1. Consider the moral hazard model from class but now assume that the principal is risk averse in wealth. How does this change the first-order conditions characterizing the optimal contract for the case where the agent is risk averse and effort is not observed?

2. Consider the moral hazard problem with three possible effort levels $E = \{e_1, e_2, e_3\}$ and two possible profit levels, $\pi_H = 10$ and $\pi_L = 0$. Let the probability of $\pi_H$ given $e$ be:

   $$f(\pi_H \mid e) = \begin{cases} 
   2/3, & \text{if } e = e_1 \\
   1/2, & \text{if } e = e_2 \\
   1/3, & \text{if } e = e_3 
   \end{cases}$$

   The agent’s cost function for effort is

   $$g(e) = \begin{cases} 
   5/3, & \text{if } e = e_1 \\
   8/5, & \text{if } e = e_2 \\
   4/3, & \text{if } e = e_3 
   \end{cases}$$

   The agent’s utility of wealth function is $v(w) = \sqrt{w}$. His reservation utility $\bar{u}$ is zero.

   (a) What is the optimal contract when effort is observable?

   (b) Show that if effort is not observable, then it is not possible to induce effort level $e_2$. (Notational suggestion: Let $v_1 = v(w(\pi_H))$ and $v_2 = v(w(\pi_L))$.) For what values of $g(e_2)$ can the principal induce effort level $e_2$?

   (c) What is the optimal contract when effort is not observable?

3. Suppose both the principal and agent are risk neutral. The agent has two possible levels of effort, $e_H$ and $e_L$. There is a probability that the project the agent is hired to carry out succeeds or fails where this probability depends on the agent’s effort. More specifically, if the agent chooses effort $e_H$, the project succeeds with probability 1. If he chooses $e_L$, the project succeeds with probability $p \in (0, 1)$ and fails otherwise. If the project succeeds, then profits are $\pi_S$. If it fails, profits are $\pi_F$ where $\pi_S > \pi_F \geq 0$. 

The agent’s payoff is $w - c$ if he chooses $e_H$ and is paid $w$, while his payoff is $w$ if he chooses $e_L$ and is paid $w$. The principal’s payoff is $\pi - w$ if profits are $\pi$ and he pays the agent $w$. The agent’s outside option is 0.

(a) Find an optimal contract for the principal.

(b) Now assume there is a limited liability constraint meaning that $w$ must be non-negative. Find an optimal contract for the principal.

4. Consider a principal who earns profits from two different tasks undertaken by the agent. His profits from task 1, denoted $\pi_1$, depend randomly on the level of effort undertaken by the agent in task 1. More specifically, if the agent puts high effort $e_H$ into task 1, then

$$\pi_1 = \begin{cases} A & \text{with probability } 1/2 \\ 0 & \text{otherwise} \end{cases}.$$

If the agent puts low effort ($e_L$) into task 1, then $\pi_1 = 0$ with probability 1. The principal’s profits from task 2, $\pi_2$, are a nonstochastic function of the agent’s effort on task 2. More specifically, if the agent puts high effort into task 2, then $\pi_2 = B$, while if he puts in low effort, $\pi_2 = C$ where $B > C$. The agent can only put high effort into at most one task. That is, the only possibilities are low effort on both tasks, $e_H$ on task 1 and $e_L$ on task 2, or $e_H$ on task 2 and $e_L$ on task 1. The agent’s payoff if he is paid $w$ and puts high effort into one of the tasks is $\sqrt{w} - g$, while his payoff if he puts low effort into both tasks and is paid $w$ is $\sqrt{w}$. The agent’s payoff if he does not work for this principal is 0. The principal is risk neutral.

Finally, assume that $A/2 > B - C > g^2$. In other words, the expected marginal value of high effort on task 1 is greater than the marginal value of high effort on task 2 and both are greater than the utility cost of high effort.

(a) What is the first–best contract?

(b) Now suppose that the principal observes both $\pi_1$ and $\pi_2$ but not the agent’s effort choice. Find the cost–minimizing contracts for inducing each possible effort choice. Under what conditions is the effort choice changed from the first–best?