

Econ 701: Problem Set 3

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1. A function f is homothetic if it can be written as $f(x) = h(g(x))$, where $h' > 0$ and g is homogeneous of degree 1. Suppose $X = \mathbf{R}_+^2$ and that $u(x_1, x_2)$ is homothetic.

(a) Show that the indifference curves are “parallel” in the sense that, along any ray from the origin, the slopes of the indifference curves the ray hits are the same. (Hint: If f is homogeneous of degree r , then any first partial derivative is homogeneous of degree $r - 1$.)

(b) Show that the income expansion path (that is, for a fixed p , $\{x(p, w) \mid w > 0\}$) is a line.

(c) Show that the income elasticity of demand for either good is 1.

2. Consider the utility function $u(x_1, \dots, x_n) = x_1 + U(x_2, \dots, x_n)$. (This functional form is called *quasi-linear*.) Show that $x_2(p, y), \dots, x_n(p, y)$ are all independent of w if w is sufficiently large. What is meant by “sufficiently large” here? Show that the indirect utility function can be written as $v(p, w) = \phi(p) + (w/p_1)$ for some function ϕ .

3. Suppose we have an agent who derives utility both from consumption (as we’ve assumed up till now) and also wealth w . That is, this consumer chooses x to maximize $u(x, w)$ (subject to the same constraints as before). More specifically, suppose $X = \mathbf{R}_+^2$ and $u(x, w) = x_1^{\alpha_1} x_2^{\alpha_2} w^{1-\alpha_1-\alpha_2}$ where $\alpha_\ell > 0$ for $\ell = 1, 2$, and $\alpha_1 + \alpha_2 < 1$. Derive the Walrasian demands and indirect utility function. Is the Lagrange multiplier still the marginal utility of wealth? Explain your answer.

4. For this problem, assume $X = \mathbf{R}_+^2$.

(a) Consider the lexicographic preferences with $(x_1, x_2) \succ (y_1, y_2)$ if $x_1 > y_1$ or if $x_1 = y_1$ and $x_2 > y_2$. Find the Walrasian demands and show that they are continuous in (p, w) .

(b) Do the same for the lexicographic preferences with $(x_1, x_2) \succ (y_1, y_2)$ if $x_1 > y_1$ or if $x_1 = y_1$ and $x_2 < y_2$.

5. Suppose the consumer's utility function is

$$u(x) = \left(\sum_{i=1}^n x_i^\alpha \right)^{1/\alpha}.$$

a) For what values of α will the preferences satisfy strong monotonicity and convexity?

b) Find the Walrasian and Hicksian demands and the indirect utility function and expenditure function.

c) Verify Roy's Identity and the Slutsky equation.

6. Suppose

$$v(p_1, p_2, w) = \frac{w}{p_1} + \frac{w}{p_2}.$$

Find the Walrasian and Hicksian demands and the expenditure function.

7. Suppose we have an economy of consumers all with the same homothetic preferences and income. There are two goods and good 1 is supplied by a monopolist who produces the good at zero cost. Suppose each consumer chooses $x_1 = 10$ and $x_2 = 5$ and suppose $p_1 = p_2 = 1$. Finally, assume that preferences are monotonic and the demand functions are differentiable. Find all six demand elasticities. That is, find all four price elasticities $((p_i/x_j)(\partial x_j/\partial p_i), i, j = 1, 2)$ and both income elasticities $((w/x_i)(\partial x_i/\partial w)$ for $i = 1, 2)$.