1. Consider the following three type version of the adverse selection model. The three types are $\theta_L$, $\theta_M$, and $\theta_H$ where $\theta_L < \theta_M < \theta_H$ and the marginal cost of effort is strictly decreasing in $\theta$. Determine which individual rationality and incentive compatibility constraints are binding. Characterize the optimal contract, giving first-order conditions where an explicit solution cannot be computed in general.

2. There are two types of people in the world, those with high income $I_h$ and those with low income $I_\ell$, $I_h > I_\ell > 0$. All people are risk neutral. The fraction of the population with high income is $\lambda$. The tax authority, the IRS, wishes to set an income tax and an auditing policy in order to maximize the amount of money it collects, minus its auditing expenses. (The IRS is risk neutral.) More precisely, the IRS sets a total tax to be paid for each income level, $T_h$ and $T_\ell$, and an audit probability for each income level, $A_h$ and $A_\ell$. If the IRS audits someone, they learn whether the person lied about their income. If so, they can take all the person’s income. If the audit shows that the person truthfully revealed income, then no penalty is imposed.

Auditing is costly. More specifically, if the IRS sets the probability of an audit to $A$, then they must pay $c(A)$ where $c(A)$ satisfies the following properties. First, $c(A) \geq 0$ for all $A \in [0,1]$. Second, $c'(A) > 0$ for all $A \in (0,1]$ with $c'(0) = 0$ and $c'(A) \to \infty$ as $A \to 1$. Finally, $c''(A) > 0$ for all $A \geq 0$. To be more specific about the timing, the IRS only sets the audit probability after income is reported.

Assume the IRS cannot tax anyone more than their income or else they will flee to Bermuda. Find the tax and audit probabilities which are optimal for the IRS. (Since you don’t have an explicit $c$ function, you will only be able to get a first-order condition for at least one of these variables, not an explicit solution.)

3. Suppose we modify the auction model discussed in class by assuming there are two objects for sale, not just one. These objects are identical. Bidder $i$’s value for one object is $\theta_i$ and has no value for a second object. The $\theta_i$’s are independent and identically distributed. Assume there are at least three bidders.
Consider the following auction: Each bidder submits a bid. The two highest bidders receive one of the objects. They each pay the third highest bid. Show that it is a dominant strategy for $i$ to bid his value.

4. Consider the independent private values auction model where each $\theta_i$ is uniformly distributed on $[0, 1]$. The auction is the “half–first–price, half–second–price” auction — that is, the high bidder gets the object and pays half his bid plus half of the second highest bid. Show that there is an equilibrium where each player bids a constant $a$ times his valuation and find $a$. 