

Econ 701: Problem Set 5

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Due: Tuesday October 13

1. Assume \succeq is a weak order over lotteries over a finite prize set Z satisfying the independence axiom. Show that the following two axioms are equivalent:

Axiom 1 For all p, q and r satisfying $p \succ q \succ r$, there exists $\alpha, \beta \in (0, 1)$ such that $\alpha p + (1 - \alpha)r \succ q \succ \beta p + (1 - \beta)r$.

Axiom 2 For all p, q , and r , the sets

$$\{\alpha \in [0, 1] \mid \alpha p + (1 - \alpha)r \succeq q\}$$

and

$$\{\alpha \in [0, 1] \mid \alpha p + (1 - \alpha)r \preceq q\}$$

are closed.

2. Prove that the axioms we gave in class for the expected utility representation imply each of the following. Note: You can use results we established in class, but be careful not to prove these claims using results we proved under the hypothesis these were true.

(a) $p \sim q$ iff $\alpha p + (1 - \alpha)r \sim \alpha q + (1 - \alpha)r$ for all $\alpha \in [0, 1]$ and all r .

(b) $p \sim q$ iff $\alpha p + (1 - \alpha)r \sim \alpha q + (1 - \alpha)r$ for all $\alpha \in (0, 1)$ and all r .

(c) $p \succeq q$ iff $\alpha p + (1 - \alpha)r \succeq \alpha q + (1 - \alpha)r$ for all $\alpha \in (0, 1)$ and all r .

(d) $p \succeq q, r \succeq s$ implies $\alpha p + (1 - \alpha)r \succeq \alpha q + (1 - \alpha)s$ for all $\alpha \in [0, 1]$.