

Econ 701: Midterm Exam

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Part 1: The following question is worth 30 points.

1. Prove any two of the following statements. Be sure to state clearly any definitions or other properties you are using in your proof. Each proof is worth 15 points.

(a) If \succeq is a preference over lotteries with a representation of the form

$$p \succeq q \text{ iff } \sum_z p(z)u(z) \geq \sum_z q(z)u(z),$$

then \succeq satisfies the independence axiom.

(b) If a consumer is (strictly) risk averse, his risk premium for a nondegenerate gamble is strictly positive.

(c) The expenditure function, $e(p, u)$, is concave in p .

(d)

$$x_i(p, w) = -\frac{\frac{\partial v(p, w)}{\partial p_i}}{\frac{\partial v(p, w)}{\partial w}}.$$

Part 2: Answer any two of the following three questions. Each question is worth 35 points.

2. A consumer has an indirect utility function given by

$$v(p_1, p_2, w) = \log\left(\frac{p_2}{p_1}\right) + \frac{w}{p_2}.$$

Find the expenditure function, the Walrasian demands, and the Hicksian demands.

3. Consider an agent who is risk averse over lotteries over wealth and prefers more money to less. She is in a job that pays the (nonrandom) amount x per year. If she gets promoted, she'll receive the (nonrandom) salary $y > x$. She will be promoted if the investments she makes on behalf of her company have a return of at least K . There are two investments she could consider, one with returns distributed according to distribution function F and one with returns distributed according to G .

(a) Suppose F FOSD G . Is this enough information to determine which she will choose? If so, which and how do you know? If not, why not?

(b) Suppose G is a mean-preserving spread of F . Now do we have enough information to determine which she will choose? If so, which and how do you know? If not, why not?

4. Assume $X = \mathbf{R}_+^3$. Suppose we see the consumer's choice on three budget sets: one with price vector $p^1 = (2, 1, 2)$ and income $w^1 = 8$, one with price vector $p^2 = (2, 2, 1)$ and income $w^2 = 8$, and one with price vector $p^3 = (1, 2, 2)$ and again income $w^3 = 8$. Denote these budget sets B_1 , B_2 , and B_3 respectively. Assume the choice correspondence is $C(B_1) = \{(1, 2, 2)\}$, $C(B_2) = \{(2, 1, 2)\}$, and $C(B_3) = \{(2, 2, 1)\}$. Show that C satisfies WARP. Can this choice correspondence be rationalized by a preference? Why or why not?