Answers to Microeconomic Theory Qualifying Exam

Department of Economics
Boston University

June 2010

1. (a) (i) Yes. (ii) $B(X, \succsim) = \emptyset$, $U(X, \succsim) = \{a, c\}$. (iii) Complete the ordering by setting $a \sim^* c$. Now $a \succsim^* y$ for all $y \in X$ and same for c. So $B(X, \succsim^*) = \{a, c\}$.

(b) The general principle is the same. Define $x \succsim^* y$ if $x \succsim y$ or if $x \not\succsim y$ and $y \not\succsim x$. That is, all noncomparability is changed to indifference. It is easy to see that this is complete. To see that $B(A, \succsim^*) = U(A, \succsim)$, fix any $x \in B(A, \succsim^*)$. So $x \succsim^* y$ for all $y \in A$. Thus for every $y \in A$, either $x \succsim y$ or $x \not\succsim y$ and $y \not\succsim x$. Either way, we cannot have $y \succ x$, so $x \in U(A, \succsim)$. For the converse, suppose $x \in U(A, \succsim)$. Then for every $y \in A$, $y \not\succ x$. Hence either $x \succ y$ or $x \succsim^* y$. So $x \in B(A, \succsim^*)$.

2. (a) FOC of $\max_e aew_o - (1 - ae)w_n - \frac{1}{2}e^2$ yields $e = a(w_o - w_n)$.

(b) (i) Since wages are non–negative and e can be chosen to be zero, the professor always nets at least zero with any feasible compensation scheme. Hence the professor must always be weakly better off with the scheme than with her outside option under any feasible contract. Hence the constraint cannot bind.

(ii) University solves $\max_{w_o, w_n} be2V - aew_o - (1 - ae)w_n = \max_{w_o, w_n} a(w_o - w_n)(2bV - a(w_o - w_n)) - w_n$. If $w_n > 0$ were optimal, lowering it and w_o to keep $w_o - w_n$ constant raises the payoff, contradiction (this relies on $w_n \leq w_o$, which is optimal as long as $bV > 0$).

We are left with $\max_{w_o} aw_o(2bV - aw_o)$ which yields $w_o = \frac{bV}{a}$.

(c) FOC of $\max_{\mathbf{e}} \mathbf{a} \cdot \mathbf{e} w_o - (1 - \mathbf{a} \cdot \mathbf{e})w_n - \frac{1}{2}\mathbf{e} \cdot \mathbf{e}$ yields $e_i = a_i(w_o - w_n)$.

(d) $\max_{w_o, w_n} \mathbf{b} \cdot \mathbf{e} 2V - \mathbf{a} \cdot \mathbf{e} w_o - (1 - \mathbf{a} \cdot \mathbf{e})w_n$ with $c = w_n = 0$ gives $\max_{w_o} \mathbf{b} \cdot \mathbf{a} w_o 2V - \mathbf{a} \cdot \mathbf{a} w_o^2$, yielding $w_o = \frac{\mathbf{b} \cdot \mathbf{a} V}{\mathbf{a} \cdot \mathbf{a}}$.

(e) (i) (A) The relative importance of each type of effort is the same for both the university's fundraising and the academic job market, though possibly the overall sensitivity of each to effort may differ (in case $\beta \neq 1$); (B) $w_o = \beta V$, essentially like (b).

(ii) (A) research is useless for generating donations but useful for generating offers, while teaching generates donations but is useless for offers; (B) $w_o = 0$.

Intuition: If perfomance measures are well aligned with objectives, as in (i), using them as a basis for compensation differs little from the "single task" case; but if they are poorly aligned (case ii) in the sense of responding very differently to different types of effort, then the best thing to do may be to avoid compensation based on the performance measure (you get what you pay for).

3. (a) TRUE — local nonsatiation is implied and thus the First Welfare Theorem applies. Disagreement about probabilities is irrelevant.

(b) FALSE — the order is incomplete and so cannot have a utility representation.

(c) TRUE — assuming subjective expected utility and the natural specification of preferences ex post, then any reallocation *within* a state is possible ex ante regardless of the number of securities. Asymmetry of information is also irrelevant.

4. (a) The normal form is

$$\begin{array}{cccc} & & 2 & \\ & & L & R \\ 1 & L & 0,0,0 & 0,0,0 \\ & R & 0,0,1 & 1,1,1 \\ & & 3:L & \end{array} \qquad \begin{array}{cccc} & & 2 & \\ & & L & R \\ 1 & L & 3,2,2 & 3,2,2 \\ & R & 4,4,0 & 1,1,1 \\ & & 3:R & \end{array}$$

From now on, ε denotes the probability that player 1 plays L, η denotes the probability that player 2 plays L and δ denotes the probability that player 3 plays L.

(b) The Nash equilibria. Pure strategies: $(\varepsilon, \eta, \delta) = (1, 0, 0)$ and $(\varepsilon, \eta, \delta) = (0, 0, 1)$.

Nash equilibria in mixed strategies: $\{(0, 0, \delta) : \delta \in [\frac{3}{4}, 1]\}$ and $\{(1, \eta, 0) : \eta \in [0, \frac{2}{3}]\}$.

(c) Sequential equilibria (in pure and mixed strategies) of this game.

I denote by p the probability of the left node in player 3's information set.

First, consider sequential rationality. Player 1 compares the expected utility $3(1-\delta)$ of playing L to the expected utility $1 - \eta + 4\eta(1-\delta)$ of playing R. Since $3(1-\delta) \geq 1 - \eta + 4\eta(1-\delta)$ is equivalent to $2 - 3\delta \geq (3 - 4\delta)\eta$, the best response correspondence for player 1 is

$$\varepsilon \quad \begin{array}{c} =0 \\ =1 \\ \in[0,1] \end{array} \quad \text{as } 2-3\delta \ \begin{array}{c} < \\ > \\ = \end{array} \ (3-4\delta)\,\eta \tag{1}$$

Similar computations show that the best response for player 2 is

$$\eta \quad \begin{array}{c} =0 \\ =1 \\ \in[0,1] \end{array} \quad \text{as } \delta \ \begin{array}{c} > \\ < \\ = \end{array} \ \tfrac{3}{4} \tag{2}$$

and for player 3 is

$$\delta \quad \begin{array}{c} =0 \\ =1 \\ \in[0,1] \end{array} \quad \text{as } p \ \begin{array}{c} > \\ < \\ = \end{array} \ \tfrac{1}{3} \tag{3}$$

- Case 1: $\eta = 0$. Then (2) implies that $\delta \geq 3/4$, and (3) implies that $p \leq 1/3$. Since $\eta = 0$ and $\delta \geq 3/4$, $2 - 3\delta < (3 - 4\delta)\,\eta$ and (1) implies that $\varepsilon = 0$. Now, since $\varepsilon = \eta = 0$, the information set of player 3 is not reached. The belief $p \leq 1/3$ that makes $\delta \geq 3/4$ sequentially rational for player 3 must therefore be obtained as the limit of posteriors obtained from a sequence $\{(\varepsilon_k, \eta_k, \delta_k)\}_{k=1}^{\infty}$ converging to $(\varepsilon, \eta, \delta) = (0, 0, \delta)$. For each element of the sequence, $p_k = \frac{\varepsilon_k}{\varepsilon_k + \eta_k(1-\varepsilon_k)}$; for instance, if $\varepsilon_k = \frac{1}{k^2}$ and $\eta_k = \frac{1}{k}$, $p_k = \frac{1}{k + \left(1 - \frac{1}{k}\right)} \to 0 < 1/3$.

- Case 2: $\eta > 0$. Then (2) implies that $\delta < 3/4$ and (3) implies in turn that $p \geq 1/3$. Since $\eta > 0$, player 3's information set is reached with positive probability and beliefs are computed by Bayes' law, $p = \frac{\varepsilon}{\varepsilon + \eta(1-\varepsilon)}$ and therefore we need $\frac{\varepsilon}{\varepsilon + \eta(1-\varepsilon)} \geq 1/3$, or

  $$\varepsilon \geq \frac{\eta}{2 + \eta}.$$

  This inequality is impossible however. Since $\eta > 0$, $\varepsilon \geq \frac{\eta}{2+\eta}$, and (1) requires that

  $$2 - 3\delta \geq (3 - 4\delta)\,\eta \tag{4}$$

  - if $\eta \in (0, 1)$, then (2) requires $\delta = 3/4$, but then (4) becomes $\frac{-1}{4} \geq 0$ which is absurd.
  - if $\eta = 1$, then (4) is equivalent to $\delta \geq 1$, which contradicts $\delta \leq \frac{3}{4}$.

  Therefore, the set of sequential equilibria is the set

  $$\left\{ ((0,0,\delta)\,, p) : \delta \in [\frac{3}{4}, 1] \text{ and } p = 1/3 \textit{ or } \delta = 1 \text{ and } p \in [0, \frac{1}{3}) \right\}.$$

  In particular, Nash equilibria in which player 1 plays L cannot be sequential.