

Econ 701: Problem Set 6

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Fall 2009

Due: Tuesday October 20

1. (a) Consider a consumer with initial wealth W who is risk averse and faces a probability p of a loss L . The consumer can insure himself against this risk at an actuarially fair premium f . Show that the consumer will buy the insurance.

(b) Now suppose that the consumer can buy any amount of insurance. Specifically, if the consumer buys C units of insurance, he pays fC and, in the event of a loss, receives compensation of C dollars. Assume that f is actuarially fair in the sense that $f = p$. Show that the consumer will necessarily find $C = L$ optimal.

(c) Now suppose that the probability of a loss depends on how much effort the consumer takes in loss prevention. Specifically, the probability of a loss is a function $p(e)$ where $e \in [0, \infty)$ is the effort undertaken by the consumer. Assume $p' < 0$ and that $c(e)$ is the utility cost of effort by the consumer where $c' > 0$. (So overall utility is the expected utility associated with the risk minus $c(e)$.) Suppose that the consumer chooses e . Afterward, the insurance company observes e and offers the consumer his choice of the number of units of insurance to buy at an actuarially fair price of $f = p(e)$. Then the consumer decides how much insurance to buy. Characterize the level of effort the consumer will take.

2. Suppose we have two consumers, 1 and 2, with utility functions u_1 and u_2 respectively. Suppose that 2's utility function is more concave than 1's in the sense that $u_2(w) = g(u_1(w))$ for all w for some strictly increasing, strictly concave function g .

(a) Show that 2's Arrow-Pratt measure of risk aversion is higher for every w .

(b) Show that for any gamble, 2's risk premium is larger than 1's if they have the same initial wealth.

(c) What can we conclude about the relationship between the two risk premiums if they have different initial wealths?

3. Suppose there are two goods, money and health. Letting w denote money and h level of health, consider an agent whose utility function is $f_1(w) + f_2(h)$. This agent is risk averse in both goods. The agent faces a risk to his health. Specifically, with probability p , the agent has low level of health \underline{h} , while with probability $1 - p$, he has a high level of health $\bar{h} > \underline{h}$. An insurance company offers to sell this consumer a policy which pays him C of compensation if his health is \underline{h} and nothing otherwise. Would the consumer be willing to purchase this policy if the price is actuarially fair?