Instructions: Answer TWO of the following questions. The questions are equally weighted. None of the questions are intentionally misleading, so if you are not sure what the question is looking for, please ask me.

1. Consider the following model of auctions with resale. There are two bidders, 1 and 2. There is a single indivisible object for sale. The object is worth $\theta_i$ to bidder $i$. The common prior is that $\theta_i \sim U[0, 1]$ where $\theta_1$ and $\theta_2$ are independent.

We model resale in the simplest possible way. Suppose bidder $i$ wins the auction but $\theta_j > \theta_i$. Then bidder $i$ sells the good to bidder $j$ at a price of $\theta_j$. (To be clear, this part of the game is taken as exogenous — we don’t analyze how bidder $i$ learns $\theta_j$ in order to set this price.) On the other hand, if $\theta_i > \theta_j$, then there is no resale — bidder $i$ keeps the object.

Suppose the seller uses a first price auction. Suppose bidder $i$ believes that bidder $j$’s strategy is to bid $\theta_j/2$.

(a) Show that for bidder $i$, the best bid less than or equal to $\theta_i/2$ is $\theta_i/2$.

(b) Show that for bidder $i$, the expected payoff to any bid greater than or equal to $\theta_i/2$ is the same as the expected payoff to bidding $\theta_i/2$.

(c) Conclude that it is a Nash equilibrium for each bidder to bid half his value.
2. Consider two agents, 1 and 2, in a public goods setting. There is a public good which can either be provided or not. The cost of providing the good is 1/2. The value to $i$ of having the public good is $\theta_i$ where the common prior is that the $\theta_i$’s are iid $U[0,1]$. More specifically, if the outcome is that the public good is provided and $i$ receives a transfer of $t_i$, then $i$’s payoff is $\theta_i + t_i$. If the public good is not provided and $i$ receives a transfer of $t_i$, then $i$’s payoff is $t_i$. As usual, the interesting case is where the agents pay money so that the transfers are negative.

(a) Suppose the public good is determined by the following procedure. The agents simultaneously make contributions toward the public good. Let $s_i$ be the contribution chosen by $i$ where $s_i \in [0, \infty)$. If $s_1 + s_2 < 1/2$, then the public good is not provided and the agents keep their money. If $s_1 + s_2 \geq 1/2$, the public good is provided and each agent pays his contribution. In particular, if $s_1 + s_2 > 1/2$, the government keeps the extra payments of $s_1 + s_2 - 1/2$. Show that there is an equilibrium of this mechanism where $\sigma_i^*(\theta_i) = \alpha \theta_i$ for $\alpha > 0$ and find $\alpha$.

(b) Let $y(\theta_1, \theta_2)$ denote the probability the public good is provided and $t_i(\theta_1, \theta_2)$ the transfer received by agent $i$ as a function of $(\theta_1, \theta_2)$. What are these functions in this equilibrium?

(c) What is the government’s expected “profit”? Recall that the government keeps contributions in excess of the cost of provision if the contributions are at least equal to the cost. So how much does the government get in expectation? (Suggestion: Compute the expected receipts of the government and then subtract the expected costs.)
3. A consumer with wealth 100 faces a risk of losing 64. If he is cautious, the loss occurs with probability 1/2. If he is not cautious, the loss occurs with probability 1. His utility function for wealth $w$ is $\sqrt{w}$. Caution costs him one unit of utility, so if he is cautious, his expected utility is $(1/2)\sqrt{100} + (1/2)\sqrt{100} - 64 - 1$, while if he is not cautious, his expected utility is $\sqrt{100} - 64$.

(a) Suppose it is impossible to buy insurance, so the consumer’s only decision is whether to be cautious or not. Will the consumer be cautious? What will his utility be?

(b) Now suppose a risk-neutral insurance company offers insurance against this kind of loss. The insurance company can make the transfer to the consumer a function of whether or not a loss occurs (as it sees this) but not whether or not the consumer was cautious. Specifically, an insurance contract is a pair of numbers $(t_s, t_L)$ where $t_s$ is what the company pays the consumer when he is “safe” (that is, there is no loss) and $t_L$ is what the company pays when there is a loss. These numbers can be positive or negative. Given that the insurance company cannot force the consumer to accept the contract, what is the optimal contract for them to offer? Be sure to explain which constraints are binding and how you know this. What is the insurance company’s expected profit and the consumer’s expected utility?