Instructions: Answer any TWO of the following questions. None of the questions are intentionally misleading, so if you are not sure what the question is looking for, please ask me.

1. A consumer with wealth 100 faces a risk of losing 64. If he is cautious, the loss occurs with probability 1/2. If he is not cautious, the loss occurs with probability 1. His utility function for wealth $w$ is $\sqrt{w}$. Caution costs him one unit of utility, so if he is cautious, his expected utility is $(1/2)\sqrt{100} + (1/2)\sqrt{100} - 64 - 1$, while if he is not cautious, his expected utility is $\sqrt{100} - 64$.

(a) Suppose it is impossible to buy insurance, so the consumer’s only decision is whether to be cautious or not. Will the consumer be cautious? What will his utility be?

(b) Now suppose a risk-neutral insurance company offers insurance against this kind of loss. The insurance company can make the transfer to the consumer a function of whether or not a loss occurs (as it sees this) but not whether or not the consumer was cautious. Specifically, an insurance contract is a pair of numbers $(t_s, t_L)$ where $t_s$ is what the company pays the consumer when he is “safe” (that is, there is no loss) and $t_L$ is what the company pays when there is a loss. These numbers can be positive or negative. Given that the insurance company cannot force the consumer to accept the contract, what is the optimal contract for them to offer? Be sure to explain which constraints are binding and how you know this. What is the insurance company’s expected profit and the consumer’s expected utility?
2. Suppose we have $I$ bidders in an independent private values auction. The value of the good to bidder $i$, $\theta_i$, is distributed uniformly on $[0, 1]$, iid across bidders. Without using things we know about first or second price auctions, compute the seller’s expected revenue for any auction with the property that the good always goes to the bidder with the highest value and any bidder whose value is 0 has an expected payoff of 0.

3. A principal hires an agent who is either a low type ($\theta_L$) or a high type ($\theta_H$), where the agent’s types are equally likely. If the agent is the low type, his cost of effort $e$ is $\alpha e$, while if he is the high type, his cost of effort $e$ is $e$. To be consistent with the interpretation, we require $\alpha \geq 1$.

   The principal’s profits (before paying the wage) if the worker puts in effort $e$ are given by $2\sqrt{e}$. The principal does not know the agent’s type, but the agent knows his own type. The outside option for either type of agent is 0. If the agent puts in effort $e$ and is paid wage $w$, his utility is $w$ minus his effort cost as defined above, while the principal’s utility is $2\sqrt{e} - w$.

   (a) Find the optimal contract for the principal as a function of $\alpha$.

   (b) Now suppose we can choose $\alpha$ subject to the constraint that $\alpha \geq 1$. What is the $\alpha$ that maximizes the high type’s payoff? The low type’s payoff? The principal’s payoff?