Econ 701: Problem Set 1

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1. Suppose we began our study of preferences with $\succ$ and defined $\succeq$ and $\sim$ from it by (a) $x \sim y$ iff $x \not\succ y$ and $y \not\succ x$ and (b) $x \succeq y$ iff $y \not\succ x$. Consider the property of $\succ$ known as *negative transitivity*: if $x \succ y$, then either $z \succ y$ or $x \succ z$. Also, we say that $\succ$ is *asymmetric* if $x \succ y$ implies $y \not\succ x$. Prove that $\succeq$ is a weak order if and only if $\succ$ is asymmetric and negatively transitive.

2. Show that if $\succeq$ is a weak order, then $x \succeq y$, $y \succ z$ implies $x \succ z$.

3. Let $X = \{x_1, x_2, \ldots, x_{100}\}$. Suppose the decision maker uses the following procedure to make a choice. He has a utility function u with the property that if $x \neq y$, then $u(x) \neq u(y)$. He also has a number $\delta > 0$. When faced with a feasible set B, he first looks at the item with the smallest subscript. (I.e., from $\{x_7, x_{10}, x_{23}\}$, he first looks at x_7.) Call this option x^*. He then compares $u(x^*)$ to the highest utility he could get from choosing any element of B. Call this highest utility level $\bar{u}$. If $\bar{u} > u(x^*) + \delta$, he chooses the point in B which maximizes u. Otherwise, he chooses x^*.

Does this procedure necessarily generate a choice rule satisfying WARP? That is, would every $\delta > 0$ and every utility function u give us a choice rule satisfying WARP? Alternatively, would the procedure necessarily give a choice rule violating WARP? If the procedure must satisfy WARP, prove this. If it must violate WARP, prove this. Finally, if it depends on u and δ, give an example where WARP holds and an example where it is violated.

4. Let $X = \{x_1, x_2, \ldots, x_{100}\}$. Suppose the decision maker uses the following choice procedure. If the number of items in the feasible set B is even, he picks the item with the smallest subscript. If the number of items in the feasible set is odd, he picks the item with the largest subscript. Answer the same questions as in Question 3 for this procedure.

5. Show that the two definitions we gave in class of WARP are equivalent.

6. Prove the following: If X is finite and $\succeq$ is a weak order, then there exists a utility function u which represents $\succeq$. (Yes, Rubinstein contains a proof of this. Try doing it without looking at his argument.)

7. Suppose we start with a strict order $\succ$ and try to represent this via the idea of just–noticeable–differences. That is, we represent $\succ$ by a utility function u and a number $\delta > 0$ such that

$$x \succ y \text{ iff } u(x) > u(y) + \delta.$$

Prove that the following conditions are necessary for the existence of such a representation: $\succ$ is irreflexive ($x \not\succ x$), $\succ$ is transitive, and

$$w \succ x,\ y \succ z, \text{ implies } w \succ z \text{ or } y \succ x$$

$$w \succ x,\ x \succ y \text{ implies } z \succ y \text{ or } w \succ z.$$

8. (Note: I may move this problem to next week's problem set, depending on how far we get in the next lecture.) We discussed three different definitions of continuity in class, two of which Rubinstein shows are equivalent. Show that the third, the one from MWG, is equivalent to the other two for a weak order on $X = \mathbf{R}^L_+$. (You can take as the result in Rubinstein that his two definitions are equivalent.)

The first definition from Rubinstein: Whenever $x \succ y$, there exists $\varepsilon_x > 0$ and $\varepsilon_y > 0$, such that for every x' within ε_x of x and every y' within ε_y of y, $x' \succ y'$.

The second from Rubinstein: If (x_n, y_n) is a sequence converging to (x, y) with $x_n \succeq y_n$ for all n, then $x \succeq y$.

The alternative definition: For each $y \in X$, $\{x \in X \mid x \succeq y\}$ and $\{x \in X \mid y \succeq x\}$ is closed. That is, if $x_n \to x$ with $x_n \succeq y$ for all n, then $x \succeq y$ and similarly on the other side.

Hint: I suggest using the fact that X is connected.