Econ 701: Answers to Problem Set 6

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1. a) This is an immediate implication of Jensen's inequality. The policy is actuarially fair iff $f = pL$ where f is the cost of the insurance. So the consumer prefers the policy iff

$$pu(W - L) + (1 - p)u(W) < u(W - f) = u(W - pL).$$

But the fact that $p(W - L) + (1 - p)W = W - pL$ and that the consumer is risk averse implies that this must hold.

b) This, again, is an implication of Jensen's inequality. Having a policy which overinsures or underinsures still leaves some risk in wealth which the consumer would prefer to avoid. More specifically, the policy in (a) is preferred to purchasing C units for $C \neq L$ iff

$$pu(W - L + C - pC) + (1 - p)u(W - pC) < u(W - pL).$$

But notice that

$$p(W - L + C - pC) + (1 - p)(W - pC) = W - pC - pL + pC = W - pL.$$

Hence the expected value of the gamble (not surprisingly) is the same as what he gets for sure with the full insurance policy $C = L$. Therefore, Jensen's inequality immediately implies that this holds.

c) Because the company observes the choice of e, the price of insurance will end up being $p(e)$. So we know from (b) that given the effort the consumer chooses and the price the insurance company offers, the consumer will fully insure. That is, the consumer will purchase L units of insurance at price $p(e)$. Hence the consumer's utility will end up being $U(W - p(e)L - c(e))$. So the first–order conditions are

$$U'(W - p(e)L - c(e))(-p'L - c') = 0$$

or $c' = -p'L$. Hence the consumer chooses his effort as if he were risk neutral since this is what a risk neutral consumer would do.

2. a) Simply calculating, we have

$$R_2(w) = \frac{-u_2''(w)}{u_2'(w)}.$$

But

$$u_2'(w) = g'(u_1(w))u_1'(w)$$

and

$$u_2''(w) = g''(u_1')^2 + g'u_1''$$

so that

$$R_2(w) = -\frac{g''(u_1')^2}{g'u_1'} + R_1(w).$$

Since g is concave, $g'' < 0$. Since g is increasing, $g' > 0$. So (as long as $u_1' > 0$), $R_2(w)$ is some positive number plus $R_1(w)$ and hence is strictly larger than $R_1(w)$.

b) Let π_i denote the risk premium for person i (where we keep in mind that this is a function of w and the specific gamble considered). Then by definition,

$$u_i(E(w+z) - \pi_i) = Eu_i(w+z)$$

where w is initial wealth, z is the gamble, and we take the gamble to be additive for simplicity. Clearly, we will have to have $\pi_2 > \pi_1$ if it is true that

$$u_2(E(w+z) - \pi_1) > Eu_2(w+z).$$

To see that this in fact will hold, notice that it holds if

$$g(u_1(E(w+z) - \pi_1)) > Eu_2(w+z)$$

But by definition of π_1, this is true if

$$g(Eu_1(w+z)) > Eu_2(w+z) = Eg(u_1(w+z)).$$

The fact that z is a random variable makes $w+z$ a random variable and hence makes $u_1(w+z)$ a random variable. Denote this random variable by x and notice that this statement is just $g(E(x)) > Eg(x)$ which must hold for any concave function g by Jensen's inequality.

c) Essentially nothing. We could easily have increasing absolute risk aversion for example, and compare these two people when 1 has a lot more wealth than 2. In this case, 1 could be more risk averse than 2 if we compare their Arrow–Pratt measures at the levels of wealth they actually have.

3. With the utility function given (which is additively separable), the cross partials are zero. Hence the function is strictly concave iff both f_1 and f_2 are strictly concave.

But then notice that insurance simply induces risk in your income. Jensen's inequality immediately implies that the consumer would not purchase insurance. That is, without insurance, the consumer's expected utility is

$$pf_1(\bar{x}) + (1-p)f_1(\underline{x}) + f_2(x_2)$$

where x_2 is initial wealth. With insurance, the first two terms in this sum do not change and we replace the third term with $pf_2(x_2 - (1-p)C) + (1-p)f_2(x_2 + C - (1-p)C)$. (That is, when you're healthy, you just pay your premium of $(1-p)C$ and when you're not healthy, you pay your premium but get your compensation of C.) But this change is desirable only if

$$f_2(x_2) < pf_2(x_2 - (1-p)C) + (1-p)f_2(x_2 + pC).$$

The expected value of your wealth given insurance is just x_2 so that the strict concavity of f_2 immediately implies that this cannot hold by Jensen's inequality. Intuitively, if money does not act, at least partially, as a substitute for health, you'd rather not insure. If money and health are not substitutes at all, insuring simply adds risk to your income without offsetting in any real sense the risk to your health.