield. What MMEU representation is this equivalent to?

3. Suppose we extend the nonadditive probability model to two–player games in the following way. Let A_i be the set of actions for player i and $u_i : A_1 \times A_2 \to \mathbf{R}$ the payoff function, $i = 1, 2$. Let's define an equilibrium in nonadditive beliefs to be a pair of nonadditive probabilities (v_1, v_2), where v_i is a nonadditive probability measure on A_i, satisfying the following property. For all a_i such that $v_i(a_i) > 0$, a_i is a best reply for player i to v_j. That is,

$$\int u_i(a_i, a_j)\, dv_j(a_j) \geq \int u_i(a_i', a_j)\, dv_j(a_j), \quad \forall a_i' \in A_i$$

where this is the Choquet integral.

(a) Show that if v_1 and v_2 are additive, then this is equivalent to the usual definition of a mixed strategy equilibrium.

(b) Consider the following game.

	b_1	b_2
a_1	$10, 10$	$-10, 9$
a_2	$9, 10$	$9, 9$

What is the set of equilibria in additive probabilities? Show that (v_1, v_2) is an equilibrium where

$$v_1(a_1) = v_1(\emptyset) = 0, \quad v_1(a_2) = v_1(\{a_1, a_2\}) = 1$$

and

$$v_2(b_2) = v_2(\emptyset) = 0, \quad v_2(b_1) = \delta, \quad v_2(\{b_1, b_2\}) = 1$$

for any $\delta \leq 19/20$.

4.

(a) Suppose we define the support of a nonadditive probability v on a finite set S to be the set of $s \in S$ such that $v(\{s\}) > 0$, the definition implicit in Problem 7. Give an example of a nonadditive probability v whose support is empty.

(b) Suppose we define *a* support of a nonadditive probability v on a finite set S to be a set $\hat{S} \subseteq S$ such that $v(S \setminus \hat{S}) = 0$ and there is no proper subset of $\hat{S}$ with the same property. Give an example of a nonadditive probability v with more than one support according to this definition.