

Econ 701: Problem Set 2

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Due: Tuesday September 22

1. (Postponed from the last problem set.) We discussed three different definitions of continuity in class, two of which Rubinstein shows are equivalent. Show that the third, the one from MWG, is equivalent to the other two for a weak order on $X = \mathbf{R}_+^L$. (You can take as the result in Rubinstein that his two definitions are equivalent.)

The first definition from Rubinstein: Whenever $x \succ y$, there exists $\varepsilon_x > 0$ and $\varepsilon_y > 0$, such that for every x' within ε_x of x and every y' within ε_y of y , $x' \succ y'$.

The second from Rubinstein: If (x_n, y_n) is a sequence converging to (x, y) with $x_n \succeq y_n$ for all n , then $x \succeq y$.

The alternative definition: For each $y \in X$, $\{x \in X \mid x \succeq y\}$ and $\{x \in X \mid y \succeq x\}$ is closed. That is, if $x_n \rightarrow x$ with $x_n \succeq y$ for all n , then $x \succeq y$ and similarly on the other side.

Hint: I suggest using the fact that X is connected.

2. Show that if u represents \succeq and u is continuous, then \succeq is continuous.

3. Show that Z is connected if for every pair of nonempty *closed* sets A and B such that $Z = A \cup B$, we have $A \cap B \neq \emptyset$.

4. Prove each of the following statements:

(a) A monotonic transformation of a quasiconcave function is quasiconcave.

(b) Preferences which are representable by a utility function are convex if and only if the representing utility function is quasiconcave.

5. Derive the Walrasian demands and indirect utility function when the utility function is:

(a) $u(x_1, x_2) = x_1^\alpha x_2^\beta$.

(b) $u(x_1, x_2) = \log(x_1) + x_2$.

(c) $u(x_1, x_2) = ax_1 + x_2$.

(d) $u(x_1, x_2) = \min\{x_1, \sqrt{x_2}\}$.