Econ 701: Extra Problems

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1. In class, we discussed how to relate objects in producer theory to analogous ones in consumer theory. Use these ideas to prove each of the following:

(a) The cost function $c(w, q)$ is homogeneous of degree 1 in w.

(b) $D_w c(w, q) = z(w, q)$ where $z(w, q)$ is the vector of conditional factor demands.

(c) $c(w, q)$ is concave in w.

(d) The conditional factor demands, $z(w, q)$, are homogeneous of degree 0 in w.

(e) The unconditional factor demands $z(p, w)$ are homogeneous of degree 0 in (p, w).

(f) The profit function is convex in (p, w).

(g) Show that aggregate demand depends only on aggregate wealth if all consumers have Gorman form indirect utility functions.

Note: (a) through (f) are intended to get you accustomed to the relationship between producer and consumer theory but all are also a good illustration of the kind of short proofs that might be asked for on the exam.

2. Suppose we observe that a firm's costs are given by the function $c(w, q)$. Suppose we try to estimate the set of input combinations sufficient to produce q by the set

$$V_q = \{z \in \mathbf{R}_+^{L-1} \mid w \cdot z \geq c(w, q), \quad \forall w \gg 0\}.$$

Under what conditions on the cost function will this turn out to be an appropriate estimate? Prove your answer. (Yes, you should explain what "appropriate" must mean in this context.)

3. Suppose $X = \mathbf{R}_+^2$ and consider the preference defined by $(x_1, x_2) \succ (x_1', x_2')$ iff one of the following holds: either (1) $x_1 > x_1'$ *and* $x_1' < 10$ or (2) $10 \geq x_1 = x_1'$ *and* $x_2 > x_2'$ or (3) $x_1 \geq 10$, $x_1' \geq 10$, *and* $x_2 > x_2'$.

Find the demand functions $x_1(p_1, p_2, w)$ and $x_2(p_1, p_2, w)$ for this preference. Verify the symmetry restrictions implied by the Slutsky equation in the range of (p, w) for which the demands are differentiable.

4. Consider a consumer with expected utility preferences with $u(x) = \log(x)$. He has initial wealth W. With probability 1/2, he loses L. Otherwise, there is no loss. He can buy insurance at p per unit. A unit of insurance pays him \$1 in the event of a loss. (So to make up the entire loss, he'd have to have L units of insurance.) Compute his demand for units of insurance as a function of p, W, and L. Show that he fully insures if $p = 1/2$. Is his demand increasing or decreasing in L? What can you say about how changes in W affect the demand for insurance?

5. Suppose a consumer's expenditure function is

$$e(p, u) = u\sqrt{p_1 p_2}.$$

Find the indirect utility function, the Hicksian demands, and the Walrasian demands. Verify the symmetry restrictions implied by the Slutsky equation.

6. Let L_1 be a lottery which pays \$10 with probability 1/2 and \$100 otherwise. Let L_2 be a lottery which pays \$10 with probability 3/4 and \$200 otherwise. Finally, let L_3 be a lottery which pays \$10 with probability 5/8, \$100 with probability 1/4, and \$200 with probability 1/8. Suppose we have a consumer for whom $L_3 \succ L_1 \succ L_2$. Could this consumer's preferences have the expected utility property? If so, give values of $u(10)$, $u(100)$, and $u(200)$ that would work. If not, state which assumption is violated.