Final Examination

Economics 703

Spring 2016

Instructions: Answer TWO of the following questions. The questions are equally weighted. None of the questions are intentionally misleading, so if you are not sure what the question is looking for, please ask me.

1. Consider two agents, 1 and 2, in a public goods setting. There is a public good which can either be provided or not. The cost of providing the good is 1. The value to \( i \) of having the public good is \( \theta_i \) where the common prior is that the \( \theta_i \)'s are iid \( U[0, 1] \). More specifically, if the outcome is that the public good is provided and \( i \) receives a transfer of \( t_i \), then \( i \)'s payoff is \( \theta_i + t_i \). If the public good is not provided and \( i \) receives a transfer of \( t_i \), then \( i \)'s payoff is \( t_i \). As usual, the interesting case is where the agents pay money so that the transfers are negative.

To specify a mechanism, let \( y(\theta_1, \theta_2) \) be the probability the good is provided as a function of the types and let \( t_i(\theta_1, \theta_2) \) be the transfer to \( i \) as a function of the types.

Suppose the mechanism is efficient in the sense that

\[
y(\theta_1, \theta_2) = \begin{cases} 
1, & \text{if } \theta_1 + \theta_2 > 1; \\
0, & \text{otherwise}
\end{cases}
\]

and is feasible in the sense that

\[
-\sum_i t_i(\theta_1, \theta_2) = 1 \text{ if } \theta_1 + \theta_2 > 1
\]

and

\[
t_i(\theta_1, \theta_2) = 0, \forall i \text{ if } \theta_1 + \theta_2 \leq 1.
\]

(a) Find the expected value of payment made by agent \( i \). That is, \( E_\theta(-t_i(\theta_1, \theta_2)) \).

(b) Use your answer to (a) to show that there is no such mechanism.
2. A principal hires an agent to work for him. The agent can be one of two types, $\theta_H$ or $\theta_L$, where the probability of each type is 1/2. For concreteness, assume $\theta_H = 2$ and $\theta_L = 1$. The principal must hire this agent (so the individual rationality constraint must be satisfied for both types). The agent’s outside opportunity is worth 0. The agent’s utility function if he receives wage $w$, puts in effort $e$, and has productivity $\theta$ is $w - (e/\theta)$. The principal’s utility in the same situation is $2\sqrt{e} - w$. Effort can be any number in $\mathbb{R}_+$. The agent knows his type, but the principal does not observe it.

(a) What is the optimal contract?

(b) Now suppose that the principal cannot commit to not renegotiate the contract. More specifically, the principal has two choices. First, he can offer only one wage and effort pair so both types of agent end up with the same effort and wage. Alternatively, he can offer two different wage–effort pairs, one for each type, but must offer a pair that won’t be renegotiated. I.e., the contract must be ex post efficient. What is the optimal contract for the principal now?
3. A worker has two possible types, $\theta_h$ and $\theta_\ell$. There are two possible employers who do not observe the worker’s type. If employer $i$ hires the worker at a wage of $w$, his payoff is $100 - w$ if the worker is type $\theta_h$ and $40 - w$ if the worker is type $\theta_\ell$. An employer gets 0 if he doesn’t hire the worker.

The worker’s payoff depends on his level of education, his type, and his wage. Specifically, education makes the worker enjoy his job more, but costs him. For either type, if he chooses education $e$, his “extra enjoyment” of his job is $4\sqrt{e}$. The cost of education is $c(e, \theta)$ where

$$c(e, \theta) = \begin{cases} 
  e/5, & \text{if } \theta = \theta_h; \\
  2e/5, & \text{if } \theta = \theta_\ell.
\end{cases}$$

Summarizing, the worker’s payoff if he gets wage $w$, chooses education $e$, and is type $\theta$ is $w + 4\sqrt{e} - c(e, \theta)$.

The time line is as in Spence. The worker knows his type and chooses $e$. The employers observe $e$ but not $\theta$, then offer wages. The worker sees the two wages offered and picks an employer to work for.

Find all separating equilibrium strategies for the worker. I.e., what is the set of $(e_\ell, e_h)$ such that there is a strategy for the employers giving us a separating equilibrium?

Hint: You don’t need to give explicit values for the range of possible education levels — implicitly (but clearly!) defined ranges are fine.