Microeconomic Theory Qualifying Exam

Department of Economics
Boston University

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Instructions. You have four hours to complete this exam, plus a 15 minute "grace period" to wrap up if needed. Answer all four questions. The questions are equally weighted.

Write on **one side** of the provided paper only. Start the answer to each question on a new sheet of paper and be sure to write your candidate number, question number, and page number on each sheet.

Be concise in your answers, and think before you write. Good luck!

1. For any binary relation $\succsim$ over some choice domain X, define the set of $\succsim$*–maximal* elements in a feasible set A by

$$B(A, \succsim) = \{x \in A : x \succsim y \text{ for all } y \in A\}.$$

Define the set of $\succsim$*–unbeaten* elements in a feasible set A by

$$U(A, \succsim) = \{x \in A : y \succ x \text{ for no } y \in A\}.$$

(a) Suppose $X = \{a, b, c\}$ and $\succsim$ is reflexive but incomplete: it strictly ranks a over b but cannot make any other comparisons (except that each item is weakly preferred to itself). (i) Is this preference $\succsim$ transitive? (ii) Find $B(X, \succsim)$ and $U(X, \succsim)$. (iii) Find a *complete* binary relation $\succsim^*$ over X such that $U(X, \succsim) = B(X, \succsim^*)$. That is, find a complete binary relation whose set of maximal elements coincide with the set of unbeaten elements under $\succsim$.

(b) Prove that for any binary relation $\succsim$ over any domain X, there exists a complete binary relation $\succsim^*$ such that $U(A, \succsim) = B(A, \succsim^*)$ for every nonempty feasible set A.

(c) State in one sentence what the claim in (b) implies about the empirical relevance of incomplete preferences.

2. Consider a professor whose effort affects her university's reputation. If the university's reputation is *enhanced*, it earns an extra $2V > 0$ in donations. Otherwise, it is *unchanged* (0 extra donations). The probabilities depend on the professor's effort. Both the university and the professor are risk neutral in income. The professor's outside option is zero. Compensating on the basis of the university's reputation is infeasible, so the university bases the professor's wage on whether she receives an outside offer, paying w_o if there is an offer and w_n otherwise, where both wages must be nonnegative.

(a) Suppose effort is one–dimensional, denoted $e \in [0, 1]$, and the professor's utility is $y - \frac{1}{2}e^2$, where y is income. The outside offer arrives with probability ae, $a > 0$. (Assume the outside offer itself is valueless to the professor.) Given a contract (w_o, w_n), what effort level is chosen by the professor? (Assume an interior solution.)

(b) The university's reputation is enhanced with probability be, $b > 0$ (where enhancement and outside offers are independent, given e). The university chooses w_o and w_n to maximize the expected value of alumni donations minus wage payments.

(i) Show that the individual rationality constraint cannot bind.

(ii) Using (a), show that $w_n = 0$ at an optimum. (Hint: it may help to rewrite the university's objective in terms of $\Delta \equiv w_o - w_n$.) Find the optimal value of w_o.

(c) Now suppose effort is multidimensional, denoted $\mathbf{e} = (e_R, e_T) \in [0, 1]^2$, where e_R is research effort and e_T is teaching effort. The university's reputation is enhanced with probability $b_R e_R + b_T e_T \equiv \mathbf{b} \cdot \mathbf{e}$, and the outside offer occurs with probability $\mathbf{a} \cdot \mathbf{e}$. Assume $\frac{1}{2} > a_i, b_i \geq 0, i \in \{R, T\}$. The professor's utility is $y - \frac{1}{2}\mathbf{e} \cdot \mathbf{e}$. Given a contract (w_o, w_n), what effort levels does the professor choose (again, assume interior solutions)?

(d) Find the optimal compensation scheme for the university. (Hint: $w_n = 0$.)

(e) Consider two special cases: (i) $\mathbf{a} = \beta\mathbf{b}$ for some constant $\beta > 0$ and (ii) $a_T = b_R = 0$, while $a_R > 0$ and $b_T > 0$. For each case, (A) describe what these mean in terms of the alignment of the university's objective and the performance measure, (B) show how the expression for w_o you derived in (d) can be simplified, and (C) offer some intuition for the results.

3. For the statements in each of parts (a), (b) and (c), indicate whether the statement is True or False, AND justify your answer fully in either case.

(a) In an Arrow–Debreu economy, where everyone has state–independent expected utility preferences with strictly increasing vNM indices, any Arrow–Debreu equilibrium allocation is Pareto optimal even if individuals disagree about the probabilities of states.

(b) Let there be I individuals and a set $\mathbb{R}^n_{++}$ of alternatives. The i^{th} individual has the utility function u_i on $\mathbb{R}^n_{++}$, $i = 1, ..., I$. Define an aggregate preference order $\succeq$ on X by: for all x and y in $\mathbb{R}^n_{++}$,

$$x \succeq y \ \text{ if } \ x \succeq_i y \text{ for all } i = 1, \ldots, I.$$

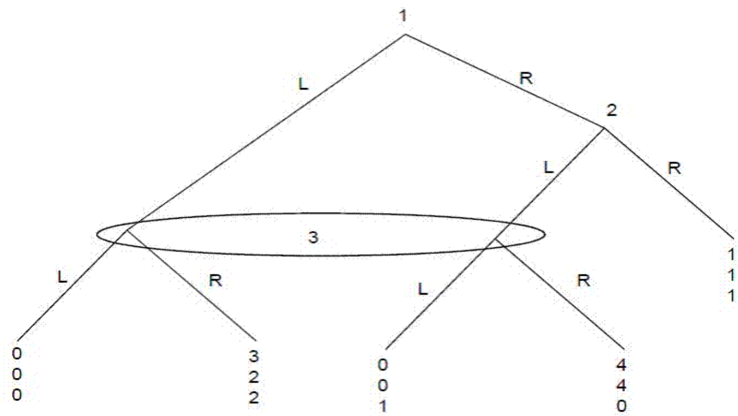
Then there exists an aggregate utility function that represents $\succeq$.

(c.i) Consider a Radner economy with symmetric information, where every consumer assigns strictly positive probability to every state, and where security markets are complete. Then any Radner equilibrium allocation is *ex post* efficient.

(c.ii) How would your answer change if security markets are incomplete? Explain.

(c.iii) How would your answer in (c.i) change if markets are complete but information is asymmetric? Explain.

4. Consider the following game:



(a) What is the normal form of this game?

(b) Find all the Nash equilibria (in pure and mixed strategies) of this game.

(c) Find all the sequential equilibria (in pure and mixed strategies) of this game.